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PROCEEDINGS AT THE MEETINGS OF THE PHYSICAL SOCIETY

SESSION 1931–32

Except where the contrary is stated, the meetings were held at the Imperial College of Science and Technology, South Kensington.

October 16, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Sydney Bennett Fulford was elected to Fellowship of the Physical Society. The President announced that Charles Norman Smyth, Denis Taylor, Nelson Laycock, Eric Graham Knowles, Ronald Osmond Jenkins, Maurice Hancock, E. H. Copsey, J. R. Tillman, Alan E. W. Austen, George Ronald Cooper, Eric Henry Dock, Charles F. Brockelsby, Frederick C. Le Manquais and Alec A. Fletcher had been admitted to Student Membership of the Society.

2. The following papers were read:

"Young's modulus for two directions in a steel bar," by G. A. WEDGWOOD, M.Sc.

"The effective mass of flexible discs and conical diaphrams used for sound-reproduction," by N. W. MCLACHLAN, D.Sc., M.I.E.E., F.Inst.P.

"Further study of diffusion for the infinite plane sheet," by A. T. MCKAY, M.Sc.

3. A demonstration of the effects of mechanical disturbance on a neon lamp was given by Miss TERESA J. DILLON, M.Sc., and Miss C. M. LOVETT, B.Sc.

November 6, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Maurice K. Taylor, Alexander Charles George Menzies, Jack Ernest Rayner Constable and P. B. Moon were elected to Fellowship of the Physical Society.

The President announced that the Council had elected A. H. M. Radwan, W. F. Hilton, A. G. Gaydon and Chetpat Ramaswami Sundaram to Student Membership of the Society.

2. The presidential address, entitled "The Expanding Universe," was delivered by Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S.

A vote of thanks to the President was proposed by Dr W. H. Eccles, seconded by Mr C. C. Paterson, and carried by acclamation.

November 20, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. The following papers were read:

"The band spectrum of zirconium oxide," by Miss F. LOWATER, Ph.D., F.R.A.S.

"Lattice-distortion of cold-drawn constantan wire," by W. A. WOOD, M.Sc., Physics Department, National Physical Laboratory.

"A remote electrically-recording accelerometer with particular reference to wheel-impact measurements," by F. AUGHTIE, Ph.D., M.Sc., Engineering Department, National Physical Laboratory.

"A remote electrically-recording load-gauge for wheel-impact measurements," by F. AUGHTIE, Ph.D., M.Sc., Engineering Department, National Physical Laboratory.

"Wireless echoes of short delay," by E. V. APPLETON, F.R.S., and G. BUILDER, B.Sc.

2. Demonstrations of (*a*) A contrivance for demonstrating the law of errors; (*b*) A new type of surface-tension-meter; and (*c*) A new type of static electrometer were given by Prof. KERR GRANT, M.Sc.

December 4, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Bernard Rhodes, Leslie H. Daniel and Donald Hewitt were elected to Fellowship of the Physical Society.

2. The following papers were read:

"Some properties of the sound emitted by airscrews," by C. F. B. KEMP, A.R.C.S., B.Sc., D.I.C.

"The latent heat of some refrigerants," by EZER GRIFFITHS, D.Sc., F.R.S., and J. H. AWBERY, B.A., B.Sc., F.Inst.P., Physics Department, National Physical Laboratory.

"The specific volumes of some gaseous refrigerants," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., and EZER GRIFFITHS, D.Sc., F.R.S., Physics Department, National Physical Laboratory.

"The basic law of the wet-and-dry-bulb hygrometer at temperatures from 40° to 100° C.," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., and EZER GRIFFITHS, D.Sc., F.R.S., Physics Department, National Physical Laboratory.

"The water-content of saturated air at temperatures up to 100° C.," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., Physics Department, National Physical Laboratory.

3. A demonstration of the effects produced when slow-velocity liquid jets fall on a barrier was given by J. H. BRINKWORTH, D.Sc., A.R.C.S., D.I.C.

January 5, 6 *and* 7, 1932.

The twenty-second Annual Exhibition of the Physical and Optical Societies was held at the Imperial College of Science.

Discourses were delivered as follows:

January 5. "Photocells: the valves which operate by light," by C. C. PATERSON, O.B.E., M.I.E.E., F.Inst.P.

January 6. "Photographic shutters and their properties," by T. SMITH, M.A., F.Inst.P.

January 7. "Reminiscences," by Sir OLIVER LODGE, D.Sc., LL.D., F.R.S., F.Inst.P.

January 15, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Frank Aughtie and A. Stanley M. Symons were elected to Fellowship of the Physical Society.

The President announced that John Spencer Roebuck, George Ernest Tunnicliffe and Arthur Cecil Challands had been admitted to Student Membership of the Society.

2. The following papers were read:

"Some thermomagnetic effects in nickel and iron," by SHIH-CHEN T'AO, M.S. (Yenching), and WILLIAM BAND, M.Sc., Yenching University, Peking.

"The electroscope capacity balance," by E. S. BROWN, D.Sc., A.C.G.I., University of Melbourne.

"On periodic movements of the negative glow in discharge tubes," by Miss W. A. LEYSHON, Ph.D.

"A cathode-ray oscillographic method of measuring inductance," by G. I. FINCH, M.B.E., and R. W. SUTTON, Imperial College of Science and Technology.

"The measurement of electrical resistance in terms of a mutual inductance and a period," by H. R. NETTLETON, D.Sc., and F. H. LLEWELLYN, B.Sc., Birkbeck College.

February 5, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Eric Norton Grindley, Cecil George Lemon and Morris E. Sions were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The collisional friction experienced by vibrating electrons in ionized air," by E. V. APPLETON, F.R.S., and F. W. CHAPMAN, M.Sc.

"Surface heating by neutralized positive rays before and after return to normal state," by M. C. JOHNSON, M.A., D.Sc., Birmingham University.

"The rapid determination of the moisture-content of seeds and other granular substances," by R. M. DAVIES, M.Sc., University College of Wales, Aberystwyth.

"Electrolytic water-transport and ionic transport numbers," by H. C. HEPBURN, Ph.D., Birkbeck College.

"On the magnetic susceptibilities of some nickel compounds," by R. A. FEREDAY, B.Sc., East London College.

February 19, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Arthur Osborne Jones and W. Grenville Symons were elected to Fellowship of the Physical Society.

2. The following papers were read:

"Electrons and light quanta," by Sir AMBROSE FLEMING, F.R.S., Emeritus Professor of Electrical Engineering in the University of London.

"The spherical shell method of determining the thermal conductivity of a thermal insulator," by S. E. GREEN.

"The optics of photometric measurements," by T. SMITH, M.A., National Physical Laboratory.

"Sir A. S. Eddington's recent theories," by W. N. BOND, M.A., D.Sc., F.Inst.P., Lecturer in Physics in the University of Reading.

"Some principles governing the design of Kerr cells," by W. D. WRIGHT, Ph.D., A.R.C.S., D.I.C.

"On the forces acting on drops in an electric field," by G. D. WEST, D.Sc., F.Inst.P.

March 4, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. D. Narayanamurti was elected to Fellowship of the Physical Society.

2. The following papers were read:

"A vacuum calorimeter for high temperatures," by L. G. CARPENTER, B.A., B.Sc., and T. F. HARLE, B.Sc., University College, Southampton.

"A new tilted electrometer," by HUGH CARMICHAEL, B.Sc., University of Edinburgh.

"On the symmetrical modes of vibration of truncated conical shells; with applications to loud-speaker diaphragms," by N. W. MCLACHLAN, D.Sc., M.I.E.E.

3. A demonstration on some lecture experiments in alternating currents was given by W. BENNETT, A.R.C.S., B.Sc.

March 18, 1932.

Annual general meeting.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. The Minutes of the preceding Annual Meeting were read and confirmed.

2. The Reports of the Council and of the Hon. Treasurer were presented and adopted.

3. The following Officers and Members of Council were elected for the year 1932–1933:

President: Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P.

Vice-Presidents (*who have filled the office of President*): Sir Oliver J. Lodge, D.Sc., LL.D., F.Inst.P., F.R.S.; Sir Richard Glazebrook, K.C.B., Sc.D., F.Inst.P., F.R.S.; Sir Arthur Schuster, Ph.D., Sc.D., F.Inst.P., F.R.S.; Sir J. J. Thomson, O.M., Sc.D., F.Inst.P., F.R.S.; Prof. C. Vernon Boys, F.Inst.P., F.R.S.; Prof. C. H. Lees, D.Sc., F.Inst.P., F.R.S.; Prof. Sir W. H. Bragg, K.B.E., M.A., F.Inst.P., F.R.S.; Alexander Russell, M.A., D.Sc., F.Inst.P., F.R.S.; Sir F. E. Smith, K.C.B., D.Sc., F.Inst.P., F.R.S.; Prof. O. W. Richardson, M.A., D.Sc., F.R.S.; W. H. Eccles, D.Sc., F.Inst.P., F.R.S.; Prof. Sir A. S. Eddington, M.A., D.Sc., F.R.S.

Vice-Presidents: A. B. Wood, D.Sc., F.Inst.P.; T. Smith, M.A., F.Inst.P., F.R.S.; J. S. G. Thomas, D.Sc.; J. Guild, A.R.C.S., D.I.C., F.Inst.P.

Hon. Secretaries: Allan Ferguson, M.A., D.Sc., F.Inst.P.; Ezer Griffiths, D.Sc., F.Inst.P., F.R.S.

Hon. Foreign Secretary: Prof. O. W. Richardson, M.A., D.Sc., F.R.S.

Hon. Treasurer: R. S. Whipple, M.I.E.E., F.Inst.P.

Hon. Librarian: J. H. Brinkworth, M.Sc., A.R.C.S.

Ordinary Members of Council: Lewis Simons, D.Sc., F.Inst.P.; J. H. Awbery, B.Sc., F.Inst.P.; Prof. J. A. Crowther, M.A., Sc.D., F.Inst.P.; W. Jevons, D.Sc., D.I.C., F.Inst.P.; Prof. W. Wilson, Ph.D., D.Sc., F.R.S.; D. Owen, B.A., D.Sc., F.Inst.P.; Miss M. O. Saltmarsh, Ph.D.; Major I. O. Griffith, M.A.; Prof. H. R. Robinson, F.Inst.P., F.R.S.; Prof. G. F. J. Temple, Ph.D., D.Sc.

4. The Duddell Medal, 1931, was presented to Prof. C. T. R. Wilson, F.R.S.

5. The following votes of thanks were moved and were carried by acclamation: To the retiring Officers and Council, proposed by Mr R. W. Paul and seconded by Prof. G. Temple; to the Honorary Auditors, proposed by Dr J. S. G. Thomas and seconded by Mr T. Smith; and to the Governors of the Imperial College of Science, proposed by Prof. W. Wilson and seconded by Mr F. J. W. Whipple.

Ordinary meeting following the annual general meeting.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. James Garton Bower, Denis Edward Alan Jones and William George Pye were elected to Fellowship of the Physical Society.

The President announced that D. G. Jackess had been admitted to Student Membership of the Society.

2. The following papers were read:

"The first spark spectrum of arsenic (As II)," by A. S. RAO, M.A., M.Sc., Solar Physics Observatory, Kodaikanal, S. India.

"The photographic measurement of the absorption coefficients of gamma-rays from radium (B + C)," by J. S. ROGERS, B.A., M.Sc., F.Inst.P., Senior Lecturer in Natural Philosophy, University of Melbourne.

"The derivation of Maxwell's equations from the equations of the quantum theory," by M. FAHMY, The Egyptian University, Cairo.

April 15, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. H. Levy, Herbert Frank Winny, Edward Burke, Stanley John Welton, Charles Henry Kemp and A. S. Radford were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The measurement of reflection coefficients for oblique incidence," by H. E. BECKETT, B.Sc., Building Research Station.

"Studies in Interferometry. (1) A new type of interference refractometer," by W. EWART WILLIAMS, M.Sc., Lecturer in Physics, King's College, London.

"A direct-reading γ-ray electroscope," by L. G. GRIMMETT, B.Sc., Assistant Physicist, Westminster Hospital, London.

May 6, 1932.

Meeting held in the Physiological Laboratory of University College, Gower Street, W.C. 1.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Cyril James Beesley, Edward William Herbert Selwyn and Leonard Ernest Sharp were elected to Fellowship of the Physical Society.

2. A series of demonstrations, including the following, was given:

The heat-production of nerve.
Galvanometer-amplification by photoelectric cell.
Photoelectric cell used as micrometer and for other purposes.
Vapour-pressure measurement by thermopile.
Muscle-heat production.
Thermoelastic phenomena.

3. A lecture was delivered by Prof. A. V. HILL, F.R.S., on "The measurement and analysis of the heat-production of nerve."*

May 20, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Donald Walter Carter, Walter Wilson, Arnold Joseph Holland, John Guyscliffe Holmes, Brian Clifford Fleming-Williams, Ronald James Cox, G. G. Blake and Henry L. Brose were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The fall of potential in a charged insulated cable," by D. K. MCCLEERY, B.Sc., A.M.I.E.E.

"On the representation and calculation of the results of gravity surveys with torsion balances," by Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., Professor of Physics in the Imperial College of Science and Technology.

"Some observations with a gravity-gradiometer," by Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., Professor of Physics in the Imperial College of Science and Technology.

"The propagation along the earth of radio waves on a wave-length of 1·6 metres," by R. L. SMITH-ROSE, D.Sc., Ph.D., A.M.I.E.E., and J. S. MCPETRIE, B.Sc., A.M.I.E.E.

"Notes on surface-tension measurement," by ALLAN FERGUSON, M.A., D.Sc., F.Inst.P., and S. J. KENNEDY, B.Sc.

3. A demonstration of an experiment on capillary adsorption was given by D. OWEN, B.A., D.Sc., F.Inst.P.

* The substance of this lecture will be found in *Chemical Wave Transmission in Nerve* (Cambridge University Press, 1932).

June 3, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., and subsequently ALLAN FERGUSON, M.A., D.Sc., in the Chair.

A discussion on vision was held, the following papers being read and discussed:*

J. GUILD, "Some problems of visual reception." F. ALLEN, "The trichromatic theory and its explanatory power." J. DREVER, "The fundamental colours and the colour triangle." J. GUILD, "The interpretation of quantitative data in visual problems." H. HARTRIDGE, "Colour vision by modified white light." S. HECHT, "A quantitative formulation of colour vision." L. C. MARTIN, F. L. WARBURTON and W. J. MORGAN, "Some recent experiments on the sensitiveness of the eye to differences in the saturation of colours." W. PEDDIE, "The essence and present position of the trichromatic theory." L. F. RICHARDSON, "The measurability of sensations of hue brightness or saturation." W. D. WRIGHT, "The significance of colour fatigue measurements." A. BRUCKNER, "Untersuchungen zur Dunkeladaptation des menschlichen Auges." R. A. HOUSTOUN, "New observations on the Weber-Fechner law." K. KOFFKA, "A new theory of brightness constancy; a contribution to a general theory of vision." D. ROAF, "The sensation of light as a photochemical process." W. S. STILES and B. H. CRAWFORD, "Equivalent adaptation levels in localised retinal areas." T. SMITH, "The colour triangle and colour discrimination." H. BANISTER, "Retinal action time." F. C. BARTLETT, "A note on the visual perception of depth." C. E. FERREE and G. RAND, "A study of the refractive conditions for the peripheral field of vision." R. A. GRANIT, "The physiological significance of the retinal synapses." J. PARSONS, "Some biological aspects of visual measurements." H. PIERON, "Les lois du temps du chroma." A. von PFLUGK, "Die Lehre von der Akkommodation im Lichte der neueren Forschung." R. S. CREED, "Visual acuity and retinal structure." E. F. FINCHAM, "The mechanism of accommodation and the recession of the near point." H. HARTRIDGE, "Visual acuity and the resolving power of the eye." H. E. ROAF, "Some experimental observations on the properties of the receiving organs in the retina." F. ROESSLER, "A subjective examination of eye astigmatism."

June 11, 1932.

Meeting held at University College, Nottingham.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. The following papers were read:

"Relation of electron collisions in gases to the Raman spectra," by H. L. BROSE.

"Electrodeless discharges," by J. S. TOWNSEND.

* Published in a separate volume.

"Intensity measurements in the spectra of electrodeless discharges," by J. E. KEYSTON.

"Directional investigation of atmospherics," by L. G. H. HUXLEY.

"Raman effect in electrolytes," by L. A. WOODWARD.

"Limiting effective cross-section of gas atoms with respect to very slow electrons," by E. H. SAAYMAN.

2. Some demonstrations were given under the supervision of N. DAVY, H. M. BROWNING and A. H. FRANKS.

June 17, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

The Seventeenth Guthrie Lecture was delivered by Prof. MAX PLANCK, on "The concept of causality."

July 8, 1932.

Special general meeting.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

The following resolutions were passed:

1. That the name of the Society be changed to "The Physical Society."

2. That, subject to the proposed change of name of the Society being approved by the Board of Trade and a new certificate of incorporation of the Society in the name of "The Physical Society" being issued by the Registrar, the regulations contained in the document submitted to the meeting and for the purpose of identification signed by the chairman thereof be approved and adopted as the articles of association of the Society in substitution for and to the exclusion of all existing articles thereof.

Ordinary meeting following the special general meeting.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Walter Edgar Flood, James Lewis Pugh Macnair and Lionel V. Cole were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The determination of refractivity temperature coefficients for liquids," by JOHN J. MANLEY, M.A., D.Sc., Oxon., Fellow of Magdalen College, Oxford.

"The axial sound-pressure due to diaphragms with nodal lines," by N. W. MCLACHLAN, D.Sc., M.I.E.E.

"The accession to inertia of flexible discs vibrating in a fluid," by N. W. McLachlan, D.Sc., M.I.E.E.

"A method for deducing accurate values of the lattice spacing from X-ray powder photographs taken by the Debye-Scherrer method," by A. J. Bradley, Ph.D., and A. H. Jay, M.Sc.

"Ionization charts of the upper atmosphere," by G. Millington, M.A., B.Sc.

"Further investigation of the arc spectrum of arsenic" by A. S. Rao, M.A., M.Sc., Solar Physics Observatory, Kodaikanal, India.

REPORT OF COUNCIL FOR THE PERIOD ENDING FEBRUARY 29, 1932

MEETINGS

DURING the period covered by the Report 9 Ordinary Science Meetings were held at the Imperial College of Science. At these meetings 40 papers were presented and 10 demonstrations given.

On June 19, 1931, a Discussion on Audition was held, to which 18 papers were contributed. On June 20, 1931, members of the Society and their friends visited Reading University at the invitation of Professor Crowther; at this meeting three papers were read and 17 demonstrations were given by the staff of the Physics Department.

A lecture entitled "Cohesion" was delivered on May 1, 1931, by Professor J. E. Lennard-Jones, D.Sc., Ph.D.

The Presidential Address entitled "The Expanding Universe" was delivered on November 6, 1931, by Sir Arthur Eddington, M.A., D.Sc., F.R.S.

EXHIBITION

The Twenty-Second Annual Exhibition arranged jointly by the Physical and Optical Societies was held on January 5, 6 and 7, 1932, at the Imperial College of Science, by courtesy of the Governing Body. The Research and Experimental Section included exhibits from 31 sources. Trade exhibits were shown by 82 firms. Discourses were given by C. C. Paterson, O.B.E., M.I.E.E., F.Inst.P., on "Photocells: the Valves which operate by Light"; by T. Smith, M.A., F.Inst.P., on "Photographic Shutters and Their Properties"; and by Sir Oliver Lodge, D.Sc., LL.D., F.R.S., F.Inst.P., on "Reminiscences."

REPRESENTATIVES ON OTHER BODIES

Dr D. Owen and Dr Ezer Griffiths have been appointed representatives on the Board of the Institute of Physics; and Sir William Bragg, Professor E. A. Owen and Professor F. L. Hopwood have been reappointed representatives of the Society on the British X-Ray Unit Committee.

THE DUDDELL MEDAL

At the Annual Meeting on March 20 the Eighth (1930) Duddell Medal was presented to Professor Sir Ambrose Fleming, F.R.S. The Council has awarded the Ninth (1931) Duddell Medal to Professor C. T. R. Wilson, F.R.S.

GUTHRIE LECTURE

Sir Richard Glazebrook, K.C.B., M.A., Sc.D., F.R.S., delivered the Sixteenth Guthrie Lecture on May 15, 1931, the subject being "Standards of Measurement, their History and Development."

SPECIAL REPORT

A report on "Band Spectra" has been received from W. Jevons, D.Sc., Ph.D., F.Inst.P., in accordance with the request of the Council, and is now passing through the press.

OBITUARY

The Council records with deep regret the deaths of two Honorary Fellows, Professor A. A. Michelson and Dr S. W. Stratton; and of the following Fellows: Emeritus Professor Archibald Barr, Dr D. W. Dye, Mr Channell Law, Colonel E. D. Malcolm, Professor J. Sampson and Mr H. Tomlinson.

MEMBERSHIP ROLL

The number of Honorary Fellows on the Roll on December 31, 1931, was 9. At the same date Ordinary Fellows numbered 773, and Students 60.

The changes in the membership of the Society are shown in the appended table:

	Total Dec. 31, 1930	Changes during 1931			Total Dec. 31, 1931
Honorary Fellows	11	Deceased		2	9
Ordinary Fellows	754	Elected		26	
		Student transfers		18	
				44	
		Deceased	5		
		Resigned or lapsed	20		
				25	
		Net increase		19	773
Students ...	51	Elected		29	
		Trans. to Fellow ...	18		
		Resigned	2		
				20	
		Net increase		9	60
Total Membership	816	Net increase		26	842

REPORT OF THE HONORARY TREASURER

THE accounts for the year ended December 31, 1931, show a balance of £55. 15*s*. 7*d*.

The reduction in the amount charged by the Institution of Electrical Engineers to the Society for "Science Abstracts" is due to the increased annual grant made by the American Physical Society.

The cost of the report of the Discussion on Audition amounted to £294. 4*s*. 2*d*.; it is anticipated that part of this expenditure will be recouped by the sale of the report.

£900 of 2½ % Consolidated Stock was purchased at a cost of £539. 12*s*. 0*d*.

The Society's investments have been valued at market prices on December 31, 1931, through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank.

(Signed) ROBERT S. WHIPPLE
Honorary Treasurer

March 9, 1932

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST DECEMBER, 1931

1930 £ s. d.	Expenditure	£ s. d.	£ s. d.
	To		
600 0 0	Stock of Publications at 1st January, 1931		779 0 0
	Science Abstracts	474 0 0	
	Less Rebate for 1930	235 0 0	
566 0 0			239 0 0
	Ordinary Publications:		
1374 12 10	"Proceedings"	1315 2 8	
80 7 7	"Bulletin" and Notices	78 13 2	
72 19 6	General	72 14 11	
			1466 10 9
116 8 5	Postage on Publications		113 15 2
	Printing and Publishing Discussion on "Audition"		294 4 2
66 4 7	Reporting at Meetings		65 1 10
33 11 2	Expenses at Meetings		37 9 1
744 10 2	Exhibition Expenses	824 5 11	
52 17 9	Optical Society Exhibition Account	67 6 0	
			891 11 11
5 19 2	Periodicals and Library		6 7 0
	Administration Expenses:		
169 11 8	Institute of Physics	170 4 4	
112 18 9	Office Expenses	105 15 5	
			275 19 9
20 0 0	Guthrie Lecture (Honorarium)		20 0 0
378 10 2	*Share of cost of printing and publishing "Photo-Electric Cells and their Applications"*		
36 0 0	*Printing List of Members*		
	Balance, being excess of Income over Expenditure, carried forward		55 15 7
£4430 11 9			£4244 15 3
	To		
	Balance carried to Accumulated Fund		£55 15 7

1930 £ s. d.	Income	£ s. d.	£ s. d.
	By		
779 0 0	Stock of Publications at 31st December, 1931		747 0 0
	Subscriptions:		
1242 1 3	Fellows*	1327 2 2	
8 3 0	(Voluntary†)	7 7 0	
28 17 6	Students	18 7 6	
102 4 0	For "Science Abstracts" and Advance Proofs	102 17 0	
			1455 13 8
819 17 8	Sale of Publications	833 14 11	
	Less Subscriptions received in advance	207 3 2	
			626 11 9
165 5 4	Advertisements in "Proceedings"		194 5 10
	Dividends from Investments and Bank Interest	249 10 6	
	Add: Income Tax refunded Jan.–April 4, 1931	15 1 11	
	Income Tax claimed April 5–Dec. 31, 1931	17 6 0	
		281 18 5	
	Less: Transferred to Duddell Memorial Fund	20 0 0	
		261 18 5	
257 10 1	*Add*: Dividends due at December 31, 1931	35 10 9	
	Income Tax claimed	11 9 10	
			308 19 0
805 10 1	Exhibitors' Payments on account of Exhibition		912 5 0
200 0 0	*Royal Society Grant for Publications*		
22 2 10	*Balance, being excess of Expenditure over Income*, carried forward		
£4430 11 9			£4244 15 3
	By		
	Balance brought forward		£55 15 7

* Eighty-eight Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total rebate being £31. 4*s*. 11*d*.

† Voluntary subscriptions are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT 31ST DECEMBER, 1931

LIABILITIES

	£ s. d.	£ s. d.
Accumulated Fund:		
As per last Balance Sheet	1895 14 11	
Entrance Fees, 1931	38 17 0	
Balance brought forward from Income and Expenditure Account	55 15 7	
	1990 7 6	
Less Decreased value of Investments	259 12 0	
		1730 15 6
Life Compositions:		
As per last Balance Sheet		2384 0 0
Duddell Memorial Trust Fund:		
As per last Balance Sheet	434 12 9	
Less Decreased value of Investments	24 0 0	
		410 12 9
W. F. Stanley Trust Fund (for the "Bulletin"):		
As per last Balance Sheet	265 0 0	
Less Decreased value of Investments	110 0 0	
		155 0 0
A. W. Scott Bequest		250 0 0
Sundry Creditors		1143 4 11
Subscriptions paid in advance:		
Members	21 15 10	
Publications	207 3 2	
		228 19 0
		£6302 12 2

ASSETS

	£ s. d.	£ s. d.
Investments at Market Value on December 31, 1931:		
£1500 Consolidated Stock 2½ %	825 0 0	
£1000 War Loan 5 % 1929/47 Inscribed "A" Account	960 0 0	
£400 War Loan 5 % 1929/47 Inscribed "B" Account	384 0 0	
£650 Funding Loan 4 % 1960/90	539 0 0	
£500 India 3½ % Stock	250 0 0	
£254. 2*s*. 9*d*. New South Wales 5 % Stock 1935/55	182 0 0	
£211 London County Consolidated 4½ % Stock	186 0 0	
£400 Lancaster Corporation 3 % Redeemable Stock	268 0 0	
£399 London Midland and Scottish Railway 4 % Debenture Stock	275 0 0	
£1000 London Midland and Scottish Railway 4 % Preference Stock	460 0 0	
£500 London and North Eastern Railway 4 % Debenture Stock	325 0 0	
£150 Southern Railway 5 % Debenture Stock	128 0 0	
£300 Southern Railway Preferred Ordinary Stock	111 0 0	
£442 Southern Railway Deferred Ordinary Stock	44 0 0	
		4937 0 0
Dividends due from Investments		35 10 9
Stock of Publications (as per Treasurer's valuation)		747 0 0
Subscriptions due		22 1 0
Sundry Debtors		279 12 5
Inland Revenue—Income Tax reclaimed for 1931		28 15 10
Cash at Bank:		
On Deposit Account	100 0 0	
On Current Account	134 8 5	
Cash in hand	18 3 9	
		252 12 2
		£6302 12 2

ROBERT S. WHIPPLE, *Honorary Treasurer.*

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the Bank Balances and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

SPENCER HOUSE, SOUTH PLACE, E.C. 2
9th March 1932.

KNOX, CROPPER & CO.,
Chartered Accountants

Examined and approved on behalf of the Society { (Signed) F. H. SCHOFIELD. (Signed) W. S. TUCKER.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1931

	£	s.	d.
128 Fellows paid £10	1280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20	20	0	0
1 Fellow paid £20. 10s.	20	10	0
14 Fellows paid £21	294	0	0
23 Fellows paid £31. 10s.	724	10	0
	£2384	0	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN")

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	111	0	0	Carried to Balance Sheet	155	0	0
£442 Southern Railway Deferred Ordinary Stock	44	0	0				
	£155	0	0		£155	0	0

DUDDELL MEMORIAL TRUST FUND

CAPITAL

	£	s.	d.		£	s.	d.
£400 War Loan 5 % 1929/47 Inscribed Stock	384	0	0	Carried to Balance Sheet	384	0	0

REVENUE

	£	s.	d.		£	s.	d.
Balance at 31st December, 1930	26	12	9	Honorarium to Medallist	20	0	0
Dividends	20	0	0	Balance carried to Balance Sheet	26	12	9
	£46	12	9		£46	12	9

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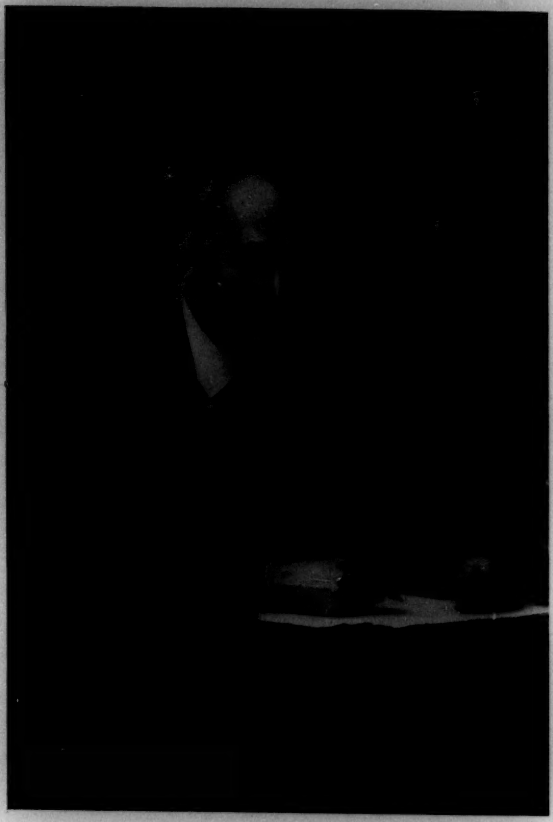

SIR ARTHUR EDDINGTON, F.R.S.

President 1930–32

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 1 *January* 1, 1932 NO. 241

THE EXPANDING UNIVERSE

Presidential Address by

SIR ARTHUR EDDINGTON, F.R.S.

PROCEEDINGS OF THE PHYSICAL SOCIETY

ERRATA

TWO PRECISION CONDENSER BRIDGES, *by* A. Campbell, M.A.

Vol. 43, *pp.* 564–568 (1931)

Page 566, equation (6): *for* $M\lambda - m\mu$ *read* $(M\lambda - m\mu)$.

Page 566, equation (7): *for* $+ m(m - \mu)$ *read* $- m(m - \mu)$.

Page 566, equations (9) and (10):
for $m(m - \mu)/\rho$ *read* $m(m - \mu)\omega^2/\rho$.

Page 567, line 28: *for* $11000\,\Omega$ *read* $1000\,\Omega$.

Page 567, line 34: *for* $500\,\Omega$ *read* $2000\,\Omega$.

expand too much they burst. On this point at least I can speak reassuringly. Our bubble of a universe is not going to burst—for the best of reasons. It burst quite a long while ago.

§ 2. ASTRONOMICAL EVIDENCE

Let us first take an astronomical view of the subject. Up to the farthest limits surveyed by our telescopes space is dotted with numerous islands—the spiral nebulae. They are so far apart that light takes about a million years to cross from one island to the next. Each island turns out to be a galaxy of stars. Naturally our own island galaxy has a particular importance in our eyes; it is estimated to

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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 1 *January* 1, 1932 NO. 241

THE EXPANDING UNIVERSE

Presidential Address by

SIR ARTHUR EDDINGTON, F.R.S.

Delivered November 6, 1931

The world's a bubble, and the life of man
Less than a span.
FRANCIS BACON.

§ 1. INTRODUCTION

THE subject which I have chosen for my presidential address is a meeting-ground of astronomy, relativity and wave mechanics. It is one of the most recent scientific developments; for the rudiments of the present ideas do not go back earlier than 1917, and it is within the last two or three years that progress has been most active. It is, I fear, not an easy subject to explain lucidly; but the importance of its reactions on other branches of physical science is so great that it deserves to be widely known. To adopt a phrase used by one of your ex-presidents with reference to former expository efforts of mine, I may perhaps hope to make the theory "pleasantly incomprehensible."

The conclusion that I have to bring before you is that the material universe is swelling up like a bubble. Moreover it is swelling up at a rate which, if not alarming to the ordinary citizen, is very disturbing to theorists. In the time which has elapsed since the oldest terrestrial rocks were formed the radius of the universe has become doubled. The simile of a bubble may suggest danger, for when bubbles expand too much they burst. On this point at least I can speak reassuringly. Our bubble of a universe is not going to burst—for the best of reasons. It burst quite a long while ago.

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contain from 5 to 50 thousand million stars. Ordinarily when we refer to the stars we mean the stars of our own island, for it is only recently that it has become possible to discern individual stars in some of the nearest of the other islands. We view our own system of stars from within; it has a flattened disc-like form, and the circuit of the Milky Way marks the plane in which it extends. The other island galaxies are seen from without, and it is easier to recognize at once their shape and character; they are usually found to be flattened and rotating like our own system.

The external galaxies are so remote that we could not expect to detect any apparent movement; but for some years we have been able in favourable cases to measure their radial velocities in the line of sight by the shift of their spectral lines. More recently the distances of some of them have been determined by a fairly reliable method. In two or three of the nearest spiral nebulae (including the great Andromeda nebula) E. Hubble was able to discover cepheid variables and to measure their magnitudes and periods. We know sufficient about the variables in our own system to understand that a cepheid variable of given period is a quite definite standard of light; we can use it in place of a standard candle. If you see a standard candle anywhere and note how bright it appears to you, you can easily calculate how far off it is; in just the same way the astronomer when he sees his "standard candle" in the midst of a nebula can calculate the distance of the nebula. That method unfortunately applies only to the nearest galaxies. For those farther off less satisfactory methods are used, and the distances assigned are to be described rather as estimates than as measures. Still they should serve well enough for statistical purposes.

When we survey the collected data as to distances and radial velocities we find an extraordinary state of affairs. The velocities are large—very much larger than ordinary stellar velocities. Also the more distant galaxies have the bigger velocities, and there is a fairly regular law of increase, the velocity being roughly proportional to the distance. The most striking thing of all is that the galaxies are with remarkable unanimity going away from us.

With regard to the last statement it is worth while examining the actual results in more detail. About 90 line-of-sight velocities of spiral nebulae have been measured, and amongst these only five are approaching us*. At first sight it may seem illegitimate to pass over the minority as insignificant. But we notice that all five exceptions are among the very nearest of the nebulae; and since we are dealing with an effect which increases with the distance, it is natural that we should have to go out some distance before it predominates over accidental irregularities (including observational error) and displays itself uniformly. The five approaching velocities are comparatively small and are at least partly attributable to our inappropriate standard of reference. The velocities as published are relative to the sun; but the sun is describing an orbit inside our galaxy with an orbital velocity of between 200 and 300 km./sec.; it is more significant to consider approach and recession with respect to our galaxy as a whole, so that a correction for the sun's orbital

* The Andromeda nebula and two satellites (close to it in space and within range of its attraction) are counted as one nebula.

motion should be applied. It is found that this considerably reduces the approaching velocities. I think it will turn out that further small corrections are required, and that ultimately these five nebulae will be found to have small velocities of recession; for even one genuine exception would be very difficult to account for.

Let us exclude the nebulae which are more or less hesitating in our neighbourhood by drawing a sphere of rather more than a million light-years radius round our galaxy; we can then say definitely that, in the vast region beyond, the nebulae are unanimously running away from us. More than 80 have been observed to be moving outwards and not one has been found coming in to take their place. It is an obvious inference that in the course of time the region will be evacuated. The nebulae will all be out of reach of our telescopes, unless we increase our telescopic power to keep pace. I find that an observer of nebulae will have to double the aperture of his telescope every 1300 million years merely to keep up with their recession. Sir James Jeans delights in telling us that we have billions of years before us in which to find out all that can be found out about the universe. I suggest, however, that there is urgency as regards the spiral nebulae; if we leave it too late, there will be none left to examine.

I do not wish to insist on these astronomical facts dogmatically. No doubt there is a possibility of error and misinterpretation. But if we ask what is the picture of the universe now in the minds of those who have been engaged in practical exploration of its large-scale features—men not likely to be moved overmuch by ideas of curvature of space or the covariance of the Riemann-Christoffel tensor—the foregoing is their answer. Their picture is the picture of an *expanding universe*. We have dwelt on the fact that the nebulae are running away from ourselves; but the rule that at double the distance the velocity is doubled means that they are running away from each other as much as they are running away from us. This is the law according to which the points on a rubber balloon recede from one another when the balloon is being inflated; the mutual recession of two points is proportional to their distance apart. There is a uniform inflation of the system of the galaxies extending at least as far as our observations extend; and they extend a good distance. The latest recruit is a nebula in Leo distant more than 100 million light-years, which is receding with a velocity of 19,500 km./sec.; that is about the speed of an alpha particle.

§ 3. DE SITTER'S THEORY

In beginning with the astronomical observations I am deviating from the historical order, for it was the theory of relativity that first led us to look for a phenomenon of this kind. In 1917 W. de Sitter found that on one of two alternative hypotheses the light of remote objects would be displaced towards the red, and he suggested the motions of the spiral nebulae as a discriminating test. At that time only three velocities of spiral nebulae had been published, and these somewhat lamely supported his theory by a majority of 2 to 1. There the matter rested until 1923 when V. M. Slipher kindly supplied me with his (then unpublished) measures of the velocities of 40 spiral nebulae for use in my forthcoming book. As the

majority had now become 37 to 3, I was able in my *Mathematical Theory of Relativity* to present de Sitter's theory with much more favourable emphasis. It should be mentioned, however, that the effect now found is not strictly the original de Sitter effect which was proportional to the square of the distance. It was not realized at first that de Sitter's hypothesis required also a larger linear effect of the same kind; that was found when the theory had been developed in the clearer form associated especially with the name of G. Lemaître (1927). When Hubble found a roughly linear relation between distances and velocities from his discussion of the observations in 1929 he was probably unaware that theory inclined to a linear rather than a quadratic relation.

§4. THE COSMICAL CONSTANT

We must go back a little before de Sitter. In 1915 Einstein had by his general theory of relativity brought the world to good order. The quantum, it is true, continued to pursue its communistic activities outside the reach of his law; but otherwise the state of things anywhere within finite distance of the observer might be considered fairly settled. Einstein thereupon became uneasy about infinity. I need not enter into the particular difficulties occasioned by having to employ boundary conditions at infinity; but Einstein saw that by far the simplest way of getting rid of these difficulties was to get rid of infinity. Instead of infinite space, take a "finite but unbounded" space—a spherical space of radius R. (We can always go back to infinite space, if it turns out that the facts of nature require it, by proceeding to the limit when R becomes indefinitely great.) Einstein effected this change by altering his law of gravitation in empty space from its original form $G_{\mu\nu} = 0$ to its more accurate form $G_{\mu\nu} = \lambda g_{\mu\nu}$, where λ is a very small coefficient called the *cosmical constant*. The added term is negligible in the ordinary applications of relativity to the solar system.

The cosmical constant λ is so fundamental in the theory of the expanding universe that we must spend a little time considering why it appears at all. The law $G_{\mu\nu} = \lambda g_{\mu\nu}$ is the expression of the fact that our measurements of length are not absolute, and that a length can only be measured relatively to another length treated as standard. If we translate it from symbols into words, the law states that the radius of spherical curvature of every three-dimensional section of the world is the same constant length—the same number of metres. Or we can put the statement the other way round—*what we call a metre at any place and in any direction is a particular constant fraction of the radius of curvature of the world for that place and direction.*

The law of gravitation is simply the ideal definition of the metre. It tells us how lengths at different places and times are to be compared. They are said to be equal if each bears the same ratio to the world-radius in its own neighbourhood. Clearly the first essential in our conception of space is that it should provide such a criterion of equality; without it space would be mere emptiness and not a metrical background. The curved space of relativity theory supplies its own standard units,

the various radii of curvature in different directions. We adopt these natural units in our measurements through the intermediary of the metre, for the metre is (if the above law of gravitation is true) a constant fraction of the natural unit.

We may perhaps ask, How is it that our practical standard, the metre bar which we move about in space and time, agrees so well with the ideal definition? Since the question relates to the behaviour of a material system of some 10^{27} particles distributing themselves according to quantum laws which are not yet fully formulated, we cannot give a very detailed answer. But what else could the metre bar do but reproduce the natural unit? It is selected for regularity of behaviour; its extension is determined by some constant set of equations. If we have the extension of our material bar on one side of the equations, what extension can we put on the other side? The only candidate for the position on the other side of the equation is the extension of the natural unit characteristic of the region where the bar is situated. Therefore we infer that the extension of the metre bar will be proportional to the extension of the natural unit contained in the space where it lies. That is what the law of gravitation asserts.

The ratio of the metre to the radius of curvature is determined by λ. If λ is zero the ratio is zero and the connection breaks down. We are left with a space which does not fulfil the first conditions of a medium of measurement; and the relativity theory is laid open to criticisms such as have been brought forward by Prof. Whitehead (mistakenly, I think, as regards the existing theory) as failing to provide a "basis of uniformity" for spatial measurement. For this reason the cosmical term $\lambda g_{\mu\nu}$ is essential in relativity theory. When it was first introduced it might have been regarded as a fancy addition, but it is now seen to be indispensable*. I see that Einstein has recently proposed to take λ equal to 0; that seems to me an incredibly retrograde proposal.

It is a curious fact that whilst the rather difficult conception that length is relative to the motion of a reference body is now a commonplace principle of physics, there is very little recognition of the much more elementary relativity of length that I have been referring to, viz. that we can only recognize the ratio of two lengths. If all lengths were altered in the same ratio the change would be undetectable and meaningless. When we speak of a linear constant of nature such as the radius of the unexcited hydrogen atom, what do we mean by saying that it is constant? We can only mean that its ratio to some other length is constant. Practically we compare it with a metre bar and say that it is a constant fraction of the international metre; but this, though true, is clearly not fundamental. It holds because the metre bar is governed by an elaboration of the same laws which govern the hydrogen atom, and both regulate their extension by reference to the same standard. The point I am here stressing is that our geometry—or description of space—must describe a standard which is everywhere available for comparison when we want to state the dimensions of a physical system. A geometry which does not supply any linear standard is useless for physical purposes. The second point is that

* Cf. H. Weyl, *Raum-Zeit-Materie*, p. 297 (English edition). "The cosmological factor (λ) which Einstein added to his theory later is part of ours from the very beginning."

although our practical measurements are made, not by direct comparison with the simple standards contained in the geometry of space but by comparison with complicated material systems, nevertheless the result is the same as if we had used the ideal standards. The interchangeability of the ideal standards and the practical reckoning is expressed symbolically by the law $G_{\mu\nu} = \lambda g_{\mu\nu}$*.

§ 5. THEORY OF THE EXPANDING UNIVERSE

The immediate result of this change in the law of gravitation was the appearance (in theory) of two universes—the Einstein universe and the de Sitter universe. Both were possible theoretically; both involved spherical space; but since de Sitter's universe required an apparent recession of distant objects whilst Einstein's did not, it was hoped that observation of the spiral nebulae would discriminate between them. They were called static universes, for unlike our "expanding universe" they remained unchanged for any length of time. But it was realized later that the changelessness of de Sitter's universe was due to the fact that he had left it entirely empty, so that there was nothing in it that *could* change†. Einstein's universe was therefore the only form of a material universe that could remain motionless. The situation has been summed up by saying that Einstein's universe contained matter but no motion and de Sitter's contained motion but no matter.

The actual universe with both matter and motion cannot correspond exactly to either of these two simple models. The only question is, Which is the best choice for a first approximation? Shall we put a little motion into Einstein's world of matter, or a little matter into de Sitter's world of motion which is waiting to move something? The question is not so urgent now, for we are no longer restricted to the two extremes. We have now the whole chain of intermediate solutions of $G_{\mu\nu} = \lambda g_{\mu\nu}$, from which we can pick the one which has the right proportion of matter and motion to correspond with what we know of our own universe. These solutions seem to have been first given by A. Friedman in 1922. They were rediscovered by Lemaître in 1927, who developed the astronomical consequences elegantly and exhaustively. His work seems to have remained unknown until last year when he called my attention to it in connection with problems then engaging attention‡. In the meantime the solutions had been again discovered by H. P. Robertson.

The intermediate solutions are expanding universes. At one end we have Einstein's universe with no motion and therefore in equilibrium; then as we proceed along the series we get more and more rapid expansion until we reach de Sitter's universe which forms the limit. It is the limit because whilst the

* This interpretation of the law of gravitation was given by the lecturer in 1921. For fuller details, see *Mathematical Theory of Relativity*, §§ 65, 66 and *The Nature of the Physical World*, chap. 7.

† The ideal changelessness of de Sitter's universe failed as soon as anything, e.g. a spiral nebula, was put into it.

‡ The original paper (which is rather inaccessible) has now been reprinted in *Monthly Notices R.A.S.* **91**, 483 (1931).

expansion has been increasing the density has been decreasing all the way along the series, so that by the time de Sitter's form is reached we are left with a rapid expansion but nothing to expand.

The meaning of this series can be better understood by beginning at the de Sitter end. Just before we remove the last vestiges of matter so as to obtain the empty de Sitter world, the expansive tendency has free play with nothing to counteract it. When matter is inserted gravitation tends to hold the mass together and so opposes the expansion. The more matter we put in, the more the expansion is counteracted. For a particular density the gravitational attraction of the matter will just balance the expansive force, and we have complete equilibrium. This corresponds to the Einstein universe. If we take still higher density gravitation will prevail over expansive force and we have a contracting universe.

A point which I think should be particularly stressed is that Einstein's universe is unstable*. It corresponds to an exact balance between gravitation and the expansive force. Now suppose there happens to be a very small expansion. The gravitation between the galaxies is weakened by the increase of distance so that it no longer counterbalances the expansive force; thus further expansion ensues. Similarly a small contraction will lead to further contraction. The slightest disturbance will therefore cause the delicately balanced Einstein world to topple into a state of continually increasing expansion or continually increasing contraction.

It will be seen that our expectation of finding an effect like that apparently manifested in the motions of the spiral nebulae has been greatly strengthened since it was first suggested as a hypothesis. Firstly the term $\lambda g_{\mu\nu}$ which gives rise to it is now understood to be an integral and necessary part of the law of gravitation and not a fancy addition. Secondly, instead of the effect appearing in one of two alternative theories, we now find that the only form of universe which does not give rise to the effect is unstable, so that even if the universe originally had this form it would not retain it.

§ 6. OBSERVED AND PREDICTED RECESSION

Attempts have been made to settle whether expansion rather than contraction would be expected theoretically, but I am not sure that they have been successful. At any rate they involve additional hypotheses and do not depend simply on the law of gravitation. Apart from this ambiguity of sign relativity theory definitely predicts the type of phenomenon observed in the motions of the spiral nebulae. But the prediction is qualitative and not quantitative; no hint is given as to the scale of the phenomenon. Theoretically we expect to find a systematic recession of sufficiently remote objects, but we have no idea as to whether "sufficiently remote" means at nebular distances or at distances 10^6 or 10^{60} times greater. It is a stroke of luck that (if the observations are to be trusted) the required distance is well within range of our telescopes. The theory gives the magnitude of the recession in

* A. S. Eddington, *Monthly Notices*, **90**, 668 (1930). This paper gives an account of Lemaître's theory, including the more elementary mathematical part.

terms of the cosmical constant λ, but hitherto we have had no knowledge of the magnitude of λ except such as may be inferred by fitting the theory to the observations.

Are we on safe ground in identifying the observed recession of the spiral nebulae with the expansion effect predicted by theory? The weak point is the absence of any prediction of the magnitude; and the question of magnitude happens to be of considerable importance. It is one thing to say that our universe cannot remain the same size for ever and that if we wait long enough it will expand indefinitely; it is another thing to say that the universe is blowing up so fast that it has doubled its diameter within geological times. Theory is responsible only for the first statement; when the spiral nebulae, not content with confirming it, proceed to add the barely credible second statement, their value as witnesses becomes suspect. For this reason some astrophysicists have not unnaturally inclined to the view that the whole notion of recession of the spiral nebulae is a misinterpretation of the red-shift of their light; they do not necessarily doubt the relativity prediction of an expanding universe, but they point out that it may well be a very much slower change altogether undetectable by astronomical observation.

A theory put forward by Zwicky as to the cause of the red-shift of the nebular light has attracted some attention. He suggests that it is not a Doppler effect but is due to cumulative loss of energy of the quantum of light on its way to us by the gravitational perturbations which it exerts on particles of matter. We cannot very well rule out this hypothesis as impossible unless we claim to know all about light quanta—a claim which I certainly shall not make. But I think the present position of Zwicky's hypothesis is not generally realized. In his original paper he supported it by a mathematical investigation intended to show that the hypothesis led to results of the right order of magnitude. There was a mathematical mistake in the paper which invalidated the result, and I understand that this investigation has been withdrawn. Dr Zwicky continues to advocate his suggestion in a more indefinite form, as he is quite entitled to do; but naturally we regard differently a suggestion which is now wholly speculative from one which appeared to be supported to some extent by a confirmatory investigation.

§ 7. THEORETICAL VALUE OF THE COSMICAL CONSTANT

I have recently obtained a result which I think clears up the situation. I have found a theoretical value of the cosmical constant from a study of the wave equation of the electron*. Using this value we can make a purely theoretical calculation of the speed of recession of remote objects; the result is found to agree with the observed motions of the spiral nebulae. Thus there seems to be no doubt that the observed motions are genuine and are due to the cosmical expansion.

For the last three years I have been much occupied with a theory of some of the natural constants which appear in physics, particularly the constant $hc/2\pi e^2$, or 137. A year ago† it seemed likely that the investigation would develop so as to

* *Proc. R.S.* A, **133**, 605 (1931).

† *Proc. Camb. Phil. Soc.* **27**, 15 (1930).

embrace two other constants, viz. the mass-ratio of the electron and proton, and the cosmical constant λ. I may say at once that I have got no further with the theory of the mass-ratio of the proton and electron*; but if I am not mistaken the theory of the cosmical constant has proved unexpectedly simple. It does not involve directly my theory of the other constants. Nor does it involve the auxiliary mathematical developments connected with my theory of 137. But I am not sure that it can be grasped without an understanding of the general conceptions that I have been applying—or misapplying—in that theory.

The theory of the cosmical constant is in fact more directly connected with the explanation of the law of gravitation to which I have already referred. We have seen that our standard metre adjusts itself as a constant fraction of the radius of curvature of space-time, so that it is an intermediary by which we compare the dimensions of a physical system with the radius of the world. Although the metre rod is a useful practical intermediary, it is a red herring in theoretical investigations; for these will naturally have a simpler and more illuminating form when the radius of the world is introduced directly.

I suppose that nowadays we should look on the wave equation for the hydrogen atom as an example of the most fundamental type of physical equation. It is of simple form because it deals with the most elementary interplay of entities. It determines the linear spread of the charge—or the probability, or whatever the stuff may be—that surrounds the nucleus. From what has already been said the spread can only be expressed as a ratio, the ratio of distances in the atom to some other standard of distance. The equation determines the ratio, and therefore both participants in the ratio must figure in the equation. The standard, as we have seen, is the radius of world curvature; and the wave equation must say just as much about the radius of the world as it says about the spread of the constituents of the atom. We have not one equation saying how the spread of the hydrogen atom is fixed and another equation saying how the curvature of the world is fixed; we have an equation saying how the spread of the hydrogen atom is fixed in relation to the curvature of the world.

The equation can be adulterated by making substitutions based on other physical equations, which will perhaps not distort its simplicity of form though they distort its simplicity of meaning. In that way we obtain the wave equation as ordinarily written, which drags in the material standard metre and gives the extension of the hydrogen atom as compared with it—a comparison which is doubtless of greater practical utility than a comparison with the world radius. Here I want to see as far as possible into the inner significance of the wave equation, and I must take the equation unadulterated.

I have said that the wave equation says as much about the world-radius R as it does about the hydrogen atom. Now the ordinarily accepted form of the wave equation never mentions R; in fact it ostensibly refers to entirely flat space-time. The ordinary wave equation agrees with experiment, so we may not reject or

* Since this was written advance has been made, and a theoretical value 1847·60 for the mass-ratio seems to follow naturally from the present theory of the cosmical constant.

substantially amend it. The two requirements are reconciled if we realize that the ordinary wave equation does indeed contain a term involving R but *it writes it in a disguised form.* There is only one possible term which could contain R in disguise; it is the term commonly attributed to the proper mass of the electron. I think we might have expected that; the well-known relation between mass and space-time curvature in relativity theory suggests that the world radius might disguise itself in a mass term.

We ordinarily regard the wave equation as describing the probability of various distances r within the atom; but it is equally an equation for the probability of various radii R of the world, since it is only the ratio of r to R that counts. The wave function ψ refers to the probability of various sets of simultaneous values of these two variables; that is to say it is a function $\psi(r, R)$. We know the term in r, for that is not disguised in the ordinary equation; it is proportional to $1/r$, and since the whole equation may be divided through by an arbitrary factor we shall suppose it so written that this term is definitely $1/r$. The mass term then becomes mc^2/e^2; but that, as I have explained, is its disguised form. Can we say precisely what is its undisguised form?

Since the equation determines only the ratio r/R, homogeneity demands that R shall occur in the form $1/R$ to correspond with $1/r$. There may be a numerical coefficient, but we shall leave this aside for the moment.

We thus reach the conclusion that the wave equation is built up round a kind of "core" consisting of two terms

$$\frac{1}{r}, \quad \frac{1}{R}.$$

We can see moreover that this corresponds precisely to the geometry of the problem. When we say that an electron is in a spherical space of radius R and at a distance r from a proton, we are effectively stating bipolar coordinates. It is distant r from one fixed point, the proton; it is distant R from another fixed point, the centre of the spherical world. The fact that one of these distances is outside ordinary physical space-time is not relevant here; that will be cared for by the way in which the remaining terms of the equation, which introduce the space-time coordinates, are linked on to the core. But we have to take note of one kind of asymmetry between our two bipolar coordinates. The datum is that there are N electrons (all the electrons of the universe) at a distance R from the centre of the world, and of these just one is at a distance r from the proton. Obviously it would be a very different problem if there were N electrons at a distance r from the proton, of which just one was in spherical space. Thus r and R are not simply interchangeable, and the number N has to be introduced.

Omitting detailed discussion it appears that this difference causes the "core" terms to be

$$\frac{1}{r}, \quad \frac{\sqrt{N}}{R}.$$

If you wonder at the square root I may remind you that in wave mechanics, if ψ is the wave function normalized so as to represent one electron, $\psi\sqrt{N}$ is the wave

function normalized to represent N electrons. I must also add that R refers not to the present radius of space but to the radius of the universe in a stationary state, i.e. arranged as an Einstein world; for the wave equation gives the stationary states (*eigen* states) of a system, and this applies just as much to the universe of N electrons as to the one electron in the atom.

We have thus penetrated the disguise of the term mc^2/e^2 and revealed it to be $\sqrt{N}/R$. The equation

$$\frac{mc^2}{e^2} = \frac{\sqrt{N}}{R}$$

is my new result. If you have not been able to follow the steps of the deduction, you will at least see that no adjustable factors have been introduced to bring about agreement with observation artificially.

Numerical results

The ordinary relativity theory of the Einstein world had already provided another relation between N and R*, so that with the new result we are able to determine N and R separately and hence the cosmical constant λ which is equal to $1/R^2$. The following are the values obtained:

Equilibrium radius of universe (R) $= 1{\cdot}010.10^{27}$ cm. $=$ 1068 million light-years
$=$ 328 megaparsecs.

Mean density in equilibrium state $= 1{\cdot}05.10^{-27}$ gm. cm.$^{-3}$.

Total mass of universe $= 2{\cdot}143.10^{55}$ gm. $= 1{\cdot}08.10^{22} \times$ mass of sun.

Number of electrons in universe (N) $= 1{\cdot}29.10^{79}$.

Cosmical constant (λ) $= 9{\cdot}8.10^{-55}$ cm.$^{-2}$.

Thus far the results are such as could not well be falsified by observation. But having determined λ and R we can at once find the limiting speed of recession of the spiral nebulae which by Lemaître's theory is $c/R\sqrt{3}$ per unit distance; this gives

528 kilometres per second per megaparsec.

This is the full recession undiminished by countervailing gravitational contraction. The correction for gravitation between the nebulae can be calculated only from estimates of the average density of matter in space; it is unlikely that it exceeds 50 km./sec. and it may well be trifling. We therefore predict a motion rather less than, but not much less than, the figure above stated.

This is in excellent agreement with the observed value which has generally been stated in round numbers as 500 km./sec. per megaparsec.

Incidentally it is a rather gratifying confirmation of our astronomical scale of distances. When we reflect on the number of intermediate stages by which we step from the standard metre to distances of a hundred million light-years, and on the possibility of unforeseen errors at each stage, it is no small achievement to find the scale confirmed to within 10 per cent.

* The total mass M of the universe is approximately NM, where M is the mass of a proton. The Einstein relation is $M = \frac{1}{2}\pi R$ in gravitational units or $GM/c^2 = \frac{1}{2}\pi R$ in c.g.s. units.

§8. THE TIME-SCALE

If the expanding universe is accepted as an established fact, its most immediate reaction is on the time-scale of evolution. Three main time-scales have been favoured at one time or another which we may distinguish as "short," "intermediate" and "long." No one now has a good word for the short Kelvin time-scale; and practically our choice lies between the intermediate scale giving the sun an age of the order 10^{10} years and the long scale giving an age of 5.10^{12} years. When there is no definite evidence one way or the other the longer time-scale naturally gets the preference. The more time allowed, the more can happen; so the policy of the evolutionist is to grab as much time as possible. This, rather than any striking success, accounts for the popularity of the long time-scale in recent years.

The hypothesis of the long time-scale came about through Einstein's theory which gave the total amount of energy in a given mass of matter. We knew just how much energy there was in the sun and could calculate how long it would maintain the radiation if it could all be released. To release it all it is necessary that protons and electrons should annihilate one another, thereby undoing the lock which fastens the energy in. This idea of the source of a star's energy seems to have been first mentioned by me in 1917*. It was the only adequate source that could be suggested at the time; but in 1920 a possible alternative was recognized in the energy released by the transmutation of hydrogen into higher elements. This alternative, however, suffices only for the intermediate time-scale. I do not think anything very decisive has been found for or against either theory (annihilation of protons and electrons, or transmutation of hydrogen) or either time-scale (long or intermediate). When it was necessary to choose one or the other, I have, like other time-grabbers, generally preferred the long scale. Last year, however, in considering the dynamics of our own rotating galaxy, I was much impressed by the strong argument that it furnished for the intermediate scale†.

With the universe doubling its radius every 1300 million years it is obvious that the long time-scale of billions of years is altogether incongruous. It is true that we cannot set any definite limit to the time occupied by the first slow development of the expansion. But if there were billions of years to choose from it is strange that the evolution of our own solar system should coincide with the relatively short interval between the "bursting of the bubble" and the complete dispersal of the galaxies.

§9. NATURE OF THE EXPANSION

For convenience we deal with a model spherical world, but it is not necessary to assume that the actual universe is at all closely spherical. If it started from an equilibrium configuration (and it is difficult to imagine any other kind of beginning which would not be unaesthetically abrupt) it must then have been spherical; but it may have developed lop-sidedly, and it may now be any shape, indeed space

* *Monthly Notices*, **77**, 611.

† *The Rotation of the Galaxy; Halley Lecture* (Oxford Univ. Press).

need not even be closed in any legitimate sense of the word. Such irregularity would not affect our calculation of the recession of the spiral nebulae, which represents a uniform expansive tendency present in every region of space; the form of the universe as a whole only affects the calculation of the countervailing gravitational attraction which is believed to be relatively small.

The expansion is relative to our ordinary standards, e.g. the standard metre or the wave-length of cadmium light, which in turn depend on the scale of atomic phenomena. Moreover it does not affect anything but the intergalactic distances. Notwithstanding the expansive tendency the atoms, the earth, the solar system, the galaxy itself, all remain constant. The reason for this paradox can be seen as follows. Suppose that the sun and planets were all given enormous electric charges of the same sign; that would introduce a strong "expansive tendency" into the solar system. But the solar system would not become an expanding system; there would be an initial readjustment, but afterwards the planets would describe periodic orbits as before under the modified field of force. It is only if the charge on the planets were made so strong as to overbalance gravitation that the planets would abandon the periodic type of orbit and recede continually. There is thus a sharp demarcation between systems which exhibit the expansion of the universe and systems which do not—corresponding to the distinction between periodic and aperiodic phenomena. If the expansion had been operative in all types of phenomena equally we should have been entirely unable to detect it.

You may perhaps have noticed an apparent contradiction. The radius of the world is increasing relatively to the metre; on the other hand we have especially emphasized the fundamental principle that the metre is defined as a definite fraction of a world-radius. It is my fault for not distinguishing more precisely between the various radii of curvature. Space-time can have an appalling variety of twistiness, and I could not specify the particular curvature referred to without becoming unduly technical. Also the curvature in the empty space between material objects, referred to in the definition of the metre, is not the same as the average curvature of a region including material objects.

Probably the radius of the universe is already increasing at a rate exceeding the velocity of light, and the rate is continually increasing. Mention of velocities greater than the velocity of light always arouses suspicion; but if any careful statement of the assertions of relativity theory on this point is consulted, it will be found that all is well.

§ 10. SEEING ROUND THE WORLD

In Einstein's spherical universe light can go round and round the world. In de Sitter's universe it cannot go round, because the journey would require an infinite time. Our expanding universe is intermediate, and we ask how light will behave in it.

In the sequence of intermediate stages between the Einstein and de Sitter forms there is a definite stage at which circumambulation ceases to be possible. Our universe is now well past this point, so that light can no longer go round the

world in finite time. The circumference of the world is expanding, and light is like a runner on an expanding track with the winning-post receding faster than he can run.

We assume that the universe was originally in the Einstein state of equilibrium. In those early days light and other radiation went round and round the world until it was absorbed. (It is worth mentioning that, in making one circuit, radiation would on the average meet with obstruction equivalent to 31 cm. of water, so that cosmic rays of moderate penetrating power could make several circuits. The average obstruction has now become less.) This merry-go-round lasted during the very early stages of expansion. But when the world had expanded to 1·003 times its original radius the bell rang for the last lap; light-waves then running will make just one more circuit before $t = \infty$; those which started later will never get round.

Somewhat later, when the increase of radius had become 1·073*, the last half-lap was announced. From that moment onwards it has become impossible for ligh to travel half-way round; so that corresponding to any star there is a region of the universe which its present radiation will never reach. And if light cannot reach no other causal influence can reach, for no signal can travel faster than light. That was the moment when *the bubble burst*. The star and its antipodal region are as disconnected from one another as the fragments of a bubble; their only causal connection is through the past time before the breakage.

After what I have just been saying, it may seem paradoxical to add that in the expanding universe we are (theoretically) able to see round the world—the process vividly, if inaccurately, described as "observing the back of one's head." It is true that the line of vision passes through the antipodal region, which has now become impassable; but the waves now reaching our eyes are those which passed through the region a long while ago, before the bubble broke.

The longer the light travels the more it becomes reddened. The reddening of the light from any object follows a simple rule; the wave-length is increased in the ratio of the radius of the world at the time of observation to the radius when the light was emitted. We have seen that light which has been round the world must have started before the expansion reached 1·003, so that at the time of its emission the radius was practically the initial Einstein radius. Unfortunately we can make only the roughest guess at the present value of the expansion; but I should be surprised if it is less than 4 : 1, and will take that figure as an illustration. In that case the light from the "back of one's head" will have its wave-length increased four-fold; it will accordingly be infra-red.

This result has an interesting bearing on the problem of the cosmic penetrating radiation. According to the statements of the experimenters the rays are coming more or less equally from all directions; this points to a source symmetrical about the earth. Now astronomers know of no celestial source with the least approach to directional symmetry about the earth, except the whole universe. In any case it

* The critical values 1·003 and 1·073 were first given by de Sitter. He pointed out a mistake in my own calculation just in time for me to give the correct values in *Monthly Notices*, **90**, 673 (1930). The exigencies of page-proofs prevented my acknowledging their source.

would seem that if cosmic rays are being generated in one system or one galaxy they should be generated in all, so that cosmic rays must have been poured out all over the universe from the beginning of evolution. We have seen that in general they will not be absorbed until they have been many times round; so I imagine that what we are now observing is the long accumulation of past ages. This seems to be the only way in which the popular theory, if it is to be accepted at all, can be put into satisfactory shape. But if so, no trust can be placed in the present attempts to determine the origin of the rays from calculations of their wave-length. The observed wave-length is much greater than the original wave-length which alone provides a clue to their origin. Presumably most of the rays were emitted during the long period when the world was still near the Einstein condition, so that a correcting factor probably greater than 4 is needed. Perhaps when accurate determinations of the wave-lengths are known and likely sources are decided on, the factor can be fixed definitely; that is to say, we may from observing the cosmic rays—which preserve remembrance of an earlier state of the world—determine how much our universe has expanded from its original condition. That would be a notable contribution to astronomy.

§ 11. CONCLUSION

All change is relative; and what we have called the theory of the "expanding universe" might also be called the theory of the "shrinking atom." It was for that reason that it was not inappropriate to investigate the problem by examining the wave equation of the atom.

"Les hommes, les animaux, les pierres grandissent en s'approchant et deviennent énormes quand ils sont sur moi. Moi non. Je demeure toujours aussi grand partout où je suis*."

I am always the same; it is the universe that is swelling. In this problem there is, I think, more justification than usual for our egocentric outlook; for I may remind you that the cosmos not only swells but it bursts, and I do not see how we can regard the bursting as the reflection of some reciprocal process undergone by ourselves. But it is a good scientific exercise to consider another point of view, though I must warn you that it has no philosophical moral in this instance. So for a last glance at the problem let us take the view of a cosmic being, whose body is composed of intergalactic spaces and swells as they swell; or rather we must now say, it keeps the same size, for he will not admit that it is he who has changed. Watching us for some few thousand million years he sees us gradually shrinking; atoms, animals, planets, even the galaxies, all share the same contraction; only the intergalactic spaces remain the same. The earth spirals round the sun in an ever-decreasing orbit. Naturally he will not accept our year as a unit of time; it is the period of a continually shrinking orbit. Presumably he will relate his units of time and length so that the velocity of light is constant. Our years will then decrease in geometrical progression in the scale of cosmic time. Owing to the property of geometrical

* Anatole France. The speaker is the dog Riquet.

progressions an infinite number of years will add up to a finite cosmical time; so that our $t = \infty$ is an ordinary finite date in the cosmic calendar. On this date stars, planets, atoms are doomed to disappear; for when $t = \infty$ the universe has expanded to infinite radius in our reckoning, and we have shrunk to zero in the reckoning of the cosmic being.

We walk the stage of life, performers of a drama for the benefit of the cosmic spectator. As the scenes proceed he notices that the actors are growing smaller and the action growing quicker. When the last act opens the curtain rises on midget actors rushing through their parts at frantic speed. Smaller and smaller. Faster and faster. One last microscopic blurr of intense agitation. And then nothing.

DIFFUSION FOR THE INFINITE PLANE SHEET

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ABSTRACT. The paper deals with a problem of diffusion of moisture into an infinite plane sheet of hygroscopic material, when subject to two different surface conditions. The method of treatment follows along the lines of a previous paper*. Further, a method is developed whereby the diffusivity and surface constants can be evaluated from experimental data. In order to facilitate the practical application of the methods propounded to this and similar diffusion problems, tables have been specially calculated giving the first four roots of each of the four equations $\left.\begin{matrix}\tan\\ \cot\end{matrix}\right\} x = \pm x \cdot \left.\begin{matrix}\tan\\ \cot\end{matrix}\right\} \lambda$. The particular design of these tables has been chosen to allow of ready interpolation.

§ 1. INTRODUCTION

THE present paper is intended as an extension of one previously published in the *Proceedings of the Physical Society*†. The problem considered is the diffusion of moisture into an infinite plane sheet of hygroscopic material, when subject to two different surface conditions.

Apart from any physical application which the derived formulae may have it is hoped that the methods are worthy of attention. The use of operational methods as adopted in this paper commends itself for several reasons, namely that (i) the necessity of effecting Fourier expansions is avoided; (ii) the final result is derived in the form of a complex integral from which moments and approximations can readily be found; and (iii) the procedure is both direct and brief.

A further purpose of the work is to draw attention to the use of moments for the evaluation of the constants of an equation. While the principle of least squares is of undoubted merit when it can be applied, it very frequently happens to be quite unmanageable and the method of moments, so ably exploited by statisticians, must be resorted to. In the present instance, were data known *a priori* to follow the derived law it would appear a very lengthy and difficult process to evaluate the constants by methods that differ from those propounded.

The evaluation of a criterion of the curve is a small but useful innovation which proves of great service in the first survey of experimental data.

In order to render this paper of practical value the roots of the four transcendental equations occurring in the theory of diffusion have been computed. The form of these tables has been chosen to allow of interpolation, a most desirable quality in practical analysis.

* *Proc. Phys. Soc.* **42**, 235 (1930). † *Ibid.*

§ 2. STATEMENT OF PROBLEM AND DEFINITION OF SYMBOLS

An infinite plane sheet of hygroscopic material, initially moisture-free, has one face maintained at saturation and the other face in the presence of a fixed humidity. It will be supposed that the absorption process is expressed by the usual partial differential diffusion equation together with a surface resistance, and that the concentration within the material is a linear function of the equivalent relative humidity. It is required to find the expression for the quantity of moisture absorbed in a given time interval and to develop a method of determining the diffusivity and surface constants from observational data.

The symbols used in this paper are the same as those in the before-mentioned paper with the following additions and modifications:

The origin of coordinates is taken at the plane in the vicinity of the arbitrary humidity.

a is the thickness of the sheet;
θ_0 the concentration *outside* the plane $x = 0$;
θ_a the concentration *at* the plane $x = a$;
$Q_0 = a\theta_0$;
$Q_a = a\theta_a$;
$(Q_a + Q_0)/Q_a = r$.

§ 3. SOLUTION OF THE DIFFERENTIAL EQUATION

We require to solve the differential equation

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t} \qquad \text{.....(1),}$$

subject to the conditions:

$$\left.\begin{array}{lll} (a)\ \theta = \theta_a & \text{when} & x = a \\ (b)\ \partial\theta/\partial x = -c\,(\theta_0 - \theta)/a & \text{when} & x = 0 \\ (c)\ \theta = 0 & \text{when} & t = 0 \end{array}\right\} \qquad \text{.....(2).}$$

Since the method of solution is parallel to that already given in the previous paper it is only necessary to give the results of the analysis. The operational solution of the problem is therefore included in the following two expressions:

$$\theta = \frac{c\,(\theta_a - \theta_0\cosh\sigma a)\sinh\sigma x + (\sigma a\,\theta_a + c\theta_0\sinh\sigma a)\cosh\sigma x}{(\sigma a\cosh\sigma a + c\sinh\sigma a)} \qquad \text{.....(3),}$$

$$Q = \frac{\sigma a\,Q_a + c\,(Q_a + Q_0)\tanh(\sigma a/2)}{\sigma a\,(\sigma a\coth\sigma a + c)} \qquad \text{.....(4).}$$

Before proceeding to the general interpretation of these last two equations it will be useful to obtain several special cases.

(i) *The concentration value at the steady state.*

We require the value of θ as $t \to \infty$. Now it is known that σ can be regarded as of the order $t^{-\frac{1}{2}}$ so all that is necessary for our purpose is to find the limit of the right-hand side of equation (3) as $\sigma \to 0$. This limit is found to be:

$$\theta_m = \{c(\theta_a - \theta_0)x + (\theta_a + c\theta_0)a\}/a(1+c) \qquad \ldots\ldots(5).$$

(ii) *The maximum amount absorbed.*

This is given by direct integration of equation (5), whence:

$$Q_m = \{Q_a + (Q_a + Q_0)c/2\}/(1+c) \qquad \ldots\ldots(6)^*.$$

(iii) *Approximation when t is small.*

When t is small, σ can be regarded as of large order; hence from equation (4),

$$Q = (Q_a + Q_0)/\sigma a - Q_0/(\sigma a + c) \qquad \ldots\ldots(7).$$

The first term is readily interpreted but the most useful interpretation of the second term depends on the order of $\sigma a/c$. Further information about the expansion of this term will be found in a paper by Sumpner†.

Returning now to equation (4) we may express Q as a contour integral by means of the Bromwich rule and we arrive at the result:

$$Q = \frac{Q_a}{2\pi i}\int_L e^{ktz/a^2} f(z).\frac{dz}{z} + \frac{c(Q_a+Q_0)}{2\pi i}\int_L e^{ktz/a^2} f(z).\frac{\tanh \frac{1}{2}z^{\frac{1}{2}}}{z^{\frac{1}{2}}}.\frac{dz}{z} \qquad \ldots\ldots(8),$$

where

$$f(z) = (z^{\frac{1}{2}} \coth z^{\frac{1}{2}} + c)^{-1}.$$

The integrand of the right-hand side of equation (8) is a single-valued function of z with poles at $z = 0$, $z = -\beta_n^2$ where $n = 1, 2, \ldots$ and β_n is the nth positive root of the transcendental equation β_n, n

$$y \cot y + c = 0 \qquad \ldots\ldots(9).$$

The poles of $\tanh z^{\frac{1}{2}}$ are zeros of $f(z)$ and therefore make no contribution to the integral. It will be noted that the pole $z = 0$ merely contributes the value of Q_m which we have already found. We only then require to consider the contributions from the poles $z = -\beta_n^2$.

Now,

$$\operatorname*{Lim}_{z \to -\beta_n^2} f(z).(z + \beta_n^2) = 2\beta_n^2/(c + c^2 + \beta_n^2),$$

whence

$$Q = Q_a\left\{\frac{1}{1+c} - \sum_1^\infty \frac{2e^{-k\beta_n^2 t/a^2}}{(c + c^2 + \beta_n^2)}\right\} + c\frac{(Q_a+Q_0)}{2}\left(\frac{1}{1+c} - \sum_1^\infty \frac{2e^{-k\beta_n^2 t/a^2}\tan \frac{1}{2}\beta_n/\frac{1}{2}\beta_n}{(c + c^2 + \beta_n^2)}\right)$$
$$\ldots\ldots(10).$$

* A point of interest perhaps worth noticing is the following: If a diffusion process somewhat like that herein contemplated plays a part in the phenomena of "sorption" we see from equation (6) that by attributing to c, the surface constant, a directional property an explanation could be offered for the hysteresis effect known to be manifested by many materials in the course of absorption and desorption of water vapour.

† *Proc. Phys. Soc.* **41**, 241 (1929).

The equation (10) is the solution sought and gives Q explicitly in terms of t. We note that when the surface constant is either infinite or zero Q can be expressed in terms of the simple diffusion function defined in the previous paper.

§ 4. DETERMINATION OF MOMENTS

The curve represented by equation (10), together with its asymptote and the quantity-axis, bounds a finite area. Let μ_n be such that

μ_n

$$\mu_n Q_m = \int_0^\infty t^n (Q_m - Q) \, . \, dt.$$

Now from equation (8) we have

$$Q = \frac{Q_a}{2\pi i} \int_M e^{ktz/a^2} F(z) \, . \, \frac{dz}{z},$$

where

$$F(z) = \frac{1 + (cr \tanh \frac{1}{2} z^{\frac{1}{2}})/z^{\frac{1}{2}}}{c + z^{\frac{1}{2}} \coth z^{\frac{1}{2}}};$$

therefore

$$\int_0^\infty t^n (Q_m - Q) \, dt = - \frac{Q_a}{2\pi i} \int_0^\infty \int_N e^{ktz/a^2} t^n F(z) \, . \, \frac{dz}{z} \, . \, dt,$$

N

where N is a contour that just avoids the origin.

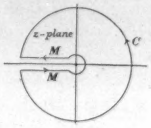

Fig. 1.

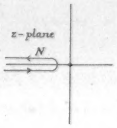

Fig. 2.

Since the real part of kz/a^2 is negative for every point of N,

$$\int_0^\infty e^{ktz/a^2} t^n \, dt = \left(\frac{-a^2}{kz}\right)^{n+1} n!,$$

so that

$$\int_0^\infty (Q_m - Q) \, t^n \, dt = - \frac{Q_a}{2\pi i} n! \int_N \frac{F(z)}{z} \left(\frac{-a^2}{kz}\right)^{n+1} dz \qquad \text{.....(11).}$$

Since the integrand of this expression is regular on and outside C and the integral round the infinite circle vanishes, we have that the integral along the path M is zero. Whence the integral along path N is equal to minus the integral round the origin. We deduce then that

$$\left.\begin{aligned} Q_m &= Q_a F(0) \\ \mu_0 Q_m &= -(a^2/k) \, Q_a F'(0) \\ \mu_1 Q_m &= +\tfrac{1}{2} (a^4/k^2) \, Q_a F''(0) \end{aligned}\right\} \qquad \text{.....(12).}$$

Hence

$$\mu_0 = \frac{a^2}{12k} \frac{(rc^2 + 5rc + 8)}{(1 + c)(2 + rc)} \qquad \text{.....(13),}$$

$$\mu_1 = \frac{a^4}{360k^2} \frac{(3rc^3 + 24rc^2 + 61rc + 16c + 96)}{(1 + c)^2 (2 + rc)} \qquad \text{.....(14).}$$

§ 5. EQUATION CONSTANTS AND CURVE CRITERION

We define the curve criterion R thus:

$$R = \mu_1/\mu_0^2 \qquad \text{.....(15).}$$ R

Whence from equations (13) and (14),

$$R = \frac{2}{5}\frac{(2+rc)(3rc^3+24rc^2+61rc+16c+96)}{(rc^2+5rc+8)^2} \qquad \text{.....(16).}$$

The maximum and minimum values of R are given when

$$3(r^3-r^2)c^4-(r^3+r^2)c^3-(39r^2-24r)c^2-(57r^2-48r)c-(88r-128)=0 \qquad \text{.....(17),}$$

and since c must be positive we are only concerned with the positive roots. When $r > \frac{16}{11}$ there is only one positive root which will give a minimum value of R. The effective maximum will be 1·2 corresponding with $c = 0$. When $r < \frac{16}{11}$ there are two positive roots of equation (17) which provide the appropriate range for R. The experimental conditions being fixed, i.e. r being known, the range for R can thus be determined. If observational values yield a value of R within the pre-determined range, two positive values of c can be derived from equation (16). The method of continued approximation or Newton's method of sequences will prove most expedient for the latter purpose. It will be noted that when $R = 1{\cdot}2$, $c = 0$ or ∞, and consideration of the physical interpretation of this shows that the ambiguity is irremovable.

We shall now discuss two special cases, namely those in which

(*a*) the concentration outside the face $x = 0$ is saturation, i.e. $r = 2$; and

(*b*) the concentration outside the face $x = 0$ is zero, i.e. $r = 1$.

Case (*a*). From equations (16) and (17) we find

$$R = \tfrac{6}{5}\{1 - c/(4+c)^2\} \qquad \text{.....(18),}$$

and the range

$$1{\cdot}125 \leqslant R \leqslant 1{\cdot}2 \qquad \text{.....(19).}$$

The values of c and k yielded by equations (13), (14) and (18) are

$$c = \left\{\frac{24}{(3 \pm 5\sqrt{(4{\cdot}8R - 5{\cdot}4)}} - 4\right\} \qquad \text{.....(20),}$$

$$k = 2a^2/15\mu_0\{1 \pm \sqrt{(4{\cdot}8R - 5{\cdot}4)}\} \qquad \text{.....(21).}$$

Case (*b*). In this case we find:

$$R = \frac{6}{5}\frac{(3c^2+15c+18)(3c^2+15c+32)}{(3c^2+15c+24)^2} \qquad \text{.....(22),}$$

and the range for R is

$$1{\cdot}2 \leqslant R \leqslant 1{\cdot}225 \qquad \text{.....(23).}$$

Writing $V = 3c^2 + 15c + 24$ we reduce equation (22) to a quadratic in V. V
R being known it is therefore a simple matter to find c. The corresponding value of k is then obtained by use of equation (13).

§ 6. EXAMINATION OF EXPERIMENTAL DATA

As is evident from the definition in § 4, $\mu_0 . Q_m$ and $\mu_1 . Q_m$ are the area and first moment respectively of the region enclosed by the diffusion curve, the asymptote and the line $t = 0$. The care to be exercised and the methods to be adopted in the derivation of Q_m, μ_0 and μ_1 from experimental data have been dealt with at length in the previous paper and need not therefore be repeated. Since the range for R is determinate when the experimental conditions are fixed the first enquiry is as to whether the ratio μ_1/μ_0^2 falls within this range. When this requirement is satisfied equation (16) must be solved by numerical methods to give the two values of c. From equation (13) two corresponding values of k are derived. It does not appear possible to remove the ambiguity and there is nothing to be done but to make tests of fit with the alternative pairs of values of the constants.

§ 7. ACKNOWLEDGMENT

In conclusion, I should like to express my thanks to the Council of the British Boot, Shoe and Allied Trades Research Association, in whose laboratories this work was done, for permission to publish the paper.

Table 1: The first four roots of the equation $\tan x = -x \tan \lambda$ for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	3·1416	6·2832	9·4248	12·5664
5	2·8936	5·8127	8·7703	11·7665
10	2·6976	5·5120	8·4453	11·4553
15	2·5434	5·3238	8·2775	11·3142
20	2·4196	5·1986	8·1781	11·2354
25	2·3176	5·1098	8·1124	11·1850
30	2·2310	5·0432	8·0655	11·1497
35	2·1559	4·9911	8·0300	11·1233
40	2·0892	4·9487	8·0018	11·1025
45	2·0288	4·9132	7·9787	11·0855
50	1·9729	4·8826	7·9590	11·0712
55	1·9204	4·8556	7·9419	11·0588
60	1·8702	4·8313	7·9267	11·0478
65	1·8214	4·8090	7·9128	11·0378
70	1·7732	4·7883	7·9000	11·0286
75	1·7249	4·7686	7·8879	11·0199
80	1·6756	4·7496	7·8764	11·0115
85	1·6246	4·7309	7·8651	11·0034
90	1·5708	4·7124	7·8540	10·9956

Table 2: The first four roots of the equation $\tan x = x \tan \lambda$ for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	—	3·1416	6·2832	9·4248
5	—	3·4344	6·8213	10·1510
10	—	3·7225	7·1859	10·5004
15	—	3·9562	7·3862	10·6588
20	—	4·1248	7·5030	10·7453
25	—	4·2446	7·5783	10·7996
30	—	4·3320	7·6308	10·8371
35	—	4·3985	7·6699	10·8649
40	—	4·4508	7·7004	10·8866
45	0·0000	4·4934	7·7252	10·9041
50	0·6837	4·5292	7·7461	10·9189
55	0·9205	4·5600	7·7641	10·9316
60	1·0798	4·5872	7·7800	10·9429
65	1·2002	4·6116	7·7943	10·9531
70	1·2973	4·6340	7·8075	10·9624
75	1·3789	4·6549	7·8198	10·9712
80	1·4498	4·6748	7·8315	10·9796
85	1·5131	4·6939	7·8427	10·9876
90	1·5708	4·7124	7·8540	10·9956

Table 3: The first four roots of the equation $\cot x = x \tan \lambda$ for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	1·5708	4·7124	7·8540	10·9956
5	1·4451	4·3488	7·2865	10·2639
10	1·3390	4·0879	6·9665	9·9432
15	1·2481	3·9044	6·7860	9·7890
20	1·1686	3·7712	6·6737	9·7008
25	1·0977	3·6704	6·5975	9·6436
30	1·0330	3·5910	6·5420	9·6033
35	0·9728	3·5264	6·4995	9·5729
40	0·9157	3·4722	6·4655	9·5490
45	0·8603	3·4256	6·4373	9·5294
50	0·8057	3·3846	6·4133	9·5127
55	0·7506	3·3478	6·3923	9·4983
60	0·6939	3·3141	6·3735	9·4856
65	0·6341	3·2827	6·3564	9·4740
70	0·5690	3·2530	6·3405	9·4633
75	0·4956	3·2245	6·3255	9·4532
80	0·4079	3·1968	6·3111	9·4435
85	0·2915	3·1692	6·2970	9·4341
90	0·0000	3·1416	6·2832	9·4248

Table 4: The first four roots of the equation $\cot x = -x \tan \lambda$ for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	1·5708	4·7124	7·8540	10·9956
5	1·7202	5·1346	8·4930	11·7968
10	1·8930	5·4807	8·8551	12·1290
15	2·0791	5·7038	9·0330	12·2711
20	2·2589	5·8437	9·1325	12·3474
25	2·4156	5·9365	9·1957	12·3950
30	2·5438	6·0022	9·2395	12·4279
35	2·6468	6·0514	9·2720	12·4522
40	2·7300	6·0899	9·2973	12·4711
45	2·7984	6·1213	9·3179	12·4864
50	2·8558	6·1475	9·3351	12·4993
55	2·9051	6·1702	9·3500	12·5104
60	2·9482	6·1902	9·3632	12·5203
65	2·9867	6·2082	9·3751	12·5292
70	3·0217	6·2248	9·3860	12·5374
75	3·0541	6·2403	9·3962	12·5451
80	3·0845	6·2549	9·4060	12·5524
85	3·1135	6·2692	9·4155	12·5595
90	3·1416	6·2832	9·4248	12·5664

YOUNG'S MODULUS FOR TWO DIRECTIONS IN A STEEL BAR

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ABSTRACT. The investigation described is a continuation of previous work* by the author on the extensions of thick cylindrical shells under internal pressure. Experiments were carried out to test whether the elastic constant E of the various steels from which the hollow cylinders were made is the same in two directions at right angles, one direction being along the axis of the original bar and the other across a diameter. The results show that for the steels dealt with Young's modulus is the same in the two directions referred to.

§ 1. INTRODUCTION

In connection with some previous experimental work on the extensions of cylindrical shells under internal pressure the question arose as to the elastic isotropy of the steel bars from which the cylinders were made, and it was thought that further work on this point was desirable. The question is a reasonable one, as it might be expected that on account of the rolling the elastic properties of a round bar are not the same across a diameter as along the axis of the bar.

In the experiments about to be described an attempt was made to test if the value of Young's modulus is the same in the directions already referred to for the materials from which the cylinders of the previous investigation were made. The work involved the measurement of the longitudinal strains of specimens of necessity very short, their length being less than 1½ in., the diameter of the original material; and this necessitated the use of a very small extensometer with which to make the measurements. The mirror extensometer with scale and telescope, as developed by Prof. E. H. Lamb† and now in frequent use, should be admirably suitable for the measurement of these very small strains; and an instrument of this kind which could deal with the extension of a 0·625 in. length bar of diameter about 0·25 in. was designed by the author, and made in the instrument shop of East London College. The accuracy of a determination of Young's modulus by means of the instrument should be well within ½ per cent.; and for this reason it was thought desirable that the testing-machine employed should be capable of measuring loads within the same degree of accuracy.

§ 2. THE TENSILE TESTING-MACHINE

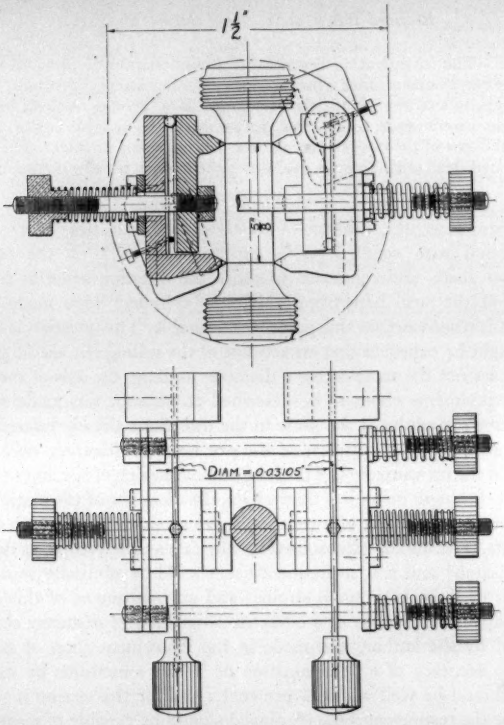
A small 1-ton testing-machine was available, and this, after some important modifications, was employed throughout the work. The machine was of the single-lever type‡ in which variable weights can be suspended from a knife-edge at a fixed

* *Proc. Phys. Soc.* **41**, 366 (1929).

† *Engineering*, **119**, 207 (1925).

‡ *Mechanical Testing*, **1**, 45 (1922).

distance from the fulcrum. The specimen was screwed into two rods, one of which was secured to the frame of the machine by a nut, and the other to the shorter arm of the lever, also by a nut. Between the lever and the upper of these two rods two degrees of freedom of movement were provided, and the alignment of the load applied to the specimen was made to coincide with the axis of the specimen. The leverage of the testing-machine was carefully determined by suspending a known



weight from the shorter arm and balancing this by weights suspended from the knife-edge at the other end; and in this way, within 0·2 per cent., the leverage was found to be 22·3. In order to reduce the jarring action set up in placing a weight on the holder suspended from the end of the longer arm, a spring was inserted between the holder and the knife-edge; and in order to avoid inaccuracies in the extension measurements due to changes of temperature in the room, and also those due to vibration effects, the whole apparatus was installed in a cellar free from sunlight and draughts, and well away from disturbances set up by machinery in the College workshop.

§ 3. THE EXTENSOMETER

The mirror extensometer is shown mounted on the specimen in the two views in the figure, from which will be seen the slight modifications in its design to meet the requirements of the present work. The mean diameter of the rollers was about $\frac{1}{32}$ in. Each mirror-holder was balanced rotationally on its roller. The knife-edges of the instrument were notched; this ensured that the instrument could be attached to the specimen correctly with respect to the axis.

One of the chief difficulties encountered in the use of an extensometer of this kind is the making certain that there is no slipping at the points of contact between the knife-edges and the surface of the specimen. To get over this difficulty without clamping the instrument too tightly to the specimen, and also to be certain as to the exact distance between the knife-edges, two fine grooves about 0·002 in. deep and 0·625 in. apart were cut into the surface of the specimen; and particular care was taken when the instrument was being set up to see that the points of contact were in these grooves.

§ 4. THE SPECIMENS

By experimenting on a pair of specimens each of the same dimensions and made from the same bar of material, one of the specimens having its axis co-axial with the bar, the other having its axis along a diameter of the bar, it should be possible to test whether Young's modulus for the material was the same in these two directions. The specimens were on this account made in pairs from bars of diameter 1·5 in., the over-all length of each specimen being slightly less than this. Gas screw-threads of 0·125 in. were cut on the ends of each specimen; and between the ends the shank was turned down to a diameter of 0·2492 in., the corners being suitably filleted as shown in the figure.

Four pairs of specimens were made from the four bars the analyses of which are given in the previous paper*. It will be convenient here to describe these bars as: no. 1, 0·09 C; no. 2, 0·28 C; no. 3, 0·43 C; no. 4, $3\frac{1}{2}$ Ni. These eight specimens were normalized. Three other pairs of specimens were also made, one pair from a bar of ordinary commercial mild steel, and the other two pairs from two bars of wrought-iron, known as Toga iron; and these six specimens were not subjected to any heat treatment. Then in order to test the effect of the length of the specimen on the value of Young's modulus, four specimens 3 in. long, screwed at the ends and turned down to a diameter of 0·2495 in., were made from the bars nos. 1, 2, 3, and 4; and these were also normalized.

§ 5. THE TENSION EXPERIMENTS

If x is the extensometer scale-reading in centimetres as read by means of the spider-line of the telescope, d the mean diameter of the extensometer rollers in inches; L the distance from the scale to the mirror facing it, and l the distance between the extensometer mirrors, both L and l being in centimetres, the extension of the specimen is given by the formula $xd/(4L + 2l)$. In all the experiments L was

x

d

L, l

xd

* *Proc. Phys. Soc.* **41**, 366 (1929).

264·2 cm., and l was 2 cm. The load was applied in equal increments of 111·5 lb.; and by taking a set of eight or ten readings the extension produced by this increment of load in terms of x cm. of the scale-reading was easily determined. And since d is 0·03105 in., the distance between the knife-edges of the extensometer 0·625 in., and the diameter of the specimen 0·2492 in., Young's modulus for the fourteen short specimens can be conveniently calculated by means of the formula

$$E = 4{\cdot}881 \times 10^7 \div x.$$

It was the usual practice to load the specimens several times before taking a set of readings. It was part of the procedure of the investigation to make four or five separate determinations of Young's modulus for each specimen; and in each of these experiments the instrument was removed from the specimen and reset in a different position. Provided the extensometer was carefully set up, it was an easy matter to make various separate determinations of Young's modulus each within 0·5 per cent. of the average of the values thus found. In all, some eighty separate determinations were made in the course of the present investigation, and in none of the experiments did the temperature in the vicinity of the specimen rise or fall by more than 0·05° C. The influence of temperature on the results is therefore practically of no account.

The readings taken in measuring the extensions of the three specimens made from bar no. 2, 0·28 C steel, are given in the table, in which is shown also the method

Table: Extensions of specimens from bar no. 2, 0·28 C

Specimen 1: Length 1½ in., co-axial with bar				Specimen 2: Length 1½ in., across a diameter of bar				Specimen 3: Length 3 in., co-axial with bar			
Load (lb.)	Scale-reading No.	Scale-reading cm.	Scale-difference (cm.)	Load (lb.)	Scale-reading No.	Scale-reading cm.	Scale-difference (cm.)	Load (lb.)	Scale-reading No.	Scale-reading cm.	Scale-difference (cm.)
—	1	19·43	(1) − (6) = 8·03	—	1	21·02	(1) − (6) = 7·97	—	1	19·52	(1) − (6) = 8·01
111·5	2	18·81	(2) − (7) = 8·01	111·5	2	19·43	(2) − (7) = 7·99	111·5	2	17·92	(2) − (7) = 8·01
223·0	3	16·21	(3) − (8) = 8·01	223·0	3	17·84	(3) − (8) = 7·98	223·0	3	16·33	(3) − (8) = 8·02
334·5	4	14·61	(4) − (9) = 8·01	334·5	4	16·24	(4) − (9) = 7·97	334·5	4	14·73	(4) − (9) = 8·02
446·0	5	13·00	(5) − (10) = 8·00	446·0	5	14·65	(5) − (10) = 7·97	446·0	5	13·12	(5) − (10) = 8·01
557·5	6	11·40	Mean = 8·012	557·5	6	13·05	Mean = 7·976	557·5	6	11·51	Mean = 8·014
669·0	7	9·80	Scale-difference for	669·0	7	11·44	Scale-difference for	669·0	7	9·91	Scale-difference for
780·5	8	8·20	load-increment of	780·5	8	9·86	load-increment of	780·5	8	8·31	load-increment of
892·0	9	6·60	111·5 lb. = 1·602 cm.	892·0	9	8·27	111·5 lb. = 1·595 cm.	892·0	9	6·71	111·5 lb. = 1·603 cm.
1003·5	10	5·00	$E = 30{\cdot}5 \times 10^6$	1003·5	10	6·68	$E = 30{\cdot}6 \times 10^6$	1003·5	10	5·11	$E = 30{\cdot}4 \times 10^6$

of arriving at the scale-difference for the given load-increment 111·5 lb. The readings here recorded are representative of those obtained from experiments on the remaining fifteen specimens.

The following is a summary of the determinations of Young's modulus, the results being given in lb. $\times 10^6$/in.^{2} for the fourteen short specimens, first for a direction co-axial with the original bar, and second for a direction at right angles to this axis: no. 1, 0·09 C, 30$\frac{3}{4}$ and 30·5; no. 2, 0·28 C, 30·5 and 30·6; no. 3, 0·43 C, 30·5 and 30·5; no. 4, 3½ Ni, 29·8 and 29·8; mild steel, 30·6 and 30·5; wrought-iron, first bar, 29·7 and 29·5 to 28·7, second bar 29·7 and 28·5 to 28·1. The corresponding results for the four longer specimens are: no. 1, 0·09 C, 30·4; no. 2, 0·28 C, 30·4; no. 3, 0·43 C, 30·5; no. 4, 3½ Ni, 29·7.

§ 6. CONCLUSION

Within the degree of accuracy of the present work the results of the investigation show that the value of the elastic constant is the same at right angles to the axis of a steel bar of diameter 1·5 in. as along the axis of the bar. In the case of a wrought-iron bar of diameter 1·5 in. it was found that the value of Young's modulus for the specimens across a diameter of the bar was not constant, and was less than the value obtained for the specimens co-axial with the bar; this is to be expected on account of the direction of the slag impurities in wrought-iron. Comparing the results of the experiments carried out on the short steel specimens with the results of the work done on the corresponding longer specimens, it is seen that the values of the elastic constant are slightly less; this, of course, is also to be expected on account of end effects. It is interesting to note that the values of Young's modulus for the bars nos. 1, 2, 3, and 4 agree with the corresponding values recorded in the author's paper dealing with the extensions of cylindrical shells under internal pressure, and verifies the accuracy of the experiments carried out by means of a larger tensile testing-machine on much larger specimens. It would also appear that in the author's previous paper his speculation as to the non-isotropic nature of the material as an explanation of certain discrepancies in the results is not justified by the present piece of work.

§ 7. ACKNOWLEDGMENTS

In conclusion the author thanks the Council of East London College and Prof. E. H. Lamb for the facilities which have been placed at his disposal in connection with the investigation here described.

DISCUSSION

Dr J. S. G. THOMAS referred to the fact that Young's modulus for a bar of wrought iron is less in the transverse than in the axial direction. The author attributed the difference to impurities in the iron. Presumably the slag inclusions would, as a result of hammering, arrange themselves parallel to the axis.

Mr T. SMITH asked whether the same extensometer was used for long as for short specimens, so that the effective lengths of all specimens were equal.

Mr A. G. WARREN: With such short specimens it is somewhat surprising that the author was able to obtain an accurate value of Young's modulus. There is a tendency in such cases for the skin to stretch more than the core. This can be shown for thin specimens by a comparison of reflection and transmission spectra.

Mr J. H. AWBERY: Mr Wedgwood's result is certainly not quite what would have been anticipated, and it is therefore all the more interesting that he should have given us the facts.

Doubtless one could find out from his earlier paper, but perhaps he will allow me to ask whether the thick cylinders used in the earlier work were simply bored out, or whether any further process was applied to them?

AUTHOR's reply: The same extensometer was employed in all the experiments described in the paper. The accuracy of the determination of Young's modulus was found to be largely dependent on the use of two finely cut scratches on the surface of the specimen; these served the purpose of registering the two pairs of knife-edges of the instrument at a distance of 0·625 in. apart on the bar. I was surprised to find so little difference between the value of Young's modulus for the shorter specimens and the longer ones; I expected that in the case of the shorter specimens there would be a tendency for the skin to stretch more than the core in the vicinity of the fillets.

In reply to Mr AWBERY: The thick cylinders of the earlier work were very carefully prepared, and were not subjected to any treatment in their preparation other than the ordinary machine operations of turning, boring, and rymering.

A REMOTE ELECTRICALLY-RECORDING ACCELEROMETER WITH PARTICULAR REFERENCE TO WHEEL-IMPACT MEASUREMENTS

By F. AUGHTIE, Ph.D., M.Sc.,
of the Engineering Department, National Physical Laboratory

Communicated by H. J. Gough, M.B.E., D.Sc., *May* 4, 1931. *Read November* 20, 1931.

ABSTRACT. The instrument was developed for the measurement of the acceleration of the rear axle of a vehicle. It was necessary for the recording to be done remotely in order accurately to phase three records, two of acceleration and one of spring load. A new method of remote recording has been developed for the purpose.

The paper describes the design of the instrument, its principle of operation, and also, though briefly, the auxiliary equipment. The undamped natural frequency is 300 ~ and the maximum reading is 25 *g*. Damping is provided by filling the casing with light mineral oil and, although less than critical, is sufficient to eliminate from the records any disturbance due to self-oscillation.

§ 1. INTRODUCTION

The increasing use of the roads for transport purposes, together with the increased loads carried by vehicles, has brought into importance the question of road-wear together with the allied problems of noise, vibration and deterioration of the vehicle. It is clear that the form and details of the vehicle suspension—springs and tyres—play an important part in the destructive action of traffic on the road, and in an endeavour to obtain definite data an extensive research has been initiated by the Ministry of Transport, the work being carried out at the National Physical Laboratory. There being no suitable instruments available for the work at the time of its commencement, attention has, up to the present, been confined to the development of such instruments, and in particular of instruments for the measurement of the wheel-impact force. The present paper describes the design and principle of operation of an accelerometer developed for the work.

§ 2. THEORETICAL CONSIDERATIONS

The measurement of the force occurring between the wheel of a vehicle and the road during the impact caused by an irregularity in the road surface may be made either from the road or from the vehicle. For the former method a section of the road can be mounted upon a force-measuring device, usually a spring of some form, arrangements being made to record either the peak value of the force or its variation with time. While this method is attractive in that the measurement is made directly, it has several serious drawbacks. The need for mounting a section

of the road upon a sprung platform precludes the use of actually existing road surfaces, and causes a discontinuity in the road surface immediately in front of the section under examination. Moreover the magnitude of the force between the wheel and road-surface depends upon the deformation of the tyre and road-material: hence, unless the stiffness of the experimental road section (as suspended) is equal to that of the actual road, the method will obviously give erroneous results. But even if this condition can be fulfilled, there is a further error arising from the fact that the mass and stiffness of the experimental road-section will be concentrated in the platform and supporting spring, while in the actual road they are distributed through a considerable volume of material and this renders it impossible to obtain a platform equivalent to the road for impacts of differing wave-form.

Thus it is clear that while measurement from the road is comparatively easy, experimentally, there are serious theoretical objections to its use.

Except in the case of sprung-rim wheels, which are little used in this country, the load imposed upon the road by the wheel is the (algebraic) sum of the force applied to the axle through the springs and that necessary to accelerate the axle and fittings. The force required to accelerate the axle can be calculated from the mass constants of the axle and its linear and angular accelerations. The acceleration of a single wheel is insufficient for the determination of the impact force because, in general, the two wheels are not mutual centres of percussion and oscillation. That is, if one wheel is running on a smooth level surface while the second encounters an obstacle, the force between the first wheel and the road will be affected by the movement of the second wheel. The linear and angular accelerations of the axle are most conveniently determined by measuring the linear acceleration of two points on the axle. For an accurate determination of the total load at one wheel it is further necessary to know the force imposed by both springs individually, but when the distance between the centre of the wheel and the point of application of the spring load is small compared with the centre-distance of the wheels, a single load is sufficient for a reasonably accurate determination of the total force.

The problem thus reduces to that of measuring a single force and two linear accelerations*, and since all three (more particularly the latter two) vary rapidly and irregularly with time, it is necessary to record their variation in such a way that the three records can be accurately phased. This essential, namely the ability to phase the records accurately, makes it desirable if not imperative to record them upon a common chart and this in turn renders it necessary to record remotely at least two and preferably all three quantities.

Apart from the difficulty of constructing a recording device to operate successfully under the conditions of vibration existing on the back axle of a vehicle, it was impossible to rely on rods or other mechanical devices for the transmission of the reading from the measuring instrument to the recording unit, on account of errors introduced by backlash, inertia and elastic strain of the parts, particularly

* It would be preferable to measure both spring loads, i.e. four quantities in all. Unfortunately standard recording apparatus has provision for only three and hence the scheme excludes the spring load on the opposite side to the wheel considered.

in view of the high magnification which will later be seen to be necessary. The transmission between the instrument and recorder-unit had therefore to be either hydraulic or electrical.

A hydraulic transmission offered attractions, but consideration showed that it could not be freed from risk of error due to thermal expansion, leakage and viscosity, and it was therefore abandoned in favour of an electrical transmission.

§ 3. PRINCIPLES OF THE METHOD

After consideration of the existing methods of remotely recording small displacements by electrical means, and an experimental trial of the only one which seemed likely to meet the conditions, they were rejected in favour of a method developed at the National Physical Laboratory for the purpose under consideration.

The fundamental principle of the method is illustrated in figure 1. Two coils P and S are mounted upon a laminated iron core, the magnetic circuit being completed by a laminated iron armature and a small air-gap whose length is determined by the quantity to be recorded. One of the coils P is supplied with alternating current of constant frequency and amplitude; the magnetic flux and hence

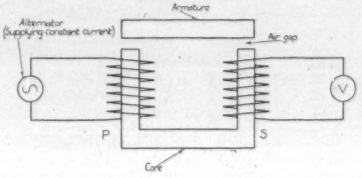


Fig. 1. Principle of operation of remote electrically-recording accelerometer.

the e.m.f. induced in the second coil S therefore depend upon the length of the air-gap, and a voltmeter connected to S will, if fitted with a suitably calibrated scale, indicate the magnitude of the air-gap and hence of the quantity to be measured. The voltmeter employed takes the form of a copper-oxide rectifier connected to an oscillograph which operates as a d.c. instrument, i.e. it does not follow the supply frequency which can therefore be raised to a value sufficiently high to give the required detail in the record. (The supply-frequency must, of course, not be so high as to render the rectifiers inoperative.)

It was considered preferable to use a constant primary current and measure the voltage (through the transformer), rather than to use a constant primary voltage and measure the current, because a power-supply of the frequency required was most conveniently obtained from a thermionic valve generator, and owing to the high internal resistance of this form of supply a constant current was obtained more readily than a constant voltage under conditions of variable load.

Trials with an experimental apparatus showed that the sensitivity of the arrangement fell as the frequency was increased owing, doubtless, to the reduced equivalent permeability of the iron core, and after consideration the supply-frequency was chosen as 1000 ~, this being about six times the anticipated natural frequency of

the accelerometer. Above that frequency accurate records would not be possible, for mechanical reasons.

The experimental apparatus showed also that there was a considerable gain in sensitivity when the rectifiers were operated with an increased current-output and consequently with a lower internal resistance, the zero of the oscillograph being correspondingly suppressed, preferably by electrical means.

It was further found that owing to variation of the primary self-inductance, the supply current varied perceptibly as the air-gap varied, even with a reasonable swamping resistance in circuit, and this change was such as to reduce the overall sensitivity. Although it was possible to use the change of inductance to give an increased sensitivity*, it was considered preferable to adopt in the final design a modification which would enable the primary current to be maintained more nearly constant.

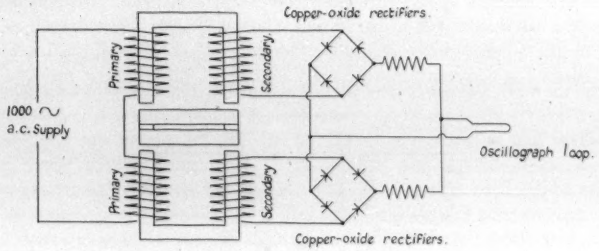


Fig. 2. Electrical connexions of remotely-recording accelerometer†.

This was achieved by a symmetrical disposition of a (double) armature between two similar cores and windings. With this arrangement, for displacements of the armature, which are small compared with the equivalent length of the magnetic path, the increase in the inductance of one winding is balanced by the decrease in that of the other. The use of two magnetic systems combined, as illustrated in figure 2, offers also several additional advantages. The output from the second rectifier can be used to set back electrically the zero of the oscillograph, thus obviating the need for a separate battery and resistance for that purpose; the sensitivity is doubled, since as the output current from one rectifier increases that from the other decreases; and when the two systems are so combined the curvatures in the individual calibration curves tend to neutralize each other, and a curve more nearly linear than either is obtained, thus facilitating the interpretation of the records.

* By connecting in series with the supply a condenser whose reactance was numerically greater than that of the winding, thus giving a decrease of the total primary circuit impedance as the inductance increased.

† Primary and secondary windings are here shown on one limb of each core; in the instrument they are distributed on both limbs.

§ 4. MECHANICAL ASPECTS OF THE PROBLEM

The fundamental form of an accelerometer comprises a mass mounted on a spring. The force necessary to accelerate the mass is transmitted through the spring and the ensuing deflection gives a measure of the force and hence of the acceleration. Since a relative movement of the mass and the system whose acceleration is to be measured is essential for the operation of the recording device, and the deflection of the spring is proportional to the acceleration of the mass itself, it follows that any accelerometer must always be inaccurate to a greater or less degree.

The extent of the error is seen from the following calculation:

Let M be the mass, and k the stiffness of the spring. M, k

Apply an acceleration $A \sin \omega t$ to the spring anchorage, then the displacement of the anchorage is $-(A/\omega^2) \sin \omega t$ and that of the mass itself is $-(A/\omega^2) \sin \omega t - x$, where x is the extension of the spring. A, ω, t x

Hence the acceleration of the mass is $A \sin \omega t - d^2x/dt^2$.

The force on the mass is kx.

Hence
$$kx = M(A \sin \omega t - d^2x/dt^2),$$
or
$$(k/M)\, x + d^2x/dt^2 = A \sin \omega t,$$
the solution of which is
$$x = P \sin (\omega_0 t + \phi) + A \sin \omega t/(\omega_0^2 - \omega^2),$$
P and ϕ being arbitrary constants while $\omega_0 = \sqrt{(k/M)}$. P, ϕ

The first term corresponds to the free oscillation of the mass on the spring, the second to the extension of the spring under the applied acceleration. Hence the observed deflection $x = (\omega_0^2 - \omega^2)^{-1} A \sin \omega t$. Only when ω^2 is small compared with ω_0^2 is this closely proportional to the applied acceleration.

The formula shows also that for unit acceleration the extension of the spring is ω_0^{-2}; hence as the natural frequency of the system is raised to permit of accurate response to acceleration of higher frequency, the displacement of the mass becomes smaller. For the case when $\omega_0 = 2000$ (corresponding to a natural frequency of just over 300 ~) the displacement in inches for an acceleration 1 in./sec.^{2} is $2{\cdot}5 \times 10^{-7}$ and for an acceleration equal to g (384 in./sec.^{2}) the displacement is approximately 0·0001 in. The magnitude of this displacement shows how futile any mechanical remote-recording device would be.

In practice it is necessary to provide damping to prevent excessive oscillation at the frequency of the system. This damping modifies the response at lower frequencies, and a treatment of the damped accelerometer is given in the appendix.

§ 5. MECHANICAL DESIGN

The design of the actual instrument presented some difficulties owing to the lack of information relating to the magnitudes and frequencies which had to be measured. American publications suggested that accelerations up to ten or twelve times gravity might be encountered, but no data were available as to the frequency

of the important components of the acceleration/time curve. The only practicable course was to design the instrument to have as high a natural frequency as possible consistent with the desired sensitivity, and to provide in the design for easy variation of this frequency by alteration of the stiffness of the springs.

The following requirements were therefore to be fulfilled in the final design: (1) A range up to 10 *g* or 15 *g*. (2) As high a natural frequency as possible. (The experimental apparatus required a displacement of 0·005 in. for full-scale deflection, but it was hoped to improve on this in the final design.) (3) The mass to be mounted so as to have a single degree of freedom. (In practice this means that the natural frequency of vibration along one axis must be much lower than that along either of the others and also than those of oscillation about any axis.) (4) The form of

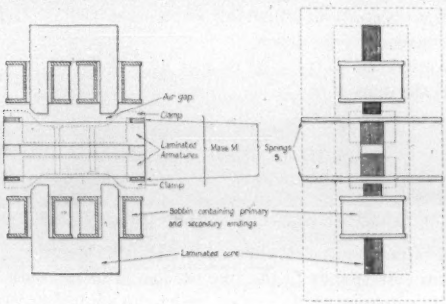


Fig. 3. General arrangement of accelerometer.

the spring to be simple and cheap so as to permit easy substitution of one spring for another of different stiffness. (5) Provision for damping the instrument, and for readily varying the degree of damping since this could not be predetermined. (6) If possible a shape which would move freely inside a cylinder 3 in. in diameter since this would enable the finished instrument to be calibrated by being mounted upon the top of a piston available in the laboratory. (7) A mechanical construction capable of withstanding the vibration to which the instrument would be subjected when mounted upon the rear axle of a vehicle. (8) The weight not to exceed about 10 to 15 lb.

The fundamental form of the design is illustrated in figure 3, the outstanding feature being the use of springs having the form of flat steel rings. Two such springs, their planes separated by a distance roughly equal to $\frac{1}{3}$ of their diameter, each clamped to the mass at the ends of a diameter, and to the frame of the instrument at the ends of a diameter at right angles, enable the armatures to be mounted with the required single degree of freedom. Their construction is simple and inexpensive, and the symmetrical shape minimizes the likelihood of their moving in the clamp and further results in any such movement having little effect upon the total stiffness.

The complete design is shown in figure 4. The body is divided into two parts 1, 2 spigotted and held together by four bolts 3. Each half is hollowed out to accommodate the laminated cores 4 and windings 5 and turned down to clear the springs 6 which are clamped to the lower half of the body by bolts 7 and fittings 8, 9, 10, 11. Clamps 12 hold the laminated armatures 13 to the springs, a loose packing piece 14 being inserted for constructional purposes. The laminations of the cores are clamped between two plates 15, these in turn being held down by cleats 16 which are made to fit both in the slot in the body and in the gap between the plates 15, thus preventing any side-movement of the cores.

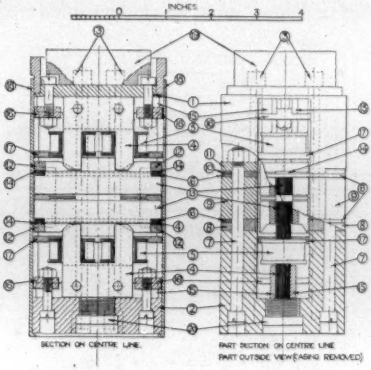


Fig. 4. Mechanical design of accelerometer.

The bobbins containing the windings are clamped with brass plates 17 suitably recessed to fit the bobbin flanges, and having a single slot for the accommodation of both limbs of the cores. Cut in this way, the plate does not link with the main flux, and any eddy currents induced in it are small and due to leakage flux only. The plate fits closely over the limb of the core and into the slot in the body, preventing side-movement of the top of the core. In addition to an oil-tight casing 18 and a terminal block 19, a plug 20 is provided in the base to permit filling the complete instrument with oil to obtain the required damping. Four tapped holes (not shown) are provided in the base for attachment to the axle.

Examination of the drawings shows how the various factors enumerated as affecting the design have been met in the finished instrument.

The choice of the dimensions of the original springs of the instrument presented some difficulty since not only was it difficult to calculate to any useful degree of accuracy the stiffness of the form of spring employed, but in addition the sensitivity of the final electrical arrangement was unknown. Rough calculations of the stiffness of the spring were made, the ring being treated as composed of eight cantilevers; the extra stiffness due to torsion was neglected. Such calculations were

of use in indicating a thickness of 0·1 in. as the maximum thickness likely to be required even for the higher ranges. The thickness specified was 0·07 in. for trial, this value being somewhat thicker than was expected to be necessary since it is clearly a simpler matter to reduce the thickness than make new springs of greater thickness. Fortunately the increased sensitivity of the final apparatus as compared with the experimental apparatus resulted in a suitable range and sensitivity of the instrument with the springs 0·07 in. thick, and these springs have therefore been retained.

The natural frequency of the suspended mass was determined by taking a record while the base was repeatedly struck with a block of rubber. The record so obtained is reproduced in figure 5, from which the natural frequency is seen to be 300 ~.

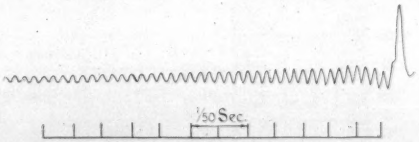


Fig. 5. Natural frequency of accelerometer (undamped) excited by striking base with block of rubber.

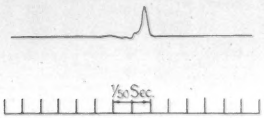


Fig. 6. As figure 5 but casing filled with oil (instrument damped).

Further experiments were made to determine a suitable damping oil; oscillations of the system were excited as before, and it was found that a light mineral oil gave a satisfactory damping-coefficient. The record obtained with the casing filled with this oil is reproduced in figure 6. The ratio of successive swings has fallen to 0·4 as compared with 0·985, when the casing was empty. At the same time the natural frequency has fallen to about 200 or 220 ~, but this figure is difficult to determine with any accuracy owing to the small number of oscillations appearing in the record.

§ 6. ELECTRICAL EQUIPMENT

On the electrical side, the auxiliaries may conveniently be described under three heads: (i) The power supply; (ii) the oscillograph; (iii) the fork and amplifier unit.

(i) The body of the experimental trailer which houses the equipment is divided into two parts, the smaller of which contains a 500-watt 12-volt petrol-electric generating set which supplies the whole of the electrical energy required, and a 12-volt battery which is floated across the lines to steady the supply in the event of slightly irregular running of the engine. (ii) The rear and larger compartment of

the trailer contains the remainder of the apparatus, and is constructed as a dark room so that records can be developed on the road if necessary. The oscillograph used for taking the records is a standard Cambridge three-element instrument, slightly modified to suit the particular conditions, and is mounted upon a sprung table to insulate it from the road shocks. The camera is driven by a $\frac{1}{8}$-h.p. 12-volt motor operated from the power supply. (iii) The appartus for converting the 12-volt supply into the 1000-cycle a.c. supply required by the accelerometer is contained, together with the rectifiers and other details, in an upright cabinet. The three upper shelves are practically identical except for slight changes in the arrangement of the components made to reduce the coupling between the circuits, and each contains a power valve, of the L.S.5*a* type, which supplies the current to

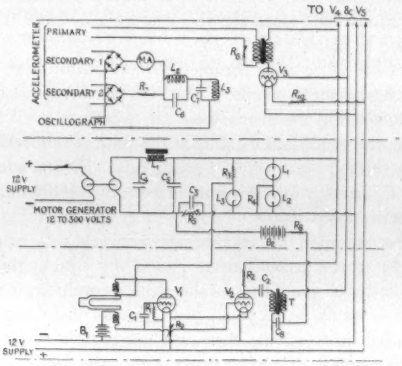


Fig. 7. Wiring diagram of amplifier for 1000~ supply.

the primary winding of the accelerometer through a suitable output transformer. The grid of the valve is excited through a single-valve amplifying stage from the grid coil of a valve-maintained tuning fork, a separate valve being used to sustain the oscillations. These valves together with the fork and coupling units are mounted on a fixed shelf in a cupboard at the bottom of the cabinet. The filaments of the valves are heated from the 12-volt supply through suitable resistances, and h.t. voltage is obtained from a small (permanent-magnet-field) motor-generator with an output of 120 mA. at 300 V.

The complete wiring diagram, excluding two power shelves which are similar to the power shelf shown, is given in figure 7. The grid of the fork-maintaining valve is self-biased with the resistance and condenser, while bias for the amplifying valve is obtained from a small battery B_1. The amplifying valve feeds the grids of the power valves through the resistance-condenser-transformer arrangement R_2, C_2, T.

The output from the motor-generator is smoothed with condensers and a

choke in the conventional manner. To minimize voltage fluctuations due to irregular running of the prime mover two neon lamps L_1 and L_2 connected in series are placed across the h.t. mains and a resistance R_5 is inserted in the negative lead. The voltage drop across this resistance, supplemented by a battery B_2, is used as grid bias for the power stages. To reduce the striking voltage of the two lamps in series, a high resistance R_4 is connected across one of them. The anode supply for the fork-maintaining valve is taken from the h.t. positive main through a resistance R_3, a third neon lamp L_3 being connected in parallel to stabilize further the voltage of the supply to this valve. A resistance R_6 is included in the output circuit to permit the adjustment of the accelerometer current to any desired value.

The rectifiers and other parts in the secondary circuits of the accelerometer are located on the same shelf as the output transformer and power valve. The essentials of the circuit have been shown in figure 2 and the full diagram of connexions is included in figure 7.

As compared with figure 2 the complete circuit exhibits two important modifications. (i) The replacement of one of the resistances of the bridge circuit by a milliammeter. By altering the primary circuit resistance until this meter shows a definite deflection, the conditions existing at the time of calibration can be repeated without reference to the primary circuit conditions. (ii) The addition of two tuned filter circuits L_2, C_6 and L_3, C_7. The output from the rectifiers is not pure d.c. but has a strong superimposed ripple of 2000 ~ together with higher harmonics. It was originally the intention to discriminate between this ripple and the lower frequencies proper to the accelerometer record, by tuning the oscillograph to a frequency of the order of 500–600 ~, under which condition it was hoped that the response to 2000 ~ would be so small as to give no visible ripple in the record. Experiments showed however that there was perceptible ripple of this frequency, and also that owing to lack of perfect symmetry in the rectifiers there was a 1000-cycle component present. To eliminate these two frequencies from the oscillograph record, two tuned filter circuits were placed in series with the instrument as shown in the diagram. Air core inductances of low d.c. resistance were used, tuned with paper condensers of capacities 2 and 4 μF respectively for the 2000 and 1000-cycle circuits. No attempt was made to construct coils of the lowest possible a.c. resistance, because a sharply tuned circuit is undesirable since the ripple is modulated by the acceleration/time wave-form. For the same reason paper condensers were used instead of those having a mica dielectric. With these filter circuits in use there is no perceptible ripple in the record of either 1000 or 2000 ~.

There are three power shelves with rectifiers, filter circuits, etc. Two of these are used for the two accelerometers which have been shown to be necessary for the complete determination of the impact-force on the axle; the third is provided for the load-gauge which is used for determining the load due to the weight of the vehicle and load. For this instrument the same method of remote recording is used as for the accelerometers.

§ 7. CALIBRATION

Two methods of calibration have been employed, the static and the dynamic.

(i) *Static.* Known weights, increasing in units equal to that of the moving system, were suspended from the moving system of the accelerometer, and a photographic record was exposed for each increment of load. A series of lines giving a scale of acceleration in units of gravity was thus obtained. For convenience in calibration and interpretation this unit of acceleration has been retained throughout. The calibration was repeated for different values of circulation current in the rectifier bridge circuit, and thus a series of curves was obtained permitting the variation of sensitivity over a small range.

(ii) *Dynamic.* The instrument was mounted on the top of a piston actuated by a crank and connecting rod of known dimensions. A photographic record was taken when the crank was rotated at a speed of approximately 350 r.p.m., the actual speed being determined from the time required for one revolution of the crank as deduced

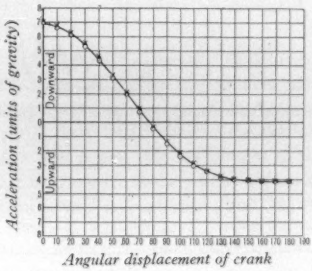

Acceleration (units of gravity)
Downward
Upward
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190
Angular displacement of crank

Fig. 8. Calibration test. Curve is plotted from calculated values (not shown). Points are plotted from record obtained from accelerometer and converted into acceleration by use of calibration curve.

from the accelerometer record. Values of acceleration determined from the record so obtained by the use of the calibration curve obtained from static loading were then compared with the calculated values of acceleration of the piston. The agreement is shown in figure 8, in which the curve is plotted from calculated values of acceleration and the isolated points are plotted from the curve of observations. It will be seen that the agreement is very good, the slight difference being due to a small error in locating the top dead-centre position of the crank in the accelerometer record. If one set of points were moved slightly to the right and the other a corresponding distance to the left they would both lie almost exactly on the calculated curve. This readjustment has not been made because the comparison can be made more easily in the figure as drawn than if the points were plotted exactly on the theoretical curve. As the agreement between the two calibrations was so satisfactory, subsequent calibrations rendered necessary by alteration of circuit conditions, etc. were made by the static method only, since this was vastly more convenient in practice.

§ 8. RESULTS OBTAINED WITH THE INSTRUMENT

As yet the instrument has not been used upon the road but it has been used for some preliminary experiments in the laboratory, with very satisfactory results*. In these experiments a long arm was hinged at one end and carried a wheel fixed at the centre of percussion of the system: the accelerometer was attached to the beam at a position as close to the wheel as possible. The wheel and beam were then raised about the hinge and allowed to fall through known heights, the tyre of the wheel striking a concrete floor on impact. Records of the acceleration of the system were taken during the fall and impact and permitted a comparison to be made of the properties of various types of tyre, etc. It will be seen that the conditions of operation of the instrument were very similar to those under which it will have to operate when upon the road.

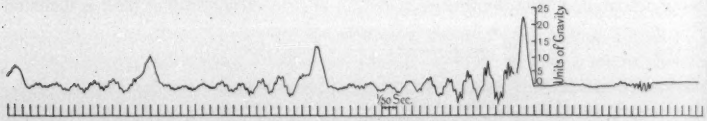


Fig. 9. Successive impacts of a solid-tyred wheel. (Height of fall 4·97 in.)

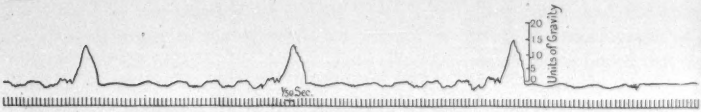


Fig. 10. Successive impacts of a pneumatic tyred wheel. (Height of fall 13·27 in.)

Typical records are reproduced in figures 9 and 10. Referring to figure 9, which is the record obtained from the impact of a solid-tyred wheel falling through 4·97 in., the first break in the curve is due to the release of the beam, and this sets up vibrations of the beam itself which die out after a few hundredths of a second. The displacement of the mean line shows that the accelerometer is falling with an acceleration equal to gravity. The impact of the tyre and ground is easily recognizable, and the shock sets up violent oscillations of the beam which persist during rebound and second fall up to the time of the second impact. This is easily recognizable by the increased amplitude of the oscillations following it.

Figure 10 is a record from a pneumatic tyre falling through 13·27 in. It will be noted that the rate of rise of acceleration and the maximum value attained are both less than the values for the solid-tyred wheel and that the time of contact of the tyre and ground is correspondingly longer. The oscillations of the beam initiated by the impact are less violent also.

* The instrument has subsequently been used upon the road with equally satisfactory results.

§ 9. CONCLUSION

The instrument and equipment described embody a new method of remotely recording small movements which is capable of rapidly and accurately following changes within the limits of the apparatus. In the particular form employed it gives a magnification of the order of 1000 with a range of movement of 0·002 to 0·003 in. at frequencies up to 300 ~. This latter figure could be increased, with a loss of sensitivity due to an increased supply-frequency and reduced oscillograph-sensitivity.

The object for which this method was devised and to which it has so far been applied is the production of an accelerometer with a natural frequency of 300 ~ and a range up to 25 *g*. The design of this instrument has been fully described in the present paper, but it is obvious that other applications are possible. In particular the load-gauge required for the completion of the experimental equipment uses the same method; it is described in the subsequent paper*.

§ 10. ACKNOWLEDGMENTS

The author desires to acknowledge the interest taken in the work by the Ministry of Transport, for whose programme of research the instrument was developed, and to tender thanks to them, and to the Director of the National Physical Laboratory, for permission to publish this paper. He wishes also to thank Mr J. H. Hyde, of the National Physical Laboratory, under whose supervision the work was performed, for much helpful advice and criticism, and Mr A. F. C. Brown for considerable assistance in the experimental work with the complete accelerometer.

APPENDIX

FREQUENCY RESPONSE OF AN ACCELEROMETER WITH DAMPING

Suppose the damping to be provided by a dash-pot.

Let y be the displacement of the frame;

z the displacement of the mass;

k the stiffness of the spring;

μ the resistance of dash-pot in units of force per unit velocity; and

M the mass.

Then the compression of the spring is $y - z$ and the velocity of the dash-pot is $\dot{y} - \dot{z}$.

The force on the mass is

$$(y - z)\,k + (\dot{y} - \dot{z})\,\mu.$$

Equating to $M\ddot{z}$ and reducing, we have

$$\ddot{z} + \frac{\mu}{M}\dot{z} + \frac{k}{M}z = \frac{\mu}{M}\dot{y} + \frac{k}{M}y.$$

* See page 45.

To determine the variation of response with frequency put

$$y = A \sin \omega t, \qquad z = C \sin \omega t + D \cos \omega t,$$

so that

$$-C\omega^2 \sin \omega t - D\omega^2 \cos \omega t + \frac{\mu\omega}{M}(C \cos \omega t - D \sin \omega t) + \frac{k}{M}(C \sin \omega t + D \cos \omega t) = \frac{\mu\omega}{M} A \cos \omega t + \frac{k}{M} A \sin \omega t.$$

Equating coefficients of sine and cosine terms and solving for C and D we have

$$C = A\left\{\frac{k}{M}\left(\frac{k}{M} - \omega^2\right) + \left(\frac{\mu\omega}{M}\right)^2\right\} \Big/ \left\{\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right\},$$

$$D = A - \frac{\mu\omega}{M}\omega^2 \Big/ \left\{\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right\}.$$

The displacement of the spring, which is the observed quantity,

$$= y - z = (A - C) \sin \omega t - D \cos \omega t = \{(A - C)^2 + D^2\}^{\frac{1}{2}} \sin (\omega t + \phi).$$

On substitution for C and D and evaluation of the coefficient,

$$(\text{Coefficient})^2 = A^2 \frac{\left[\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2 - \frac{k}{M}\left(\frac{k}{M} - \omega^2\right) - \left(\frac{\mu\omega}{M}\right)^2\right]^2 + \left[\frac{\mu\omega}{M}\omega^2\right]^2}{\left[\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right]^2}$$

$$= A^2 \frac{\omega^4}{\left\{\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right\}}.$$

The applied acceleration $= -A\omega^2 \sin \omega t$. Hence

$$\frac{\text{applied acceleration}}{\text{observed deflection}} = \sqrt{\left\{\left(\frac{k}{M} - \omega^2\right)^2 - \left(\frac{\mu\omega}{M}\right)^2\right\}} \times \frac{\sin \omega t}{\sin (\omega t + \phi)}.$$

Hence on comparison of amplitudes only,

$$\text{Applied acceleration} = \sqrt{\{(\omega_0^2 - \omega^2)^2 + \alpha^2\omega^2\}} \times (\text{observed deflection})$$

$$= \sqrt{\left[\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \left\{\frac{\alpha\omega}{\omega_0^2}\right\}^2\right]} \times \omega_0^2 \times (\text{observed deflection}),$$

where $\omega_0^2 = k/M$ and $\alpha = \mu/M$.

But $\omega_0^2 \times$ (observed deflection) is the value of acceleration obtained from static loading, i.e. is the observed acceleration.

Hence the true acceleration is obtained by multiplying the observed values by the factor

$$\sqrt{\left[\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \left\{\frac{\alpha\omega}{\omega_0^2}\right\}^2\right]}.$$

DISCUSSION

For discussion see page 49.

A REMOTE ELECTRICALLY-RECORDING LOAD-GAUGE FOR WHEEL-IMPACT MEASUREMENTS

By F. AUGHTIE, Ph.D., M.Sc.,
of the Engineering Department, National Physical Laboratory

Communicated by H. J. Gough, M.B.E., D.Sc., *July* 23, 1931. *Read November* 20, 1931.

ABSTRACT. The paper is supplementary to the previous paper and describes the load-gauge designed for measuring that component of the wheel-load which arises from the sprung-weight.

§ 1. INTRODUCTION

In the preceding paper* it was shown that the impact-force between the wheel of a vehicle and the road can be determined from a knowledge of the linear and angular acceleration of the axle, and the load imposed upon it by the vehicle body. The present paper supplements the one cited and describes the load-gauge designed for measuring that component of wheel-load which arises from the sprung-weight.

There were two alternative positions for the measuring spring: (i) between the chassis and the suspension spring, and (ii) between the suspension spring and the axle. The former position gave greater space for the instrument and considerably simplified the conditions to be met, in that, by being located in the spring shackle, the instrument would be called upon to withstand pure compression loads only. This location was however inferior to (ii) for theoretical reasons. The suspension spring is attached at its centre to the axle and at its ends to the chassis; consequently a portion of the spring moves with the axle, thereby increasing its moment of inertia. The equivalent mass of the spring, i.e. that proportion of the mass which effectively moves with the axle, depends upon the deflection curve of the spring, which in turn is affected by friction between the leaves, etc., and hence can neither be measured nor calculated with any high degree of accuracy. Thus, at the expense of increased complexity and cost, it was considered desirable to insert the measuring unit between the axle and suspension spring if possible.

§ 2. DESIGN OF APPARATUS

This location was achieved by the use of a measuring spring of the form shown in figure 1. The top centre face carries the suspension spring, while the ends are carried from the axle by two split brackets, one on either side. To permit free deflection of the spring the hole in its centre is made of larger diameter than the axle.

* See page 31.

The deflection is observed and recorded by the use of an electrical equipment operating upon the same principle as the accelerometer and attached to the three lower faces of the spring. In this way errors due to slipping or distortion of the loading clamps are entirely eliminated.

The spring was machined from a solid forging of nickel-chrome steel, as it was not considered possible to build up the form required without grave risk of error due to relative movement of the parts. After hardening the spring was ground all over to a depth of approximately 0·005 in. to remove the surface metal and scale.

Under normal load the deflection is approximately 0·005 in. and hence the suspension system of the vehicle is inappreciably affected by the insertion of the instrument. The mass of the axle is increased somewhat, but not to any serious extent. Further, measurements will during the early work be confined to a trailer, the rear axle of which is considerably lighter than that of a self-propelled vehicle.

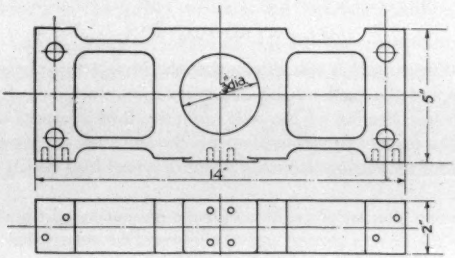


Fig. 1. Measuring spring.

With reference to figure 2, which shows a part section and inverted plan of the complete arrangement, the spring 1 carries an aluminium bridge casting 2 attached by studs 14; this in turn carries the armature 3 mounted on a steel facing 4 and clamped with studs 15 and a pad 5. The cores 6, fitted with wound bobbins, are mounted in the two halves 7, 8 of the body, which are held on to the centre face of the spring by four nickel steel studs 9. The mode of fixing the cores and bobbins is exactly the same as that employed in the accelerometer. Entry of rain and dirt is prevented by a rubber diaphragm 10 which is clamped between the body and a distance piece 11 and permits relative movement of the body and aluminium bridge. Weather-proofing is completed by a sheet-brass cover (not shown) which is fitted to the bottom of the bridge.

The spring itself is carried from the axle 12 by split brackets 13 to which it is held by four bolts 18. The brackets themselves are clamped on the axle by bolts 20.

The suspension spring of the vehicle (not shown) is held down to the top face of the measuring spring 1 by four bolts 16 anchored in a plate 17. Connexion to the windings is made through a terminal block 19; and to permit easy separation of the two halves of the body 7 and 8, plug and socket connectors are interposed in the leads to the winding on the upper half 7.

One feature of the design, of value in view of the fact that the maximum load is not known to any high degree of accuracy, is that in the event of failure of the measuring spring the vehicle will fall on one side by an amount equal to the clearance between the axle and the centre-hole of the spring. Thus although such a mishap would wreck the load-gauge it would not cause disaster to the whole vehicle.

§ 3. CALIBRATION

The stiffness of the spring unmounted was determined from mechanical measurements before the remainder of the instrument was assembled and the nominal value was approximately 550,000 lb./in. On assembly some discrepancy was apparent between the mechanical and electrical measurements; it found that the

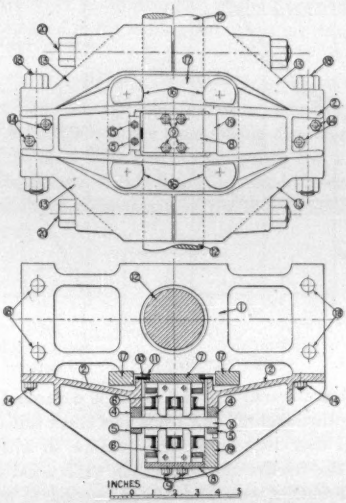


Fig. 2. Complete load-gauge. Above: inverted plan. Below: part section with supporting brackets 13 removed.

load necessary to bring the armature to the central position, as deduced from electrical measurements, was different from that predicted from the measured stiffness of the spring and length of the air-gaps. This disagreement was ultimately found to have three contributory causes: (i) slight differences between the areas of the gaps, (ii) a slight error in the length of the gaps and (iii) a change in the stiffness of the spring when clamped. It so happened that all these causes had cumulative effects, and the discrepancy was therefore more difficult to trace than it would have been if due to a single cause.

A feature of more serious importance was that the calibration curve showed a

hysteresis loop, the width of which amounted to 5 or 6 per cent. of the total load-variation. While this was not so large as to prohibit the use of the instrument—particularly as it would represent a much smaller proportionate error in the wheel load—it was considered too great to be accepted without some attempt at reduction of its magnitude. After several experimental calibrations the hysteresis was found to be purely mechanical and was traced to the influence of the clamping brackets 13, figure 2, and figure 3 shows the calibration curves obtained with the bolts 18 tight and slack respectively.

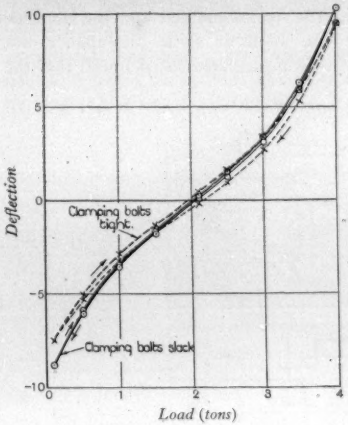


Fig. 3. Preliminary calibration.

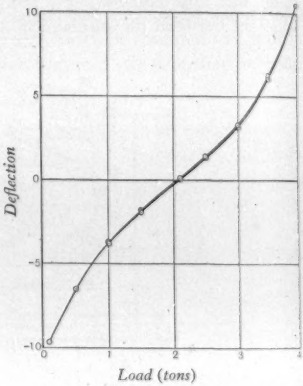


Fig. 4. Calibration with rubber sheet between spring and clamps.

It is clear that when the centre of the spring is depressed under load the ends must approach each other slightly, and while this effect was not overlooked in the design, it was hoped that the magnitude would be so small as not to demand special provision for this relative movement. The calibration showed that this hope was not realized, and while it would have been possible to fit an elastic mounting at one end, a solution, at once much cheaper, simpler, and quite satisfactory, was obtained by interposing between the spring and the faces of the clamps 13 a piece of thin sheet rubber about $\frac{1}{16}$ in. thick. This permitted the slight breathing movement required, and reduced the hysteresis to less than 1 per cent. A layer of tinfoil was inserted between the spring and the rubber to prevent the sulphur from the latter from diffusing into the steel. The final calibration curve is reproduced in figure 4. It may be added that the shape is due to the electrical recording device and that the load-deflection curve of the spring is practically straight over the working range*.

* As in the case of the accelerometer, it proved difficult to predict the stiffness of the spring owing to the large radii left in the corners. The deflection was actually greater than had been anticipated and the air-gaps had therefore to be lengthened. This circumstance is responsible for the increased curvature of the calibration curve as compared with that of the accelerometer.

§ 4. CONCLUSION

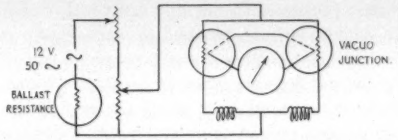
To avoid much repetition in describing the mode of operation, this paper is presented with the previous one instead of being delayed until experimental results have been obtained. It is hoped to publish shortly results obtained with the complete equipment.

§ 5. ACKNOWLEDGMENTS

The author wishes again to acknowledge the kindness of the Ministry of Transport and the Director of the Laboratory in granting permission to publish this paper. Acknowledgment is gratefully made of the help given by Mr J. H. Hyde, of the National Physical Laboratory, by advice and criticism of the design, and of that given by Mr A. F. C. Brown, also of the Laboratory, who carried out the bulk of the work involved in the calibration of the instrument.

DISCUSSION

Mr A. F. DUFTON: The measurement desired is that of the force between one of the back wheels of a vehicle and the road during the impact caused by an irregularity in the road surface. The instruments which the author has described will record only one component of this force. I should like to know whether it is proposed to ignore the other components in the investigation of the destructive action of traffic on a road.



A somewhat analogous method of remotely recording small displacements by electrical means was developed some years ago at the Building Research Station. As in the author's method, displacement of the armature, which in this case is pivoted, increases the inductance of one coil and decreases that of the other. Both coils are supplied with alternating current and in series with each coil is placed the heater of a vacuojunction. The two thermojunctions are opposed and the resultant current is measured on a galvanometer as shown in the accompanying figure. Steadiness in the supply is secured by means of an iron ballast resistance. The calibration of the apparatus showed that the change in galvanometer current was sensibly proportional to the displacement of the armature.

Mr R. S. WHIPPLE: The fact that a small and rapid mechanical movement can be transmitted to an oscillograph opens up a large field of usefulness. Mr E. B. Moullin used a very similar method for recording the variations in the torque of a steamship shaft, the sensitive element of the device being mounted in the tunnel of the ship, and the recording oscillograph in the engine room.

AUTHOR'S reply: Mr Dufton has drawn attention to a point which was not stressed in the paper, namely, that the instruments record only the force in a vertical direction or, more accurately, in a direction perpendicular to the average contour of the road. It is probably clear that this is all that was attempted. Measurement of forces in a direction parallel to the road (tractive effect and skidding) has been the subject of two other investigations. Since the vertical force is so much larger than the other two it is probable that it plays the principal part in breaking up the road, but now that equipment is available for measuring the various quantities concerned we may hope for definite evidence on this point in the future.

The arrangement of remote recording described by Mr Dufton is of interest and has several points in common with that described in the paper, notably the conversion from a.c. to d.c. and the balanced arrangement of two similar circuits. I imagine, however, that the lag of the thermojunction would render it impossible to follow changes more rapid than, say, 1 ~. Also the output power available for operating the indicating instrument is much less than that given by a copper-oxide rectifier.

Mr Whipple refers to the Moullin torsion-meter, the forerunner of many methods of remote recording. Practically all the successful methods of remote recording introduce an a.c. carrier which is modulated by the wave-form to be recorded*.

The differences between the various methods lie in the manner of modulation and demodulation. Apart from the use of the copper-oxide rectifiers, I consider that the most important feature of the method described in the present paper lies in the fact that the equipment was designed as a whole; for instance, the exact form of electrical arrangement employed was determined in part by the nature of the source of a.c. supply, and there are other similar interrelations. I should perhaps mention that a copper-oxide rectifier has also been used in America for remote-indicating purposes†, though only in conjunction with a deflection instrument for visual observation. It is rather curious that, for recording, use was made of an oscillograph with an a.c. field system, since this obviously calls for a very much larger power-supply.

* Even broadcasting may be included.

† "A magnetic strain gauge," *Proc. Am. Soc. Testing Materials*, **30**, 1041–47. Reference to this paper and the *Annual Report* of the N.P.L. for 1929 and 1930 shows that the ideas were conceived quite independently.

THE BAND SPECTRUM OF ZIRCONIUM OXIDE

By F. LOWATER, Ph.D., F.R.A.S.

Communicated by Prof. A. Fowler, F.R.S., *September* 2, 1931. *Read November* 20, 1931.

ABSTRACT. (i) The spectrum of ZrO has been photographed from λ 2600 to λ 8800, and bands have been found to extend from λ 3200 to λ 7600. (ii) The most prominent bands have been analysed into three systems, all having the same lower electronic state; the blue system α is probably due to a transition $^3\Pi \rightarrow {}^3\Pi$; the yellow system β and the red-infra-red system γ are probably due to transitions $^3\Sigma \rightarrow {}^3\Pi$. (iii) Analysis of the remaining bands is in progress.

§ 1. INTRODUCTION

After publication of the band systems of titanium oxide* it was deemed desirable to analyse the band systems of the analogous molecule, zirconium oxide, the metal of which occurs in the fourth column and fifth row of the periodic table, whereas titanium is in the same column but the fourth row.

Kayser† gives a brief account of the earlier investigations of the band spectrum of compounds of zirconium. Hagenbach and Konen in 1905 observed that the spark spectrum of Zr was full of bands. In 1910 Eder and Valenta published the wave-lengths of thirty-two heads, distributed through the red, orange and yellow regions, λ 6612 to λ 5552, and of one other head at λ 4640, all being degraded toward the red. In the same year Bachem also observed this band spectrum, but published the wave-lengths of only eight heads ranging from λ 6508 to λ 5718 in the visible region; however, he also attributed to the same source five other bands in the ultra-violet, ranging from about λ 3090 to λ 2178, degraded toward the further ultra-violet; he questioned the two of shortest wave-length.

In 1922 Merrill‡ published the results of his investigation of a group of long period variable stars, of which the spectra were in part characterized by strong absorption bands in the blue and red regions; they differed however from stars of class *M*, since their spectra did not contain the titanium-oxide bands with heads at $\lambda\lambda$ 4761, 4954, 5168 Å. He remarked that the most characteristic feature of their spectra was a complicated structure between λ 4630 and λ 4660, consisting of emission and absorption lines and probably containing one or more band-heads. He identified this part of the spectrum as due to zirconium, by comparing it with that obtained from a zirconium compound in the arc. He concluded that these stars probably formed a third branch of the giant sequence *BAFG*, and were related to stars of class *M* more nearly than *N*. The International Astronomical

* F. Lowater, *Nature*, **123**, 644 (1929); *Proc. Phys. Soc.* **41**, 557 (1929); *Phys. Rev.* **33**, 701 (1929); *Nature*, **123**, 873 (1929); *Astrophys. J.* **70**, 1 (1929).

† *Handbuch d. Spec.* **6**, 864.

‡ *Astrophys. J.* **56**, 457 (1922).

Union (1922) suggested that this class should be designated *S*. Examples of these stars are R Andromedae, T Camelopardalis, R Lyncis, T Sagittarii and R Cygni. The following year Merrill* pointed out an interesting parallel relationship, namely, that while Ti and Zr occupy analogous positions in the periodic table, TiO bands predominate in the *M*-type stars and Zr in a closely related type. He further remarked that stellar observations indicate that the ZrO bands are produced at a higher temperature than the TiO bands.

Merrill's conclusion in regard to temperature was confirmed by Dr A. S. King†, of Mount Wilson Observatory, from his experiments on the bands of Ti and Zr produced in the electric furnace. He found that the bands of TiO appeared at a temperature of about 1900° C.; from a mixture of Ti and Zr in a stream of oxygen TiO bands were very strong at 2200° C., but no ZrO bands appeared; while at 2550° C. the TiO bands were so strong as to make doubtful the existence of the ZrO bands. With Zr alone in the oxygen stream no ZrO bands appeared at a temperature of 2200° C., but at 2550° C. the whole of the ZrO bands appeared, although they were faint compared with those of TiO at that temperature. He concluded that the lower limit of temperature for the appearance of ZrO bands was between 2400° C. and 2500° C. In these experiments King showed that the emitter of the bands was an oxide of Zr.

Not only is the ZrO spectrum characteristic of *S*-type stars, but from his work on "Molecular Spectra in Sunspots," R. S. Richardson‡ obtained evidence that ZrO is included in the sunspot spectrum. He arrived at this conclusion by comparing the sunspot spectrum with the laboratory spectrum of the ZrO band at λ 6474; although he studied other bands, he drew his conclusion from this band because it is very strong and more favourably placed than others for identification, since the sun's spectrum in this region has comparatively few atomic lines and the only other molecular lines belong to a weak TiO band at λ 6478.

Although these ZrO bands are thus well known, no analysis of their structure has yet appeared.

The most prominent bands of the spectrum, and therefore doubtless those most characteristic of the molecule, have been analysed by the writer into three systems of triplets, the systems being designated α, β and γ and the members of the triplets *a*, *b* and *c*. These bands are all degraded toward the infra-red, and therefore the heads must be those of *R* branches. A second branch is very obvious in systems β and γ. Systems α and γ are analogous to the α and γ systems of TiO, but are displaced toward the shorter wave-lengths, and the intervals between the members of the triplets are greater. System β of ZrO, in the yellow, is a triplet system also and, as will be shown later, has the same lower electronic state as systems α and γ.

Less conspicuous bands, of which about half occur in the regions occupied by systems α, β and γ, have not yet been analysed into a system related to the latter; in the ultra-violet region there is a system differing in general appearance from

* *Publ. Astron. Soc. Pac.* **35**, 218 (1923).
† *Publ. Astron. Soc. Pac.* **36**, 140 (1924).
‡ *Astrophys. J.* **73**, 216 (1931).

α, β and γ, for the bands are much weaker and the triplets not easily recognizable. Comparison of these less conspicuous bands with those of titanium oxide and of hafnium oxide measured by Meggers* and by King† shows that they are not due to these oxides, which might be present as impurities.

Search has been made for vibrational isotope displacements, but with two exceptions none has been found; hence those two are probably accidental coincidences between observed wave-numbers and calculated isotopic displacements.

§ 2. EXPERIMENTAL PROCEDURE

Arcs. The source of the spectrum was chemically pure zirconium oxide on the positive pole of either a Pfund arc, or a similar one with copper poles. A current of about 5 amp. was found most effective in maintaining the bands from the iron arc, but about 9 amp. were required with that of copper. Comparison of this current of 5 amp. with that most effective in maintaining the TiO bands, namely, 2·5 amp., shows that it is in accordance with the results obtained by Dr King in his investigation of these bands by means of the furnace.

Spectrographs. Spectrograms of the red and infra-red regions were obtained by means of a Littrow spectrograph having a dense glass prism of 30°, the dispersion of which ranged from 8·6 Å./mm. at λ 5600 to 35·5 Å./mm. at λ 8700. With this spectrograph panchromatic, kryptocyanin and neocyanin plates were used and the spectrum photographed from λ 4300 to λ 8800. Spectrograms were also taken by means of a 10 ft. Rowland grating in the first order from λ 2600 to λ 6600, in the second order from λ 2600 to λ 3400 and from λ 5030 to λ 6600. With this instrument the photographic plates used were Imperial Ordinary and Ilford panchromatic. Thus with the two instruments the spectrum has been photographed from λ 2600 to λ 8800 and bands have been found to extend from λ 3200 to λ 7600.

Wave-lengths and wave-numbers. The positions of the heads of the bands were measured in the usual manner, and their wave-lengths were determined in international angstroms by reference to those of iron lines, recommended as secondaries by the International Astronomical Union‡ wherever available; and otherwise to wave-lengths published by Meggers and Kiess§. Many iron lines appeared within the band spectrum and were used as standards; they enabled one to avoid errors that might otherwise arise from a slight shift between the band and a comparison spectrum. In cases where these iron lines were not sufficiently strong, the lines of the adjacent comparison spectrum were used, any shift being checked or corrected by means of the iron lines which appeared in both the band and comparison spectra. The wave-lengths of the heads of bands which were hidden by iron lines in the band spectrum were determined from spectrograms taken with the copper arc. The numerous atomic Zr lines included in the spectrum were identified by

* *Scientific Papers, Bur. Stan.* No. 8,151 (1928).
† *Astrophys. J.* **70**, 113 (1929).
‡ *Trans. Int. Astr. Union*, **3**, table 1 (1929).
§ *Scientific Papers, Bur. Stan.* No. 479 (1924).

reference to the tables of the wave-lengths of Zr lines published by King* and by Kiess†. The wave-numbers of the heads, reduced to vacuum, were obtained by the use of Kayser's‡ table.

§ 3. STRUCTURE, MULTIPLICITY AND INTENSITY

From the red-infra-red region shown on plate 2 it is evident that the sequences are triple and that system γ is, in the main, formed of three sequences for which $\Delta v = 0$, three for which $\Delta v = +1$ and three for which $\Delta v = -1$. The intensity in the sequences decreases rapidly, but before the sequence ($\Delta v = 0$) of the first member (a) has faded out, that of the second (b) has overlapped it; similarly, the second is overlapped by the third (c). Systems α and β likewise exhibit this triple constitution and overlapping.

This overlapping and complexity of structure make it difficult to follow the sequences to bands of the higher quantum numbers and to form a correct estimate of the relative intensities. It is, however, fairly certain that the intensity of the third member of a triplet is greater than that of the second and that of the second greater than that of the first.

As has been mentioned above, the prominent heads belong to R branches; it will be seen that in the ($v' - v'' = 0$) sequences of the β and γ systems each sub-band has a second branch beginning near the R head, but without rotational analysis it cannot be determined with certainty whether these are Q or P branches. However, close examination of enlarged prints in the second and third orders suggests P branches, for the lines of the R branch, having become comparatively weak, can be easily traced amid the much stronger lines of the second branch; also the lines of the latter are apparently not packed together sufficiently closely to form a Q head. This opinion is formed from the a, b and c members of the 0, 0 band in system γ and the c member of the 0, 0 band in system β. (See plate 2, first, third and fourth strips.) Nevertheless, in system β the b member of the 0, 0 band presents the appearance of a head at the beginning of the second branch. In other cases overlapping of bands renders the character of the second branch obscure, so that its determination must await rotational analysis, which will be a formidable problem even when spectrograms of sufficiently high resolving power have been secured.

§ 4. OBSERVATIONAL DATA

The data of all the true heads are collected in table 1, but held in reserve are some apparent heads which need further investigation to determine their true character; they may be weak heads or mere coincidental grouping of band-lines due to overlapping of several branches. A study of enlarged prints of third-order spectrograms has given much aid in the ascertainment of the constitution of those analysed, and of others included in the table.

* *Astrophys. J.* **65**, 86 (1927).
† *Scientific Papers, Bur. Stan.* No. 548 (1927).
‡ *Tabelle d. Schwingungszahlen* (1925).

Column 1 contains the wave-lengths with estimated intensities in parentheses, column 2 the wave-numbers reduced to vacuum. In column 3 are the vibrational quantum numbers, v', v'', with prefixed letters indicating the system, and suffixed letters the branches and members of the triplets. Prefix α indicates the blue system which extends into the violet and yellow, β the yellow and γ the red-infra-red system. R has its usual significance, while X indicates the second branch of undetermined character. The last column, $o-c$, contains the differences between the observed wave-numbers and those calculated from equations (2) for system α, (3) and (4) for system β, and (5) and (6) for system γ.

§ 5. VIBRATIONAL ANALYSIS

The vibrational analysis of the three systems is exhibited in tables 2 to 6, which consist of v' and v'' progressions, arranged in triple form to show the consistency of the triplets.

The wave-numbers of the origins of bands in a system are given by the equation:

$$\begin{aligned}\nu_0 &= \nu_e + \nu_v \\ &= \nu_e + \{w_e'(v'+\tfrac{1}{2}) - x_e'w_e'(v'+\tfrac{1}{2})^2 + \ldots\} - \{w_e''(v''+\tfrac{1}{2}) - x_e''w_e''(v''+\tfrac{1}{2})^2 \\ &\qquad + \ldots\} \quad \ldots(1),\end{aligned}$$

where v, the vibrational quantum number, takes successive positive integer values, 0, 1, 2, ...;

ν_e is the "system-origin," a wave-number which would arise from an electronic transition alone;

w_e is the frequency (reduced to wave-number units) of vibrations of infinitesimal amplitude about the equilibrium positions of nuclei;

$x_e w_e$ is a coefficient which takes into account the fact that the vibrations are anharmonic, even when $v = 0$;

and the superscripts $'$ and $''$ distinguish quantities pertaining to the upper and lower electronic states respectively.

When band-*head*, instead of band-*origin*, data are used, the equation which applies has the same form but slightly different coefficients, and an additional term in $(v'+\frac{1}{2})(v''+\frac{1}{2})$. No reliable determination of the coefficients of this term was possible in the present band systems.

Equations (2) to (6) have been deduced from the data in tables 2 to 6 respectively. As the separation of the triplets changes progressively as v'' increases, the coefficients were calculated separately for each of the three members of the triplets; otherwise unduly large residuals $(o-c)$ would have been obtained. However, for system α and the second branch of system β insufficient data were available to make this course feasible.

Equations for the three systems follow in succession.

For the R branches in system α

$$\left.\begin{matrix}(a)\\(b)\\(c)\end{matrix}\right\}\ \nu_{\text{head}} = \begin{cases}21698{\cdot}1\\21614{\cdot}3\\21601{\cdot}4\end{cases} + \{820{\cdot}58\,(v'+\tfrac{1}{2}) - 3{\cdot}306\,(v'+\tfrac{1}{2})^2\} - \{937{\cdot}20\,(v''+\tfrac{1}{2}) - 3{\cdot}346\,(v''+\tfrac{1}{2})^2\} \quad \ldots(2).$$

For the R branches in system β

$$(a)\quad \nu_{\text{head}} = 18053{\cdot}0 + \{846{\cdot}20\,(v'+\tfrac{1}{2}) - 3{\cdot}675\,(v'+\tfrac{1}{2})^2\} - \{937{\cdot}47\,(v''+\tfrac{1}{2}) - 3{\cdot}633\,(v''+\tfrac{1}{2})^2\} \quad \ldots(3a),$$

$$(b)\quad \nu_{\text{head}} = 17806{\cdot}0 + \{844{\cdot}26\,(v'+\tfrac{1}{2}) - 3{\cdot}642\,(v'+\tfrac{1}{2})^2\} - \{936{\cdot}11\,(v''+\tfrac{1}{2}) - 3{\cdot}931\,(v''+\tfrac{1}{2})^2\} \quad \ldots(3b),$$

$$(c)\quad \nu_{\text{head}} = 17529{\cdot}2 + \{845{\cdot}71\,(v'+\tfrac{1}{2}) - 3{\cdot}594\,(v'+\tfrac{1}{2})^2\} - \{937{\cdot}10\,(v''+\tfrac{1}{2}) - 3{\cdot}717\,(v''+\tfrac{1}{2})^2\} \quad \ldots(3c).$$

For the unidentified branches, X, in system β

$$\left.\begin{matrix}(a)\\(b)\\(c)\end{matrix}\right\}\ \nu_{\text{head}} = \begin{cases}18038{\cdot}7\\17790{\cdot}1\\17510{\cdot}7\end{cases} + \{846{\cdot}28\,(v'+\tfrac{1}{2}) - 3{\cdot}409\,(v'+\tfrac{1}{2})^2\} - \{937{\cdot}17\,(v''+\tfrac{1}{2}) - 3{\cdot}417\,(v''+\tfrac{1}{2})^2\} \ldots(4).$$

For the R branches in system γ

$$(a)\quad \nu_{\text{head}} = 16088{\cdot}9 + \{855{\cdot}85\,(v'+\tfrac{1}{2}) - 3{\cdot}183\,(v'+\tfrac{1}{2})^2\} - \{937{\cdot}00\,(v''+\tfrac{1}{2}) - 3{\cdot}383\,(v''+\tfrac{1}{2})^2\} \quad \ldots(5a),$$

$$(b)\quad \nu_{\text{head}} = 15791{\cdot}7 + \{852{\cdot}50\,(v'+\tfrac{1}{2}) - 3{\cdot}025\,(v'+\tfrac{1}{2})^2\} - \{935{\cdot}34\,(v''+\tfrac{1}{2}) - 3{\cdot}106\,(v''+\tfrac{1}{2})^2\} \quad \ldots(5b),$$

$$(c)\quad \nu_{\text{head}} = 15483{\cdot}8 + \{853{\cdot}27\,(v'+\tfrac{1}{2}) - 3{\cdot}200\,(v'+\tfrac{1}{2})^2\} - \{935{\cdot}05\,(v''+\tfrac{1}{2}) - 3{\cdot}200\,(v''+\tfrac{1}{2})^2\} \quad \ldots(5c).$$

For the unidentified branches, X, in system γ

$$(a)\quad \nu_{\text{head}} = 16075{\cdot}0 + \{856{\cdot}99\,(v'+\tfrac{1}{2}) - 3{\cdot}225\,(v'+\tfrac{1}{2})^2\} - \{939{\cdot}62\,(v''+\tfrac{1}{2}) - 3{\cdot}683\,(v''+\tfrac{1}{2})^2\} \quad \ldots(6a),$$

$$(b)\quad \nu_{\text{head}} = 15782{\cdot}5 + \{856{\cdot}73\,(v'+\tfrac{1}{2}) - 3{\cdot}562\,(v'+\tfrac{1}{2})^2\} - \{940{\cdot}55\,(v''+\tfrac{1}{2}) - 3{\cdot}867\,(v''+\tfrac{1}{2})^2\} \quad \ldots(6b),$$

$$(c)\quad \nu_{\text{head}} = 15467{\cdot}4 + \{857{\cdot}18\,(v'+\tfrac{1}{2}) - 3{\cdot}333\,(v'+\tfrac{1}{2})^2\} - \{939{\cdot}70\,(v''+\tfrac{1}{2}) - 3{\cdot}675\,(v''+\tfrac{1}{2})^2\} \quad \ldots(6c).$$

The coefficients of $(v''+\frac{1}{2})$ in equations (2) to (6) show that all three systems have the same lower electronic state; as in the analogous case of TiO, this is probably the ground state of the molecule and is a $^3\Pi$ state. In order to arrive at a plausible interpretation of the three upper electronic states, it is necessary to make a few assumptions. Firstly, noting that in ZrO the triplet separations in system γ are greater than in the two other systems, we may assume that in the upper electronic

states of this system the three sub-states are very near together and that this is a $^3\Sigma$ state. This is in analogy with the red system γ of TiO, which is also ascribed to a $^3\Sigma \rightarrow {}^3\Pi$ transition. Assuming that the separations are zero in a $^3\Sigma$ state, we have for those in the ground state of ZrO

$$R_a - R_b = 292{\cdot}1 \text{ and } R_b - R_c = 313{\cdot}4,$$

or

$$X_a - X_b = 293{\cdot}1 \text{ and } X_b - X_c = 314{\cdot}5.$$

Secondly, it may be assumed that the blue system α of ZrO arises, as that of TiO, from a transition $^3\Pi \rightarrow {}^3\Pi$, and that the relatively small separations observed,

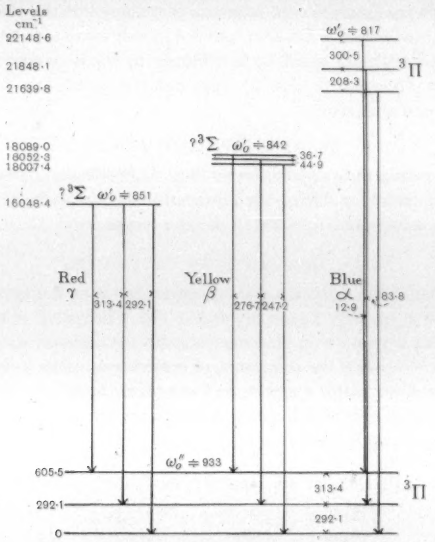


Fig. 1. Provisional energy-level diagram for ZrO.

namely, $R_a - R_b = 83{\cdot}8$ and $R_b - R_c = 12{\cdot}9$, are the differences between the separations in the upper and lower $^3\Pi$ states; in this way we obtain the separations in the upper $^3\Pi$ state as

$$292{\cdot}1 - 83{\cdot}8 = 208{\cdot}3 \text{ and } 313{\cdot}4 - 12{\cdot}9 = 300{\cdot}5,$$

less of course than the separations in the lower $^3\Pi$ state. With these assumptions for systems γ and α we obtain, from the observed separations in the yellow system β, its separations in the upper state; thus from $R_a - R_b$ and $R_b - R_c$ we get

$$292{\cdot}1 - 247{\cdot}2 = 44{\cdot}9 \text{ and } 313{\cdot}4 - 276{\cdot}7 = 36{\cdot}7,$$

or from $X_a - X_b$ and $X_b - X_c$ we get

$$293{\cdot}1 - 248{\cdot}6 = 44{\cdot}5 \text{ and } 314{\cdot}5 - 279{\cdot}4 = 35{\cdot}1, \text{ respectively.}$$

Although not negligible, these separations are so much smaller than those in either of the $^3\Pi$ states, that they suggest a $^3\Sigma$ state for system β. The interpretation of systems γ and β as due to transitions $^3\Sigma \rightarrow {}^3\Pi$ is in conformity with the fact that other heads are observed in these bands and that these may be heads of Q branches.

These tentative assumptions and assignments are shown in a provisional energy-level diagram in figure 1. It would have been more in conformity with other band spectra had the separations in the upper $^3\Pi$ state been more nearly equal to one another and had those in the upper ($?\,^3\Sigma$) state of system β been still smaller. However, this may be the case, since the separations are based on the assumption that the upper ($?\,^3\Sigma$) state of system γ has zero, and not merely small separations*. Repeated attempts on the analysis, based on confidence in the separation of the triplets forming system γ (evident in plate 2), have however yielded no separations which appear to be more satisfactory.

§ 6. ACKNOWLEDGMENTS

The author thanks most appreciatively Prof. A. Fowler for his valuable guidance, suggestions and criticisms during the prosecution of this work; also Dr W. Jevons for his valuable advice and criticism in its later stages.

§ 7. DESCRIPTION OF PLATES

All strips except the tenth are grating spectrograms of the first order, covering a range of 2280 Å. namely, λ 4300 to λ 6580. The tenth strip is a prism spectrogram taken with a kryptocyanin plate and includes the region from λ 6200 to λ 7700. Unbroken lines —— show the sequences of system α on plate 1, broken lines – – – those in system β on plates 1 and 2, and unbroken lines —— those in system γ on plate 2. To avoid overcrowding, the X heads are not marked on the plates, nor are the rather outlying R heads.

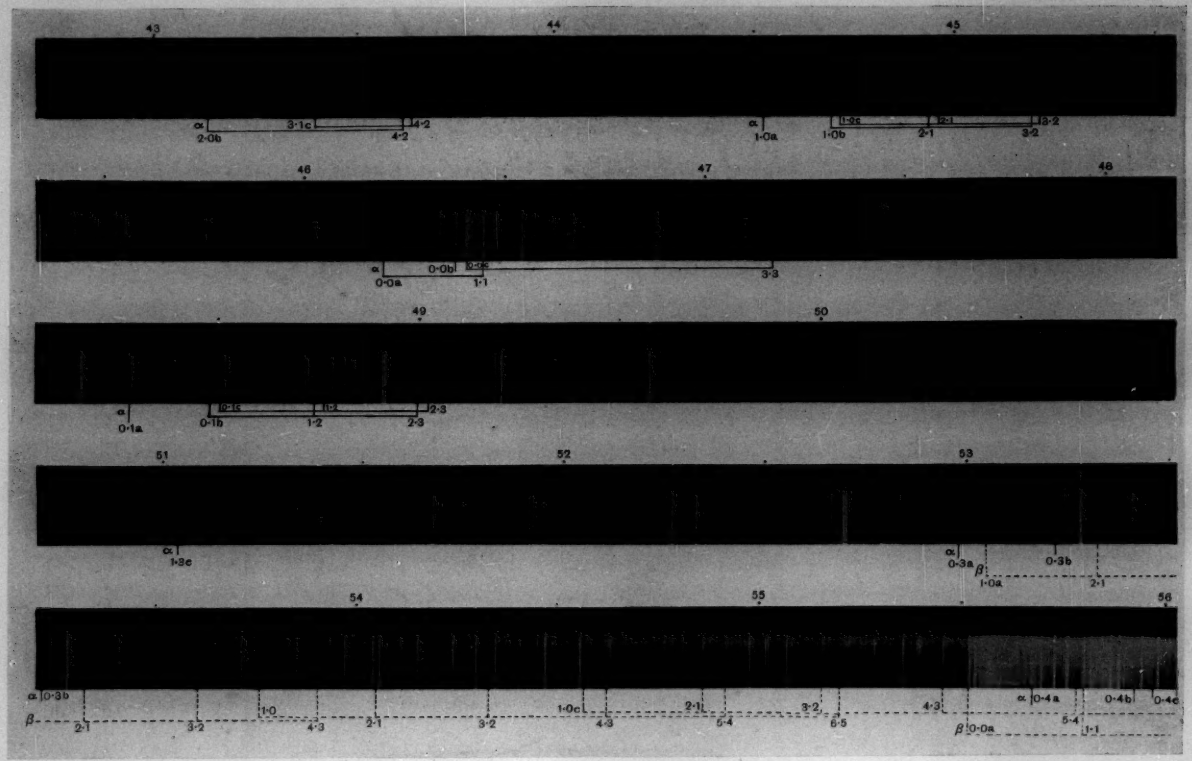
Notes to table 2. (i) Data for what appear to be R heads of further bands of system α are:

a	18482·4	18387·5	18291·7	20709·4
b	18400·3	18302·5	18208·0	20624·8
c	18387·5	18291·7		20614·0

all of which give only small values of $(o-c)$, the greatest being + 1·6.

(ii) A satisfactory explanation of the incompleteness of this table is not forthcoming; it may be that the missing band-heads are merely obscured by overlapping branches. Attention may be drawn to the similar character of the progressions of system α of TiO, as analysed by Christy; he found the blue-green system to consist of eleven sequences containing thirty-nine bands, but of these only ten had all three components of the triplets, three had two and twenty-six had only one component. Table 2 might be extended if, as in the case of TiO, confidence could be placed in the assignment of observed heads to positions in the table involving quantum numbers from 5 to 9; then the system would consist of ten sequences containing thirty bands, seven having three components of the triplets, seven having two, and sixteen only one component. Both spectra, ZrO and TiO, possess some bands not yet included in the analyses.

* It may be well to emphasize that these deductions are on band-head data only.



Bands of zirconium oxide from λ4300 to λ5600.

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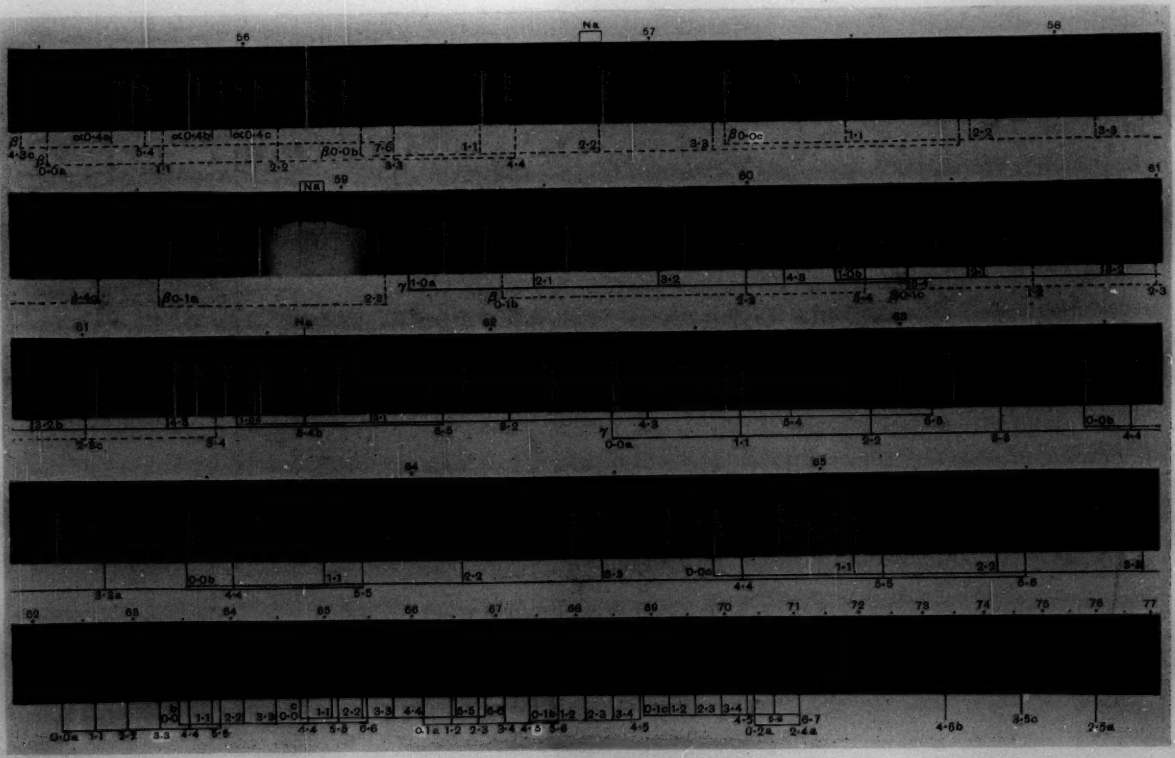

Bands of zirconium oxide from λ 5500 to λ 7700.

Table 1

λ	ν	v', v''	$o-c$
3472·43 (5)	28790·1		
3491·84 (5)	28630·0		
3491·97 (6)	28629·0		
3506·22 (5)	28512·6		
3508·24 (5)	28496·2		
3511·93 (2)	28466·3		
3515·79 (2)	28435·0		
3589·81 (3)	27848·7		
3593·07 (2)	27823·7		
3598·92 (2)	27783·6		
3682·43 (6)	27148·3		
3726·31 (3)	26828·6		
4257·11 (2)	23483·0		
4287·81 (2)	23315·1		
4313·32 (3)	23177·5	α 2, o R_b	+ 0·3
4340·18 (2)	23034·1	α 3, 1 R_c	− 0·6
4361·95 (3)	22919·1	α 4, 2 R_b	+ 1·2
4363·04 (2)	22913·4		
4364·38 (4)	22906·4	α 4, 2 R_c	− 1·4
4365·14 (4)	22902·4		
4368·16 (3)	22886·5		
4375·08 (3)	22850·3		
4452·21 (1)	22454·5	α 1, o R_a	+ 0·7
4460·39 (2)	22413·3		
4469·38 (8)	22368·6	α 1, o R_b	− 1·4
4471·53 (10)	22357·4	α 1, o R_c	+ 0·3
4474·87 (3)	22340·8		
4493·79 (7)	22246·7	α 2, 1 R_b	− 0·1
4496·24 (8)	22234·6	α 2, 1 R_c	+ 0·7
4519·26 (6)	22121·3	α 3, 2 R_b	− 2·5
4521·26 (6)	22111·5	α 3, 2 R_c	− 0·4
4534·41 (8)	22047·4		
4542·55 (7)	22007·9		
4545·15 (5)	21995·3		
4593·53 (1)	21763·7		
4610·25 (2)	21684·7		
4619·82 (15)	21639·8	α o, o R_a	
4637·79 (18)	21556·0	α o, o R_b	
4640·56 (20)	21543·1	α o, o R_c	
4644·68 (10)	21524·0	α 1, 1 R_a	+ 0·7
4716·86 (2)	21194·6	α 3, 3 R_c	+ 0·8
4736·90 (12)	21105·0		
4740·49 (3)	21089·0		
4769·35 (2)	20961·4		
4771·25 (2)	20953·0		
4797·16 (1)	20839·8		
4798·09 (1)	20835·8		
4827·38 (3)	20709·4	? α 8, 8 R_a	− 0·3
4827·52 (8)	20708·5	α o, 1 R_a	− 0·8
4847·19 (5)	20624·8	α o, 1 R_b	− 0·7
		? α 8, 8 R_b	− 1·1
4849·72 (3)	20614·0	? α 8, 8 R_c	+ 1·0
4850·15 (6)	20612·2	α o, 1 R_c	− 0·4
4863·08 (4)	20557·4		
4873·53 (1)	20513·3	α 1, 2 R_b	− 2·4
4876·12 (3)	20502·4	α 1, 2 R_c	− 0·4
		β 3, o R_a	+ 0·5
4899·76 (3)	20403·5	α 2, 3 R_b	− 2·4
4902·47 (3)	20392·2	α 2, 3 R_c	− 0·8
5103·85 (1)	19587·6	α 1, 3 R_c	+ 1·9
5156·34 (2)	19388·2		
5185·00 (7)	19281·1		
5212·19 (4)	19180·5		
5298·26 (2)	18868·9	α o, 3 R_a	+ 0·5
5304·63 (1)	18846·2	β 1, o R_a	0·0
5305·96 (2)	18841·5		
5308·42 (2)	18832·8	β 1, o X_a	0·0
5322·03 (2)	18784·6	α o, 3 R_b	0·0

Table 1 (*continued*)

λ	ν	v′, v″	o−c
5332·50 (2)	18747·1	β 2, 1 R_a	− 0·4
5360·70 (3)	18649·1	β 3, 2 R_a	+ 0·3
5364·34 (2)	18636·5	β 3, 2 X_a	− 0·9
5375·68 (2)	18597·1	β 1, 0 R_b	− 0·1
5377·26 (2)	18591·7		
5379·50 (3)	18583·9	β 1, 0 X_b	− 0·3
5389·25 (2)	18550·3	β 4, 3 R_a	+ 0·4
5390·36 (2)	18546·5		
5392·96 (2)	18537·6	β 4, 3 X_a	− 2·1
5404·35 (4)	18498·5	β 2, 1 R_b	− 0·2
5407·14 (3)	18488·9		
5408·16 (2)	18485·5	β 2, 1 X_b	− 1·0
5409·05 (3)	18482·4	? α 3, 6 R_c	− 0·1
		? α 4, 7 R_a	− 0·6
5422·04 (1)	18438·1	β 5, 4 X_a	− 1·0
5433·18 (3)	18400·3	? α 4, 7 R_b	+ 1·1
		β 3, 2 R_b	− 0·4
5436·96 (8)	18387·5	? α 4, 7 R_c	+ 1·2
		? α 5, 8 R_a	+ 0·7
		β 3, 2 X_b	− 1·3
5439·40 (7)	18379·2	γ 5, 2 R_a	+ 0·7
5456·49 (8)	18321·7	β 1, 0 R_c	− 0·3
5461·61 (5)	18304·3	β 1, 0 X_c	− 0·5
5462·21 (5)	18302·5	? α 5, 8 R_b	− 0·5
		β 4, 3 R_b	+ 0·9
5465·44 (6)	18291·7	? α 5, 8 R_c	+ 1·6
		? α 6, 9 R_a	+ 1·0
		β 4, 3 X_b	+ 0·6
5465·95 (4)	18290·0		
5485·71 (6)	18224·4	β 2, 1 R_c	+ 0·7
5490·28 (3)	18209·0	? α 6, 9 R_b	+ 2·1
		β 2, 1 X_c	+ 1·9
5491·67 (6)	18204·4	β 5, 4 R_b	− 2·0
5502·84 (3)	18167·4		
5515·33 (6)	18126·3	β 3, 2 R_c	+ 0·7
5519·56 (3)	18112·4	β 3, 2 X_c	+ 3·0
		β 6, 5 R_b	+ 2·2
5538·80 (5)	18049·5		
5539·35 (6)	18047·7		
5545·16 (6)	18028·8	β 4, 3 R_c	+ 1·0
5547·42 (2)	18021·2		
5551·74 (10)	18007·4	β 0, 0 R_a	
5553·10 (10)	18003·0		
5556·09 (3)	17993·3	β 0, 0 X_a	
5562·67 (2)	17972·0		
5566·93 (3)	17958·3	α 0, 4 R_a	− 0·4
		γ 3, 0 X_c	+ 0·6
5575·74 (5)	17929·9	β 5, 4 R_c	− 0·2
5580·09 (5)	17915·9	β 1, 1 R_a	− 0·2
5581·74 (6)	17910·6		
5584·56 (2)	17901·6	β 1, 1 X_a	− 0·9
5592·53 (2)	17876·0	α 0, 4 R_b	+ 1·9
5596·49 (2)	17863·4	α 0, 4 R_c	+ 0·3
		γ 4, 1 R_c	+ 0·3
5608·87 (2)	17824·0	β 2, 2 R_a	− 0·6
5610·05 (10)	17820·2		
5612·61 (3)	17812·1	β 2, 2 X_a	+ 0·5
5623·97 (2)	17776·1		
5629·00 (12)	17760·2	β 0, 0 R_b	
		γ 5, 2 R_c	− 2·0
5629·53 (14)	17758·5		
5633·92 (6)	17744·7	β 0, 0 X_b	
5634·88 (5)	17741·7	γ 2, 0 R_a	+ 0·7
5636·97 (5)	17735·1	β 3, 3 R_a	+ 2·0
		β 7, 6 R_c	− 0·6
5658·13 (9)	17668·8	β 1, 1 R_b	− 0·2
5663·06 (4)	17653·4	β 1, 1 X_b	− 0·5
5664·19 (3)	17649·9		
5665·52 (3)	17645·7	γ 3, 1 R_a	− 1·9

Table 1 (*continued*)

λ	ν	v', v''	$o-c$
5666·80 (4)	17641·8	β 4, 4 R_a	+ 0·3
5670·67 (2)	17629·7	β 4, 4 X_a	− 0·2
5681·08 (4)	17597·4		
5687·43 (5)	17577·4	β 2, 2 R_b	− 0·8
5692·30 (4)	17562·6	β 2, 2 X_b	− 0·4
5716·61 (1)	17488·0	β 3, 3 R_b	− 0·2
5718·11 (20)	17483·5	β 0, 0 R_c	
5724·05 (11)	17465·3	β 0, 0 X_c	
5748·14 (16)	17392·1	β 1, 1 R_c	− 0·2
5753·80 (6)	17375·0	β 1, 1 X_c	+ 0·5
5776·05 (1)	17308·1	β 5, 5 R_b	− 1·5
5778·46 (10)	17300·8	β 2, 2 R_c	− 0·7
5783·83 (3)	17284·8	β 2, 2 X_c	+ 1·2
5809·18 (7)	17209·4	β 3, 3 R_c	− 1·4
5814·49 (4)	17193·7	β 3, 3 X_c	+ 1·0
5839·79 (2)	17119·2	β 4, 4 R_c	− 1·2
5854·30 (1)	17076·7	β 0, 1 R_a	− 0·5
5860·09 (12)	17059·8		
5867·98 (7)	17036·9	γ 3, 1 R_c	+ 1·4
5908·52 (10)	16920·0		
5911·45 (4)	16911·6	β 2, 3 R_a	+ 2·6
5916·44 (4)	16897·4	γ 1, 0 R_a	− 0·5
5917·67 (4)	16893·9	β 2, 3 X_c	− 1·0
5923·33 (4)	16877·7		
5939·08 (4)	16833·0	β 0, 1 R_b	+ 1·0
5946·99 (6)	16810·6	γ 2, 1 R_a	− 0·2
		β 3, 4 X_a	− 0·3
5951·85 (3)	16796·8	γ 2, 1 X_a	+ 0·7
5977·68 (6)	16724·3	γ 3, 2 R_a	+ 0·2
5983·29 (4)	16708·6	γ 3, 2 X_a	+ 0·4
5999·44 (3)	16663·6	β 2, 3 R_b	− 2·2
6008·50 (4)	16638·5	γ 4, 3 R_a	+ 0·7
6014·46 (2)	16622·0	γ 4, 3 X_a	− 0·6
6021·29 (6)	16603·1	γ 1, 0 R_b	+ 0·4
6026·04 (2)	16590·1	γ 1, 0 X_b	+ 0·7
6028·37 (1)	16583·6	β 3, 4 R_b	+ 0·1
6039·13 (4)	16554·1	β 0, 1 R_c	+ 0·3
		γ 5, 4 R_a	+ 2·2
6053·82 (5)	16513·9	γ 2, 1 R_b	− 0·1
6059·33 (2)	16498·8	γ 2, 1 X_b	− 1·2
6070·01 (6)	16469·9	β 1, 2 R_c	− 0·2
6086·85 (5)	16424·3	γ 3, 2 R_b	− 1·2
6091·53 (3)	16411·7	γ 3, 2 X_b	+ 1·4
6099·85 (3)	16389·3	β 2, 3 R_c	+ 2·6
6020·10 (3)	16335·1	γ 4, 3 R_b	− 2·0
6125·36 (1)	16321·1	γ 4, 3 X_b	− 0·1
6131·54 (1)	16304·6	β 3, 4 R_c	+ 1·2
6137·22 (4)	16289·5	γ 1, 0 R_c	− 0·3
6153·92 (8)	16245·3		
6170·20 (4)	16202·5	γ 2, 1 R_c	+ 1·0
6175·18 (2)	16189·4	γ 2, 1 X_c	+ 1·2
6188·01 (4)	16155·8		
6188·67 (1)	16154·1	β 0, 2 R_a	− 0·2
6200·88 (1)	16122·8		
6203·94 (4)	16114·3	γ 3, 2 R_c	+ 1·0
6210·20 (4)	16098·1	γ 3, 2 X_c	− 2·3
6222·78 (2)	16065·6		
6229·40 (18)	16048·4	γ 0, 0 R_a	
6235·10 (6)	16033·8	γ 0, 0 X_a	
6238·92 (6)	16024·0	γ 4, 3 R_c	− 1·2
6260·89 (16)	15967·8	γ 1, 1 R_a	+ 0·2
6266·52 (6)	15953·4	γ 1, 1 X_a	+ 1·3
6272·98 (4)	15937·0	γ 5, 4 R_c	− 0·1
6292·79 (14)	15886·6	γ 2, 2 R_a	− 0·7
6298·95 (4)	15871·2	γ 2, 2 X_a	− 0·1
6308·70 (2)	15846·8	γ 6, 5 R_c	− 2·0
6312·46 (1)	15837·3	β 1, 3 R_b	+ 1·2
6317·63 (2)	15824·3		
6324·33 (6)	15807·6	γ 3, 3 R_a	+ 0·2

Table 1 (*continued*)

λ	ν	v′, v″	o−c
6331·35 (1)	15790·1	γ 3, 3 X_a	− 1·3
6344·91 (18)	15756·3	γ 0, 0 R_b	
6351·23 (6)	15740·7	γ 0, 0 X_b	
6356·26 (6)	15728·2	γ 4, 4 R_a	+ 0·4
6362·51 (5)	15712·7	γ 4, 4 X_a	+ 0·3
6378·32 (16)	15673·8	γ 1, 1 R_b	+ 0·1
6384·57 (4)	15658·4	γ 1, 1 X_b	+ 0·9
6387·84 (4)	15650·4	γ 5, 5 R_a	+ 1·7
6394·07 (5)	15635·2	γ 5, 5 X_a	+ 0·8
6395·99 (4)	15630·4	β 0, 2 R_c	− 1·2
6412·29 (12)	15590·8	γ 2, 2 R_b	− 0·4
6419·28 (2)	15573·8	γ 2, 2 X_b	− 1·1
6446·52 (3)	15508·0	γ 3, 3 R_b	− 0·8
6452·50 (2)	15493·6	γ 3, 3 X_b	+ 0·6
6473·67 (20)	15442·9	γ 0, 0 R_c	
6480·71 (4)	15426·2	γ 0, 0 X_c	
6508·15 (18)	15361·1	γ 4, 4 R_b	− 0·4
6515·15 (4)	15344·6	γ 1, 1 R_c	+ 0·1
6542·98 (10)	15279·4	γ 1, 1 X_c	+ 0·2
6549·72 (2)	15263·6	γ 5, 5 R_b	0·0
6578·20 (4)	15197·5	γ 2, 2 R_c	+ 0·1
6584·92 (2)	15182·0	γ 2, 2 X_c	+ 0·4
6613·07 (3)	15117·4	γ 6, 6 R_b	+ 1·0
6619·64 (1)	15102·4	γ 3, 3 R_c	0·0
6645·26 (2)	15044·2	γ 3, 3 X_c	− 0·7
6649·52 (2)	15034·5	γ 4, 4 R_c	+ 1·8
6678·00 (5)	14970·4	γ 0, 1 R_a	− 0·8
6686·58 (2)	14951·2	γ 4, 4 X_c	− 0·6
6693·95 (1)	14934·8	γ 0, 1 X_a	+ 0·8
6710·87 (2)	14897·1	γ 1, 2 R_a	0·0
6717·99 (2)	14881·3	γ 5, 5 R_c	+ 0·5
6742·49 (4)	14827·2	γ 2, 3 R_c	− 0·2
6751·52 (1)	14807·4	γ 6, 6 R_c	− 1·0
6777·21 (4)	14751·3	γ 3, 4 R_a	− 0·4
6785·86 (1)	14732·5	γ 3, 4 X_a	0·0
6812·53 (3)	14674·8	γ 4, 5 R_a	+ 4·5
6820·93 (1)	14656·7	γ 0, 1 R_b	0·0
6847·33 (3)	14600·2	γ 0, 1 X_b	− 0·5
6848·92 (3)	14596·8	γ 5, 6 R_a	− 1·0
6854·80 (1)	14584·3	γ 1, 2 R_b	+ 0·5
6884·80 (2)	14520·8	γ 1, 2 X_b	+ 0·1
6887·79 (4)	14514·5	γ 2, 3 R_b	+ 0·4
6897·81 (1)	14493·4	γ 2, 3 X_b	− 0·9
6923·73 (5)	14439·1	γ 3, 4 R_b	− 1·5
6931·83 (5)	14422·2	γ 3, 4 X_b	+ 1·0
6959·90 (6)	14364·1	γ 4, 5 R_b	− 1·5
6968·93 (2)	14345·5	γ 0, 1 R_c	+ 0·3
6996·31 (6)	14289·3	γ 0, 1 X_c	− 0·4
7005·55 (3)	14270·5	γ 1, 2 R_c	+ 0·3
7033·16 (5)	14214·4	γ 1, 2 X_c	+ 2·8
7043·13 (4)	14194·3	γ 2, 3 R_c	+ 0·7
7071·28 (4)	14137·8	γ 2, 3 X_c	− 0·1
7079·55 (3)	14121·3	γ 3, 4 R_c	+ 1·2
7109·83 (2)	14061·2	γ 3, 4 X_c	− 1·9
7127·18 (2)	14026·9	γ 4, 5 R_c	+ 1·7
7335·62 (2)	13628·4	γ 0, 2 R_a	− 0·7
7426·31 (2)	13461·9	γ 5, 6 R_c	+ 0·4
7468·24 (3)	13386·4	γ 6, 7 R_c	− 0·7
7484·41 (3)	13357·4	γ 2, 4 R_a	+ 0·5
7541·49 (4)	13256·3		
7597·51 (2)	13158·6	γ 4, 6 R_b	+ 4·2
		γ 3, 5 R_c	+ 1·4
		γ 2, 5 R_a	+ 1·1

Table 2. *R* heads of bands in violet-blue system α

v'' / v'		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4
0	*a*	21639·8 *83·8*	931·3	20708·5 *83·7*		·····		18868·9 *84·3*	910·6	17958·3
	b	21556·0 *12·9*	931·2	20624·8 *12·6*		·····		18784·6	908·6	17876·0
	c	21543·1	930·9	20612·2		·····		·····		17863·4
	$\Delta'a$	*814·7*		*815·5*						
	$\Delta'b$	*812·6*								
	$\Delta'c$	*814·3*								
1	*a*	22454·5 *85·9*	930·5	21524·0		·····		·····		
	b	22368·6 *11·2*		·····		20513·3 *10·9*	*914·8*	·····		
	c	22357·4		·····		20502·4		19587·6		
	$\Delta'a$									
	$\Delta'b$	*808·9*								
	$\Delta'c$							*804·6*		
2	*a*	·····		·····		·····		·····		
	b	23177·5	930·8	22246·7 *12·1*		·····		20403·5		
	c	·····		22234·6		·····		20392·2		
	$\Delta'a$									
	$\Delta'b$									
	$\Delta'c$			*799·5*				*802·4*		
3	*a*			·····		·····		·····		
	b			·····		22121·3 *9·8*	*916·7*	·····		
	c			23034·1	*922·6*	22111·5		21194·6		
	$\Delta'a$					*797·8*				
	$\Delta'b$					*794·9*				
	$\Delta'c$									
4	*a*					·····				
	b					22919·1				
	c					22906·4				

Table 4. Unidentified heads (X) of bands in yellow system β

v'' / v'		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4
0	*a*	17993·3 *248·6*								
	b	17744·7 *279·4*								
	c	17465·3								
	$\Delta'a$	*839·5*								
	$\Delta'b$	*839·2*								
	$\Delta'c$	*839·0*								
1	*a*	18832·8 *248·9*	*931·2*	17901·6 *248·2*						
	b	18583·9 *279·6*	*930·5*	17653·4 *278·4*						
	c	18304·3	*929·3*	17375·0						
	$\Delta'a$			·····						
	$\Delta'b$			*832·1*						
	$\Delta'c$			*834·0*						
2	*a*			·····	····	17812·1 *249·5*	*918·2*	16893·9		
	b			18485·5 *276·5*	*922·9*	17562·6 *277·8*	····	·····		
	c			18209·0	*924·2*	17284·8	····	·····		
	$\Delta'a$					*824·4*				
	$\Delta'b$					*824·9*				
	$\Delta'c$					*827·6*				
3	*a*					18636·5 *249·9*	····	·····		
	b					18387·5 *275·1*	····	·····		
	c					18112·4	*918·8*	17193·7		
	$\Delta'a$							····		
	$\Delta'b$							····		
	$\Delta'c$							····		
4	*a*							18537·6 *245·9*	*906·9*	17629·7
	b							18291·7	····	·····
	c							·····		*808·4*
	$\Delta'a$									····
	$\Delta'b$									····
	$\Delta'c$									
5	*a*									18438·1
	b									·····
	c									·····

Table 3. *R* heads of bands in yellow system β

v' \ v''		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4	$\Delta''a$ $\Delta''b$ $\Delta''c$	5	6
0	*a*	18007·4 247·2	930·7	17076·7 243·7	922·6	16154·1							
	b	17760·2 276·7	927·2	16833·0 278·9		.....							
	c	17483·5	929·4	16554·1	923·7	15630·4							
	$\Delta'a$	838·8		839·2		....							
	$\Delta'b$	836·9		835·8		....							
	$\Delta'c$	838·2		838·0		839·5							
1	*a*	18846·2 249·1	930·3	17915·9 247·1	.....	.....		.....					
	b	18597·1 275·4	928·3	17668·8 276·7	.....	.....		15837·3					
	c	18321·7	929·6	17392·1	922·2	16469·9		.....					
	$\Delta'a$			831·2		.....							
	$\Delta'b$			829·7		.....		826·3					
	$\Delta'c$			832·3		830·9		.....					
2	*a*	.....		18747·1 248·6	923·1	17824·0 246·6	912·4	16911·6 248·0					
	b	.....		18498·5 274·1	921·1	17577·4 276·6	913·8	16663·6 274·3					
	c	.....		18224·4	923·6	17300·8	911·5	16389·3					
	$\Delta'a$					825·1		823·5					
	$\Delta'b$					822·9		824·4					
	$\Delta'c$					825·5		820·1					
3	*a*	20502·4				18649·1 248·8	914·0	17735·1 247·1	904·4	16583·6 279·0			
	b	.....				18400·3 274·0	912·3	17488·0 278·6	904·8	16304·6			
	c	.....				18126·3	916·9	17209·4		....			
	$\Delta'a$							815·2		....			
	$\Delta'b$							814·5		814·6			
	$\Delta'c$							819·4					
4	*a*							18550·3 247·8	908·5	17641·8			
	b							18302·5 273·7	....	.....			
	c							18028·8	909·6	17119·2			
	$\Delta'a$									....			
	$\Delta'b$									....			
	$\Delta'c$									810·7			
5	*a*									.....			
	b									18204·4 274·5	896·3	17308·1	
	c									17929·9		.....	
	$\Delta'a$											804·3	
	$\Delta'b$											....	
	$\Delta'c$											.....	
6	*a*											18112·4	
	b											.....	
	c												
	$\Delta'a$												
	$\Delta'b$												
	$\Delta'c$												
7	*a*												.....
	b												.....
	c												17735·1

Table 5. R heads of bands in red-infra-red system γ

v' \ v''		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4	$\Delta''a$ $\Delta''b$ $\Delta''c$	5	$\Delta''a$ $\Delta''b$ $\Delta''c$	6	$\Delta''a$ $\Delta''b$ $\Delta''c$	7
0	a	16048·4 292·1	931·0	15117·4 290·2	923·1	14194·3										
	b	15756·3 313·4	929·1	14827·2 312·7		·····										
	c	15442·9	928·4	14514·5		·····										
	$\Delta'a$	849·0		850·4		849·9										
	$\Delta'b$	847·9		846·6												
	$\Delta'c$	846·6		846·6												
1	a	16897·4 294·2	929·6	15967·8 294·0	923·6	15044·2 292·9										
	b	16603·2 313·7	929·4	15673·8 312·7	922·5	14751·3 312·2										
	c	16289·5	928·4	15361·1	922·0	14439·1										
	$\Delta'a$	844·3		842·8		842·4										
	$\Delta'b$			840·1		839·5										
	$\Delta'c$			841·4		840·3										
2	a	17741·7	931·1	16810·6 296·7	924·0	15886·6 295·8	916·2	14970·4 295·6	909·2	14061·2	902·6	13158·6				
	b	·····		16513·9 311·4	923·1	15590·8 311·4	916·0	14674·8 310·7		·····		·····				
	c	·····		16202·5	923·1	15279·4	915·3	14364·1		·····		·····				
	$\Delta'a$			835·1		837·7				835·9						
	$\Delta'b$					833·5										
	$\Delta'c$			834·4		834·9										
3	a			17645·7	921·4	16724·3 300·0	916·7	15807·6 299·6	910·5	14897·1 300·3		·····				
	b			·····		16424·3 310·0	916·3	15508·0 310·5	911·2	14596·8 307·5		·····				
	c			17036·9	922·6	16114·3	916·8	15197·5	908·2	14289·3	902·9	13386·4				
	$\Delta'a$							839·9								
	$\Delta'b$							838·8								
	$\Delta'c$			826·5				826·5								
4	a			·····		·····		16038·5 303·4	910·3	15728·2 302·0	901·0	14827·2 306·4	892·4	·····		
	b			·····		·····		16335·1 311·1	908·9	15426·2 308·8	905·4	14520·8 306·4		13628·4		
	c			17803·4		·····		16024·0	906·6	15117·4	903·0	14214·4		·····		
	$\Delta'a$									825·9		823·2				
	$\Delta'b$											823·8				
	$\Delta'c$									819·6		820·1				
5	a					18379·2		·····		16554·1	903·7	15650·4 305·8	899·1	14751·3		
	b					·····		·····		·····		15344·6 310·1	896·7	·····		
	c					17760·2		·····		15937·0	902·5	15034·5		14137·8		
	$\Delta'a$											812·3		813·4		
	$\Delta'b$											·····		·····		
	$\Delta'c$											·····		15263·6 312·4	890·0	·····
6	a											15846·8	895·6	14951·2		14061·2
	b															
	c															

Table 6. Unidentified heads (X) of bands in red-infra-red system γ

v' \ v''		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4	5
0	a	16033·8 293·1	931·4	15102·4 295·0							
	b	15740·7	933·3	14807·4 314·0							
	c	15426·2 314·5	932·8	14493·4							
	$\Delta'a$			851·0							
	$\Delta'b$	849·4		851·0							
	$\Delta'c$			851·2							
1	a	·····		15953·4 295·0		·····					
	b	16590·1	931·7	15658·4 313·8	925·9	14732·5 310·3					
	c	·····		15344·0	922·4	14422·2					
	$\Delta'a$			843·4							
	$\Delta'b$			840·4		841·3					
	$\Delta'c$			844·8		841·4					
2	a	·····		16796·8 298·0	925·6	15871·2 297·4	917·1	·····			
	b	·····		16498·8 309·4	925·0	15573·8 310·2	918·1	14056·7 311·2			
	c	·····		16189·4	925·8	15263·6		14345·5			
	$\Delta'a$					837·4					
	$\Delta'b$					837·9		836·9			
	$\Delta'c$					834·5		836·5			
3	a	·····		·····		16708·6 296·9	918·5	15790·1 296·5	908·8	14881·3 297·0	
	b	·····		·····		16411·7 313·6	918·1	15493·6 311·6	909·3	14584·3 313·8	
	c	17958·3		·····		16098·1	916·1	15182·0	911·5	14270·5	
	$\Delta'a$							831·9		831·4	
	$\Delta'b$							827·5			
	$\Delta'c$									831·9	
4	a							16622·0	909·3	15712·7	
	b							16321·1		·····	
	c							·····		15102·4	
	$\Delta'a$										
	$\Delta'b$										
	$\Delta'c$										
5	a										15035·2
	b										·····
	c										·····

LATTICE-DISTORTION OF COLD-DRAWN CONSTANTAN WIRE

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Communicated by Dr G. W. C. Kaye, *September* 1, 1931. *Read November* 20, 1931.

ABSTRACT. The space-lattice of constantan is found to undergo large distortion as a result of cold-working. A method is described for determining the variation in distortion quantitatively as constantan wire is cold-drawn, and the variation across the section of the wire during the drawing. It is found that (*a*) the distortion on drawing increases quickly to a steady maximum, which is maintained despite further drawing; (*b*) orientation does not begin until the maximum distortion appears; (*c*) just below the surface in the less drawn wires is a region of diminished distortion, but as the wire is further drawn the degree of distortion evens out across the section (this is explained in terms of the action of the die on the surface); (*d*) the temperature-coefficient of electrical resistance exhibits changes in drawing similar to the variation in distortion, and the two properties are practically proportional.

§ 1. INTRODUCTION

THE space-lattice of constantan was found to be very susceptible to distortion as a result of cold-working. The chief object of the present work was to determine (i) the change in degree of distortion of constantan wire as its diameter was gradually reduced by cold-drawing, (ii) the change in degree of distortion across the section of a wire at a given stage of the drawing, and (iii) the relation, if any, between the degree of distortion thus produced and the variation in the value of the temperature-coefficient of electrical resistance which results when constantan wire is hard-drawn[1]. The point where crystal orientation is introduced by the drawing was also sought.

The term "lattice-distortion of constantan" is used herein solely in the sense of a permanent, irregular displacement of atoms from their normal positions at the face-centres and corners of the cubic unit cells which form the basis of the constantan lattice. A number of the cells therefore cease to be perfect cubes. This type of distortion is recognized by its influence in broadening the lines of the X-ray diffraction spectrum of the distorted material[2, 3]. It differs from "distortion by slip," where the grains of a worked metal are deformed by movement along slip planes, inasmuch as this process leaves the breadth of the diffraction lines unaltered[2]. On this account distinction between the two types could be made by experiment. It differs also from the change in size of a unit cell, sometimes referred to as distortion[4], which occurs when some of the atoms of a metal are replaced by atoms of another in order to form an alloy of the solid-solution type.

This change in size does not affect the breadth of the diffraction lines and is presumably the same therefore, on the whole, for every cell. In the last two cases no irregularity is introduced into the lattice.

§ 2. EXPERIMENTAL PROCEDURE

It was necessary first to secure an appropriate range of specimens, and secondly, to select a method of estimating quantitatively the degree of distortion.

Two sets of constantan wire were finally used. The first was obtained by cold-drawing a wire, previously annealed and thus rid of distortion, as shown by its X-ray spectrum, from an initial diameter of 1·645 mm. by twenty-eight steps to 0·726 mm. At each step of the drawing a length was severed from the parent wire and its X-ray photograph secured. The second set (kindly supplied by the Research Department of Metropolitan Vickers-Electrical Co., Ltd.) was obtained from a wire of diameter 0·762 mm. drawn by twelve steps to 0·102 mm. Measurements of the temperature-coefficient of electrical resistance of these specimens between 0° and 40° C. were made in the Electrotechnics Division of the National Physical Laboratory. By measuring the distortion from the X-ray photographs of the severed specimens the change in degree of distortion with the percentage reduction of diameter of the initial wire could thus be determined.

The variation of distortion across the section of a wire was obtained by taking an X-ray photograph of the surface of the specimen, then etching down the wire slightly by electrolytical dissolution in dilute hydrochloric acid[5], and finally re-photographing the new surface. By repetition of the process of etching and photographing, the degree of distortion was determined from the outside to the core of typical wires.

The estimation of distortion was based on the change in breadth of the (331) line of the X-ray spectrum. This line under the conditions of experiment was a doublet. The separation of the components due respectively to the α_1 and α_2 constituents of the analyzing copper $K\alpha$ radiation, was of the order of 1 mm. For any wire the breadth of the line in a Debye photograph will depend on its diameter, its absorption, the radius of the camera, the divergence of the incident beam, and lastly, the degree of lattice-distortion in the wire. These complications were allowed for experimentally by the following procedure. Consider the specimens drawn from the wire of diameter 1·645 mm. First the breadth of the (331) line in the photographs of these specimens was measured, as described below, with the aid of a Moll microphotometer. This breadth was plotted against the known percentage reduction of diameter of each specimen. Next, a number of the specimens were annealed until free of distortion, and then photographed under exactly the same conditions as before. The wires were annealed for 1 hour at about 650° C. in an electric furnace through which was circulating purified argon. This atmosphere was chosen to prevent change in diameter of the wires as a result of oxidation. The measurements of the breadths of the (331) line were now repeated and again plotted, on the same diagram as before, against the percentage reduction of

diameter. Now the only factor distinguishing this curve from the first is that of lattice-distortion, which is present in the first only. The other factors have remained unchanged. Therefore, for a given value of the abscissae (the percentage reduction of diameter) the difference in height of the ordinates (line-breadth) gives the change in breadth of the (331) line caused solely by distortion. Finally, the second curve was subtracted in this way from the first and a resultant third curve was obtained giving the change in breadth of the (331) line for each percentage reduction in diameter as the initial wire was drawn down. This change in breadth was taken as a measure of the degree of lattice-distortion.

The process was repeated for the wire drawn down from the initial diameter of 0·762 mm. and also, with one modification, for the specimens obtained by etching down the wires. This modification was necessitated by the fact that the wire at any stage of the etching could not be annealed, since it was always required for further etching. It involved therefore the numerical deduction of the breadth of line which these specimens would give if they were annealed from the second curve described above. This curve shows the breadth of an undistorted line for a wire of known diameter in the range 1·645 mm. to 0·726 mm. The corresponding graph of the second set of wires gives a curve for the range 0·762 mm. to 0·051 mm. The diameter of the etched wires fell in one or other of these ranges.

Measurements of the (331) line were made as follows. The photometer records give the distribution of intensity across the width of the line. The line appears as two neighbouring peaks, approximately triangular, set on a line of continuous background. One peak corresponds to the α_1 and the other to the α_2 wave-length. On broadening, the two peaks coalesce. But since the intensity of the α_1 peak is known to be about twice that of the α_2, the point of maximum intensity of the compound line gives the position of the α_1 peak. The point of minimum intensity of the α_1 line is taken as the point where the trace of the peak descends to the line of continuous background on the side of the peak opposite to that complicated by the presence of the α_2 component. The distance between the first point and the second measured along the base line of continuous background gives a measurement of half the breadth of the α_1 line uncomplicated by the α_2 component. This is the value used in constructing the graphs. If B_1 is the value obtained for a distorted wire and B_2 the corresponding value after the distortion was removed by annealing, then $B_1 - B_2$ is the quantity taken as a measure of the distortion. This quantity is denoted by S.

B_1
B_2
S

The value of S as it stands is an arbitrary expression of length. It can, however, be compared with other work when the experimental data which are stated herewith are considered. The radius of the circular camera used was 5 cm.; the magnification factor introduced by the Moll microphotometer is such that 1 cm. on the film corresponds to 7 cm. on the photometer record; the value of the reflection angle θ for the (331) line was 70·7°. Further interpretation of S can be made only when the exact position of the atoms after distortion is known. This is not known at present.

If we assume, however, the change in breadth of the line to follow from the

variation of spacing about the normal value, and that the increase in breadth on distortion corresponds to the maximum change of lattice parameter, then S may be related to the fractional change of spacing of the (311) planes as follows. From the Bragg relation $2d \sin\theta = \lambda$, we have by differentiation

$$\delta d/d = -\cot\theta\,\delta\theta,$$

d, θ, λ where d is the spacing of the planes, and θ the angle at which the wave-length λ is reflected by those planes. Now S measured in mm. on the photometer record corresponds to a distance $S/7$ mm. on the photograph, and this distance subtends an angle $\delta\theta = S/350$ at the centre of the camera, the radius of the camera being 50 mm. This angle is the angle subtended by the change in the breadth of the line in the direction of increased spacing, so that for the maximum fractional increase of spacing

$$\begin{aligned}\delta d/d &= (S/350)\cot\theta \\ &= 0{\cdot}001\,S, \text{ since } \cot\theta = 0{\cdot}3502,\end{aligned}$$

and the percentage increase in spacing is $0{\cdot}1\,S$, where S is measured in mm. This value may be taken as giving the order of magnitude of the distortion.

The photographs were also watched for the appearance of crystal-orientation as shown by irregular distribution of intensity along the spectral lines. Orientation first appeared at the core of the wire, as was shown by the etched specimens. The diameter of the wire when orientation at the core was first found was noted for each set of drawn wires.

§ 3. RESULTS

Orientation was observed to begin at the core in the first set of wires drawn down from the initial diameter of 1·645 mm. when the reduction was 42 per cent. In the second set of wires drawn from the diameter of 0·762 mm. the presence of orientation was first noted at the reduction of 61 per cent. Some difference between the two sets was to be expected, since the wires obtained from different sources were drawn by different stages. (There appears to be room for research on the effect of different speeds of drawing on the rate of production of distortion and orientation.)

The results of the distortion measurements are recorded on the accompanying graphs. Figure 1 shows, in the upper curve, the way in which the half-breadth B_1 of the (331) line, as measured on the photometer records, varies as the diameter of the wire was drawn from 1·645 mm. The influence of the distortion is striking. The lower curve gives the variation of the half-breadth B_2 of the (331) line for the corresponding undistorted specimens. This is virtually a curve which calibrates the circular camera by giving the breadth of the line as a function of the diameter of normal wire specimens photographed at the centre of the camera under the particular experimental conditions employed.

Figure 2 is obtained by subtracting the second curve from the first in figure 1,

in the manner already described. The resultant value S, the actual broadening due to distortion, and the quantitative measure thereof, is plotted against the percentage reduction of diameter on the same scale as figure 1. On the right side of the graph the ordinates are expressed in terms of the equivalent percentage change in size of the (331) spacings. The first point of interest is that the curve is made up of two

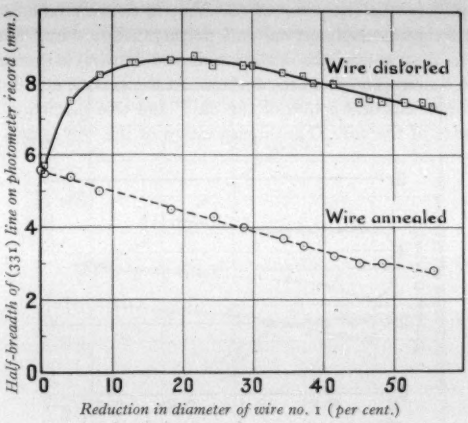

Fig. 1. Effect of drawing on half-breadths B_1 and B_2. (Initial diameter of wire 1·645 mm.)

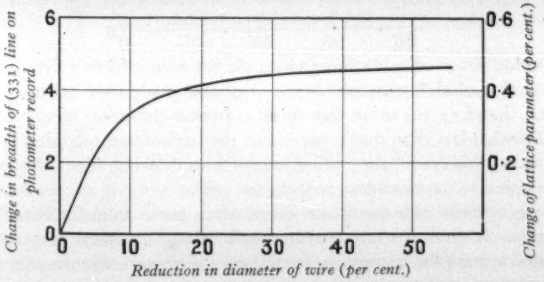

Fig. 2. Effect of drawing on distortion S. (Initial diameter of wire 1·645 mm.)

parts, (i) the quick rise in degree of lattice-distortion in the early stages of drawing, and (ii) the region of a maximum value to which the degree of distortion steadies up as the wire is further drawn. The second point of interest is that orientation only begins to make its appearance when the distortion has practically attained the steady maximum value.

Figure 3 shows the variation in degree of distortion across the section of three typical specimens at which the external diameter had become 1·578 mm., 1·500 mm.

and 0·852 mm. respectively. The value of S for each specimen is plotted against the decrease in diameter as the surface was etched away. It appears that in the less drawn wires there is a region of diminished distortion just under the surface. This diminution disappears, however, on further drawing, and in the later stages the distortion is practically constant across the wire. This observation agrees with that of Burgers[3] on the sharpness of the $K\alpha_1$, α_2 doublet reflected at different depths within tungsten single-crystal and polycrystalline wire. The explanation advanced here is that, at first, the distortion due to drawing is greatest at the core of the wire and decreases towards the surface. At the surface additional distortion is caused by the mechanical action of the die[5] and this distortion will decrease toward the centre of the wire. The superposition of the two effects will produce

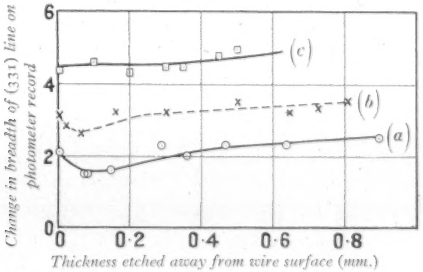

Fig. 3. Variation of S across section. Initial diameters: (*a*) 1·578 mm.; (*b*) 1·500 mm.; (*c*) 0·852 mm.

the diminution observed below the surface. As the wire is further drawn the steady maximum value of distortion will spread over the whole wire. It must be noted that at first, therefore, the mean distortion taken over the whole section of the wire will be somewhat less than that measured at the surface and recorded in figures 1 and 2, but that this discrepancy will diminish with drawing. The influence of this point is referred to in connection with figure 4 (*b*).

Figure 4 (*a*) deals with the second set of wires drawn from the initial diameter of 0·762 mm. It gives the temperature-coefficient of electrical resistance of the wires plotted against the percentage reduction of diameter. Figure 4 (*b*) gives the change in degree of distortion of the same wires plotted against the percentage reduction of diameter on the same scale as figure 4 (*a*). The first observation of note is that in each case we have a similar type of curve. The maximum of the distortion is, however, reached before that of the temperature-coefficient. This is to be expected since, as discussed in connection with figure 3, the mean value of the distortion throughout the wire is less at first than the distortion at the surface recorded in the graphs. It is the mean value which should be taken into account in the comparison of figure 4 (*b*) with the temperature-coefficient curve. This consideration would increase the resemblance. Unfortunately the application of an

accurate numerical correction based on figure 3 is complicated by the difficulty of maintaining a constant diameter during the etching. We emphasize, however, the similarity in the early rise and the attainment of a steady maximum at virtually the same reduction. The second point of note is that the orientation again does not occur until the distortion has reached its maximum value. Further drawing therefore, whilst increasing the amount of orientation, leaves the temperature-coefficient of resistance unaltered. This contradicts the view, sometimes advanced[6], that orientation affects electrical properties in metals based on a cubic lattice.

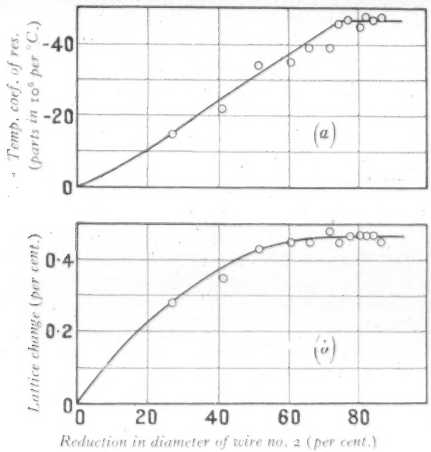

Fig. 4. Lattice change and temperature-coefficient of resistance.
(Initial diameter of wire 0·762 mm.)

§ 4. DISCUSSION OF RESULTS

The above results give rise to the following considerations.

(i) Orientation begins when distortion in the constantan is a maximum. This suggests that the process of deformation produces, in order, lattice-distortion, slip, and orientation as a result of slip. The range occupied by distortion may be negligible as in pure aluminium[2], small as in pure copper, or large as in constantan and steel.

(ii) The cold-drawing of constantan produces a maximum value of lattice-distortion which further drawing merely maintains. It is suggested that the maximum distortion attainable is a definite quantity for one metal or alloy, and different for different metals or alloys. It is hoped to follow up this point.

(iii) The variation of the temperature-coefficient of resistance of constantan is practically proportional to the change in degree of distortion produced by cold-work. It would follow that a material capable of exhibiting a large stable change in

lattice-distortion will also be capable, as a result of some treatment, of possessing a wide range in the value of given physical properties influenced by the lattice-disturbance. Thus the effect of alloying iron with other elements may well be merely one means of varying the range of distortion which the iron lattice will withstand, and consequently the range of certain properties—a point so characteristic of steel. On the other hand, a metal which orients quickly on being rolled or drawn would have fairly constant properties as far as the distortion factor is concerned, whatever the heat or work treatment.

The degree of distortion a lattice will exhibit before it slips or breaks down appears to be a quantity of fundamental physical importance.

§ 5. ACKNOWLEDGMENTS

In conclusion the author wishes to record his thanks to Dr G. Shearer for help in securing specimens, and to Dr G. W. C. Kaye for his interest and for providing facilities necessary for the research.

REFERENCES

(1) R. S. J. Spillsbury, *J. Sci. Inst.* **6**, 357 (1929).
(2) V. Dehlinger, *Z. für Krist.* **65**, 615 (1927).
(3) W. G. Burgers, *Z. für Phys.* **58**, 11 (1929).
(4) W. L. Fink and K. R. van Horn, *Inst. of Met.* **44**, 241 (1930). (Discussion.)
(5) W. A. Wood, *Phil. Mag.* **11**, *Suppl.* 610 (1931).
(6) R. W. Drier and C. T. Eddy, *Trans. Am. Inst. Met. Eng., Inst. Met. Sec.* p. 140 (1930).

DISCUSSION

Dr G. Shearer referred to the fact that selective orientation begins to make its appearance only when distortion has practically attained its maximum value (p. 71). It would be interesting to find out whether a similar rule holds in the case of other alloys and metals, and if so, whether the point where distortion is a maximum and selective orientation begins is masked by changes in other physical properties.

Dr E. H. Rayner: Methods of studying the internal structure and stability of nickel-copper alloys are of special interest from the point of view of their use for standard resistances. The reason is that a very small temperature-coefficient is desired in such resistances in order to be able to attain accuracy of measurement of the order of a few parts in a million. Most pure metals have coefficients of the order of 4000 parts per million per degree centigrade, but it happens that if nickel be added to copper to the extent of Cu 66 to Ni 34, the coefficient becomes zero and thereafter somewhat negative, until at a ratio of Cu 45 to Ni 55 it is zero again, rising at 100 per cent. Ni to a value of 6700, that for copper being 4300. Constantan is the term given to the alloy having zero coefficient with the larger proportion of copper. It is more ductile and cheaper than the other.

The disadvantage of the copper-nickel alloy is its large thermo-e.m.f. against copper, about 40 μV./° C. Weston, of America, found that the addition of a suitable proportion of manganese would reduce this to about 2 μV., while retaining a low temperature-coefficient. Secular stability of resistance has, however, proved difficult to attain, and any information on the internal structure of manganin and other alloys suitable for use in the making of standard resistances cannot fail to be of value.

Mr B. P. DUDDING: There is one aspect of this paper to which I would like to call attention. The author claims to have established that distortion of the lattice takes place by cold-working. In similar studies made on tungsten it was never possible to say definitely that the diffusion in the ray pattern was produced by lattice-distortion, since the presence of very small crystallites produced by cold-work would cause similar diffusion. If we assume that the author can establish his claim to differentiate between these two effects, then it would be of interest to know if the anomalous behaviour of nickel-chrome alloys with regard to change of resistivity, on being cold-worked, can be shown to be associated with a similar lattice-distortion.

AUTHOR's reply: Work on the lines suggested by Dr Shearer is in progress, and the relation between onset of orientation and distortion appears to hold for a number of other metals.

Mr Dudding wonders whether it is possible to distinguish the diffusion caused by distortion from that caused by the presence of very small crystallites. This is merely a matter of accuracy of measurement, and when the broadening is large, as for constantan, then there should be no difficulty in differentiating between the two effects. For, firstly, the change in breadth of a line depends on its angle of diffraction in a way which is different in the two cases. Also, secondly, the presence of distortion may, and usually does, produce a shift in the position of maximum intensity of the spectral lines; such a shift cannot occur in the case of the small-crystal effect. This aspect of the question has been treated in many recent researches such as those referred to in the text, and, on that account, was not given prominence in the paper.

WIRELESS ECHOES OF SHORT DELAY

By E. V. APPLETON, F.R.S. and G. BUILDER, B.Sc.

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ABSTRACT. An account of a simple method of producing short pulses of radio-frequency energy is given, together with notes on its application in the investigation of wireless echoes of short delay. Details of simultaneous visual and photographic methods of delineating such echoes are also described. The discussion of sample records and results serves as a basis for drawing conclusions concerning the relative advantages of the frequency-change and group-retardation methods of investigating the ionized regions of the upper atmosphere.

§ 1. INTRODUCTION

One of the two methods most commonly used for exploring the electrical structure of the upper atmosphere is that involving the measurement of the time required for a brief wireless signal to travel upwards to the ionized region and back. This quantity is most conveniently determined by arranging for an emitting station to send out very short pulses of radio-frequency energy and measuring, at a point a short distance away, the difference between the times of arrival of a particular signal pulse via the ground and via the atmosphere. It is readily seen that such measurements give the quantity $\int \frac{ds}{U}$ for the atmospheric wave track, where U is the group velocity with which the waves travel along an element of path ds. Moreover, since, for the portion of the wave track within the ionized region, U is less than the velocity c of electromagnetic radiation in free space, the equivalent path, $c\int \frac{ds}{U}$, of the atmospheric waves is greater than the actual path s. The equivalent height of the reflecting region, obtained by simple triangulation from the atmospheric wave equivalent path and the ground path, is therefore greater than the actual height at which deviation takes place.

In a previous communication* a comparison was made of the two methods principally used for equivalent-height measurement, and it was shown that both the frequency-change method and the group-retardation method (that mentioned above) give substantially the same result. Up to the present, however, the frequency-change method has been used predominantly in England while the group-retardation method has been similarly used in America. Although, as has been mentioned, the two methods should give similar results, somewhat divergent conclusions have been drawn by the British and American investigators from their observations. It therefore seemed to us desirable to develop the use of the group-retardation method in this country and compare it with the frequency-change method we have previously used exclusively.

* E. V. Appleton, *Proc. Phys. Soc.* **41**, 43 (1928).

The present communication gives a brief account of a simple method of producing the short pulses required in the group-retardation experiment together with notes on possible methods of delineating the short-delay echoes which occur. Some sample records are also discussed and serve as a basis for drawing conclusions as to the relative advantages of the frequency-change and group-retardation methods of measuring equivalent heights. It is hoped to give a more detailed account of observations made by the echo method in a future communication.

§ 2. THE PRODUCTION OF SHORT RADIO-FREQUENCY PULSES

Various methods of producing the short radio-frequency pulses necessary for the group-retardation experiment have been previously described. Breit and Tuve* and Dahl and Gebhard† used an alternating voltage of 500~, amplified by a transformer, and applied to the grid of a set of amplifying tubes which linked a master oscillator to the oscillator proper. By applying also simultaneously a fixed negative grid-bias to the amplifying tubes it was arranged that these tubes functioned during part only of each positive half-cycle of the grid potential. In this way radio-frequency pulses having a duration of about 0·0007 sec. spaced 0·002 sec. apart were obtained. As a result of our own experiments we believe that such pulses, because of their relatively long duration and close spacing in time, could hardly have been very satisfactory.

Later Tuve and Dahl‡ obtained shorter pulses by using a multivibrator circuit in an unbalanced condition. A transformer in the plate-supply circuit of the multivibrator translated the square-topped current pulsations into voltage pulses, alternately positive and negative, of very short duration. These voltage pulses were applied to the grids of amplifying tubes already strongly negatively biassed so that only positive pulses were effective. Pulses of 0·00025 sec. duration were obtained in this way.

More recently Goubau§ and Goubau and Zenneck‖ have described experiments in which the low-frequency pulses applied to the grids of the amplifying tubes were produced by means of a highly-saturated iron-cored choke, connected in series with a condenser and the secondary of a transformer, the primary of which was fed by a 500-cycle generator. Pulses of duration 0·0001 sec. were obtained¶.

In the experiments to be described no special modulating device has been used at all. It has been found that if the grid leak of an ordinary continuous-wave triode transmitter is increased to a high value, the generator automatically produces the desired short pulses of radio-frequency energy alternating with uniform periods of quiescence. This property of a triode oscillator has been previously used in pro-

* *Phys. Rev.* **28**, 554 (1926).
† *Proc. Inst. Rad. Eng.* **16**, 290 (1928).
‡ *Proc. Inst. Rad. Eng.* **16**, 794 (1928).
§ *Phys. Zeit.* **31**, 333 (1930).
‖ *Zeit. für Hochfrequenztech.* **37**, 207 (1931).

¶ Since the above was written E. L. C. White has published (*Proc. Camb. Phil. Soc.* **27**, 445 (1931)) an account of two novel methods of producing the radio-frequency pulses. Short flashes of light of duration 0·00025 sec. were obtained by mechanical means and were used to modulate a wireless transmitter via a photo-electric cell and amplifier.

viding a linear and unidirectional time-base for cathode-ray oscillographic delineation of wave-form, and an account of its action has been already given*. Briefly the action depends on there being a difference between the mean negative grid potentials for starting and stopping the generation of oscillations. When oscillations start, the partial insulation of the grid causes the mean grid potential rapidly to assume a negative value numerically greater than that required for stopping the oscillations. The oscillations are thereby rapidly quenched and do not start again until the negative grid potential has fallen (by the leakage of electrons from grid to filament via the high-resistance leak) to the appropriate value for the re-starting of oscillations.

§ 3. THE EMITTING STATION

The sending station used in these experiments was located in the Electrical Engineering Department at East London College, Mile End Road, and consisted of a series-fed valve oscillator of tuned-grid tuned-anode type. This type of circuit was preferred because with it the excitation can be readily adjusted to any desired extent.

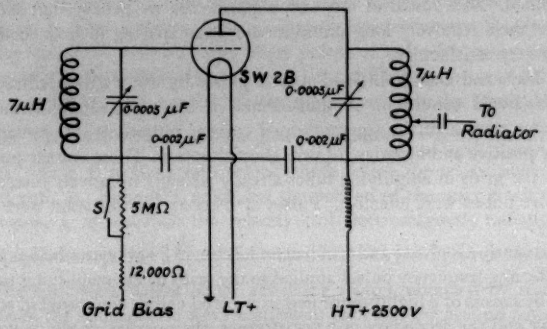


Fig. 1. Emitter circuit.

Figure 1 shows the circuit with the values of the components used. Small inductance/capacity ratios were used in the tuned circuits and, with only 1 μF across the anode supply and switch S closed, a pure steady c.w. signal was generated. A Mullard SW 2 B (200 watts anode dissipation) was employed, the requisite filament and plate voltages being obtained from a 500-watt Newton motor-generator yielding 2500 and 12·5 volts for the anode and filament supplies respectively.

The radiating system was a half-wave Hertz antenna suspended horizontally between poles standing about 20 ft. above the buildings. This was fed through a single wire from a tapping on the transmitter plate inductance, a d.c. stopping

* E. V. Appleton, R. A. W. Watt and J. F. Herd, *Proc. Roy. Soc.* A, **111**, 672 (1926).

condenser being inserted in the feeder. To carry out with the same system experiments using the frequency-change method a small variable condenser (not shown in diagram) was tapped across a few turns of the anode circuit inductance. Variations of the capacity of this condenser gave the required small variations of emitted frequency.

To convert the system into a pulse-generator, switch S is opened so that a high resistance is introduced into the grid circuit. It is found that, with values as shown, pulses of duration of 0·0001 sec. or less, spaced in time 0·02 sec. apart, are generated. It will readily be seen that the duration of the pulse produced depends on the time required for the grid condenser to charge up negatively so that small values of this condenser favour the production of very short pulses. The periods of quiescence between successive pulses is determined by the time-constant of the grid circuit and thus by both the value of the grid condenser and that of the grid leak.

To check the nature of the signal pulses emitted, a local monitor oscillograph circuit was used. This consisted of a coil and rectifier, loosely coupled to the transmitter, the output of which was applied to one pair of deflecting plates of a cathode-ray oscillograph. A linear time-base voltage was applied to the other pair of plates and synchronized with the frequency of occurrence of the pulses. With the monitor system used in this way it was found that the optimum adjustment of the excitation was such that the mean anode current was a minimum. Such conditions gave the shortest pulses to be obtained with given grid-circuit constants. In fact this adjustment is really the only one necessary, and after we had learned this we found that a monitoring system was superfluous.

§ 4. THE RECEIVING STATION

The receiving apparatus was housed in the wireless hut on the roof of the East Wing of King's College, London, at approximately 5 km. from the sending station.

The receiving assembly was designed to permit measurements by both the frequency-change method and pulse-retardation method. It is shown schematically in figure 2. On account of the high "electrical noise-level" at King's College, especially in the day-time, a small frame was used almost exclusively as the antenna system. The radio-frequency amplifier was a two-stage screened-grid valve set of conventional design. This was followed by a balanced detector stage in which circuit an Einthoven galvanometer could be included for work with the frequency-change method.

For observations by the group-retardation method the signal output of the detector stage was fed into a low-frequency amplifier for magnification to a suitable level for recording with a high-speed oscillograph. Since the signals to be amplified consist, as has been mentioned, of very short pulses of about 10^{-4} sec. duration, it is necessary for the amplifier to have a uniform response over as wide a range of frequencies as possible. The detector and amplifier constants indicated in the diagram have been found suitable, but care must be taken, in assembling the set,

to minimize stray capacities so as to reduce high-frequency losses. Decoupling elements in the feed circuits were found useful in ensuring stability.

For photographic recording a Duddell three-element high-frequency oscillograph, made by the Cambridge Instrument Company, was used, the scale sensitivity of which is about 0·3 mm./mA. One of the vibrators was included in the anode circuit of a triode of high mutual conductance, the grid circuit of which was connected to the output stage of the low-frequency amplifier. To obtain a time-base on the record another vibrator was fed in a similar way from a low-frequency triode oscillator. The third vibrator was not used.

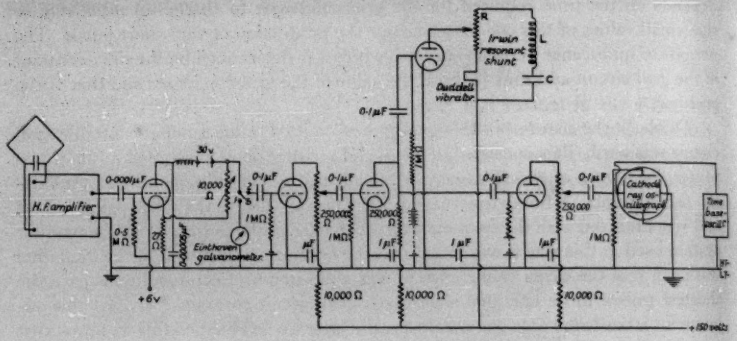


Fig. 2. Receiver circuits.

Much difficulty has been experienced owing to unsatisfactory damping of the oscillograph vibrators. The large changes in the viscosity of the damping oil, due to variations of temperature, make it impossible to maintain the moving systems in a critically damped condition. To obviate this difficulty the Irwin resonant-shunt method* of damping has been employed. In this method the electrical circuit, consisting of a series resonant shunt and the vibrator, is critically damped by means of the resistance R, figure 2, the most suitable tapping point on which depends on the natural damping of the vibrator. Although this method does not appear to be generally satisfactory with a bifilar suspension if the natural damping of the vibrating system is low, we have found that it works very well indeed with a vibrator about one-half damped in oil. In this connection it should be noted that the temperature changes of oil-viscosity affect not only the damping but also the natural frequency of the vibrator, so that a change of oil-temperature necessitates a slight adjustment of the shunt circuit, as well as an alteration of the tapping point on R, to restore optimum conditions.

For photographic recording of the received signals a rotating drum camera carrying a strip of sensitive paper about 20 in. long was used. The camera was

* J. T. Irwin, *Oscillographs*, p. 91.

belt-driven by a synchronous a.c. motor driven from the mains. Illumination was provided from a 60-volt 6-amp. arc with automatic feed.

For the visual observations a von Ardenne cathode-ray oscillograph, connected to the low-frequency amplifier as shown in figure 2, was used. The time-base, which was linear and unidirectional, was synchronized in stroke-frequency with the frequency of pulse-emission of the sending station, so that a stationary screen image was produced showing the ground pulse and any echo pulses present. A more detailed account of the circuit used for the production of the time-base voltage is given in an appendix. For visual work the oscillograph anode voltage was about 1500 volts, but this was increased to 3000 volts when it was desired to take photographs of the screen image.

§ 5. SOME TYPICAL RECORDS

In the plate, (*a*) to (*g*), are reproduced some typical records of the received pulses obtained with the Duddell oscillograph. They were all obtained at King's College with the sending station at East London College. The wave-length used throughout was 80 metres. Except where the contrary is stated the time scale shown under the record is that produced by an alternating current of 1115 ~. In each case the first impulse (marked *G*) is that received direct via the ground, the subsequent pulses being due to waves reflected by the upper atmosphere.

Record (*a*) (0337 G.m.t. July 16, 1931) shows the reception of the ground signals *G*, *G* without any echoes. It is of interest in showing the sharpness of the pulses used.

Record (*b*) (0355 G.m.t. July 16, 1931) is to be compared with record (*a*) and shows the reception of a ground signal *G* followed by a singly-reflected signal F_1.

Records (*c*) (June 15, 1931) are of interest in confirming results previously obtained in England with the frequency-change method of measuring equivalent heights, in that they indicate reflections from two regions at different heights in the upper atmosphere. Record (*c*1) taken at 1830 G.m.t. illustrates a singly-reflected pulse E_1 from the lower of these two regions (region *E*). Record (*c*2) taken at 1850 G.m.t. shows a singly-reflected pulse E_1 from the lower region and a singly-reflected pulse F_1 from the upper region (region *F*). Record (*c*3) taken at 1910 G.m.t. shows that as sunset (2020 G.m.t.) was approached the singly-reflected pulse E_1 from the lower region was less intense while that from the upper region F_1 was much more marked. A pulse F_2 indicates double reflection from the upper region.

Record (*d*) (2300 G.m.t. June 18, 1931) illustrates a case of multiple reflection up to the fifth order. The first order echo F_1 shows signs of being composite.

Multiple reflection up to echoes of the eighth order has been observed at times.

Record (*e*) (2115 G.m.t. July 15, 1931) illustrates simultaneous singly-reflected signals from both regions *E* and *F* together with an echo *S* of comparatively long delay, the origin of which is doubtful since it cannot be related to the others in a simple manner.

Record (*f*) (0430 G.m.t. July 16, 1931) illustrates a phenomenon frequently observed during the night hours, namely the splitting of an echo from region *F* into two components F_1' and F_1''. It will be seen that in the second-order echoes only F_2' is appreciable.

Record (*g*) (2247 G.m.t. July 15, 1931) shows the splitting of both first and second-order echoes from region *F*.

Records (*f*) and (*g*) illustrate a phenomenon which was first observed some years ago using the frequency-change method. In this case the splitting of the down-coming waves into two components of slightly different equivalent path is shown as the superposition of two sets of interference fringes of slightly different period. For example, a typical early-morning record of such interference maxima and minima is shown in record (*h*) taken at 0330 G.m.t. on April 26, 1929, with a wave-length of 100 m. Here it will be seen that, in addition to the large number of fringes produced by interference with the ground waves and the down-coming waves, there are about $3\frac{1}{2}$ fringes due to interference between the two down-coming waves F_1' and F_1''. Record (*g*) is of interest also in showing the rapid variation in the intensity of the down-coming waves which very frequently takes place within a fraction of a second. Such variations of intensity are very strikingly shown on the cathode-ray oscillograph during visual observations.

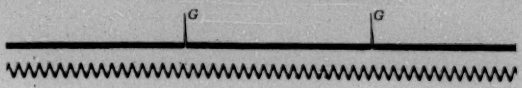
Record (*i*) is a snap photograph of the echo pattern commonly observed on the cathode-ray oscillograph screen. The ground signal *G* and a singly-reflected signal from region *F* are shown. The base-line frequency when this record was taken was 80 ~, so that the whole length of the base corresponds to a time of about 12 milliseconds.

§ 6. SOME EQUIVALENT-HEIGHT MEASUREMENTS

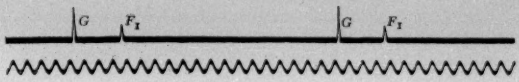
In figure 3 is shown the variation of equivalent heights obtained for the night July 15–16, 1931, using a wave-length of 80 metres and employing the group-retardation method.

It is seen that reflection took place at times from the Kennelly-Heaviside layer (region *E*) at a height of about 110 km., but that during the greater part of the night the waves penetrated to region *F*. It will further be seen that the reflected signals from region *F* were very frequently split into the two components F_1' and F_1''; examples of this occurrence have been given above. It is not definitely certain to what this splitting of the echo into two components is due; but a possibility is that we are dealing with the rays, elliptically polarized in opposite senses, which travel in the ionized region with different group-velocities, owing to the magnetic field of the earth.

The absence of echoes during the period from 0208 to 0335 G.m.t. is probably due to electron limitation, the 80-metre waves having penetrated both regions *E* and *F*. If this is so, since we are dealing with a case of approximately normal incidence, it is possible to obtain a superior limit to the ionization content of region *F*. This is found to be $3{\cdot}5 \times 10^5$ electrons per cm.[3]



(a)



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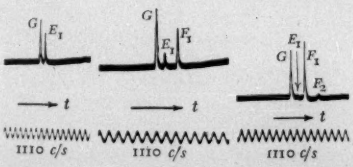
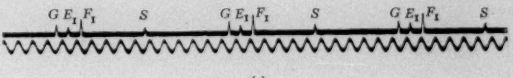
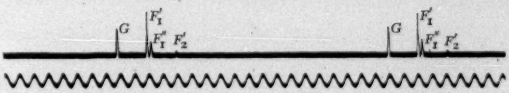


(c 1) (c 2) (c 3)

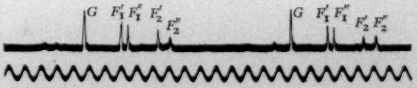
(d)



(e)

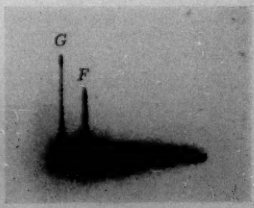


(f)



(g)

(h)



(i)

To face page 82

600
500
400
300
200
100
0
160
Equivalent Heights (feet)

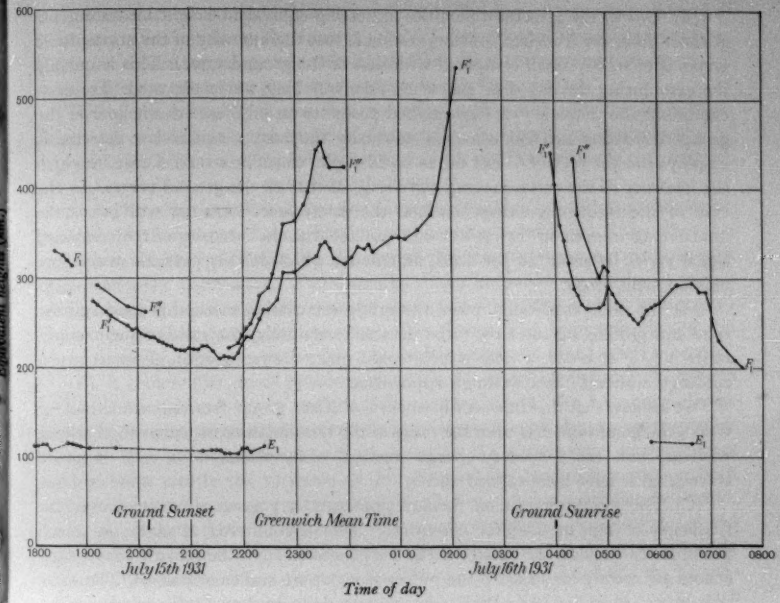


Fig. 3.

§ 7. DISCUSSION OF RESULTS

Our experience with the use of the group-retardation method has enabled us to compare the relative advantages and disadvantages of this method and the frequency-change method previously used. These points are discussed immediately below under various heads (A) to (E).

(A) So far as accuracy of the determination of the echo-time $\int \frac{ds}{U}$ is concerned, there is perhaps little to choose between the two methods. In the case of the frequency-change method we count the number Δn of interference maxima and minima produced by a continuous change of emitter frequency Δf, and the retardation time is given simply by $\Delta n/\Delta f$. Now Δn can usually be estimated to 1 per cent. and Δf to a much greater degree of accuracy, so that the overall accuracy is to about 1 per cent.

In the group-retardation method the retardation time has to be measured directly by means of a high-speed oscillograph. It is our experience that the photographic records obtained, when examined under a microscope, permit the determination of this time to about 1 per cent.

(B) One of the greatest difficulties in making equivalent-height measurements at short distances from the emitting station is that the intensity of the atmospheric waves is relatively small compared with that of the ground waves. This is notably the case during the day-time and when relatively long waves are used. For such conditions the frequency-change method possesses an inherent advantage over the group-retardation method when, as is usually the case, a square-law detector is employed in the receiver. Let us, as an example, consider a typical case in which the intensity of the down-coming wave is $\frac{1}{10}$ of that of the ground waves. In the case of the frequency-change method the interference maxima will be to the interference minima in the ratio $(1{\cdot}05/0{\cdot}95)^2$, so that the intensity of the combined signal varies by about 20 per cent., an amount which readily permits an accurate measurement of Δn.

For the same conditions, when the group-retardation method is used and the echo and ground signals have to be detected separately, the ratio of their amplitudes is $(\frac{1}{20})^2$ or 0·002, a value which would make the recognition of the echo exceedingly difficult if not wellnigh impossible.

We believe that this inherent insensitivity of the group-retardation method for weak echo conditions has been the cause of the restriction of its use to short waves, whereas, with the frequency-change method, observations on a wide range of wave-lengths have been carried out.

(C) The group-retardation method possesses very great advantages over the frequency-change method for conditions, often encountered at night, in which multiple echo signals are obtained. As has been seen from the records above, such echoes are readily identified if the pulses sent out are sufficiently short. The same convenient separation of multiple signals can be delineated on a cathode-ray oscillograph and their variations in intensity can be watched continuously, accurate records being taken by means of the high-speed oscillograph when desired. It is true that, in the case of the frequency-change method, a long experience in examining the records may enable us to unravel a case in which, say, two sets of down-coming waves are present, but it is certain that more complicated cases where there are many echoes would defy even the most experienced. Moreover, during the course of an experiment with the frequency-change method, the nature of the changes that are occurring are not known until the Einthoven galvanometer records are developed, whereas, with the dual representation scheme we have adopted for the group-retardation method, we are aware of the nature of the intensity and retardation of the echoes all the time, and no interesting features are necessarily missed.

(D) Another advantage of the group-retardation method with the quasi-stationary delineation of the received echoes described above is to be found in connection with the measurement of the maximum ionization-content of the Kennelly-Heaviside layer (region *E*). As has been mentioned there are two ionized regions in the upper atmosphere at which waves are reflected, the upper region *F* being richer in ionization than the lower region *E*. If, therefore, a series of mean frequencies are used for equivalent-height determinations, a discontinuity is found

in the curve representing equivalent height as a function of frequency. The discontinuity corresponds to the frequency which just penetrates the lower region. When the critical frequency is known it is possible, if certain assumptions are made, to calculate the maximum ionization-content in this lower region.

To find the critical frequency it is necessary to obtain data for the equivalent-height curve for a selected range of frequencies in which the discontinuity is expected. Such observations have been made with the frequency-change method, but this method is clearly somewhat laborious.

Since, however, it is found that the variation of ionization with time is usually gradual, the change-over from reflection at one region to reflection at the other can very conveniently be studied by the use of the group-retardation method with quasi-stationary delineation. For example, if the ionization in the lower region is increasing, the pulses can be sent out on a radio-frequency slightly higher than that corresponding to the critical value. The change-over from reflection at the upper region to reflection at the lower can subsequently be noted when it occurs.

(E) It is clear that, in all problems in which it is necessary to know the relative phases of the ground waves and the down-coming waves, the frequency-change method possesses enormous advantages when compared with the group-retardation method. The photographic records obtained in the prosecution of the former method show clearly the variation of the phase of the down-coming waves both when the emitted frequency is constant and when its value is continuously altered. From such records it has been possible to study the temporal variation of the optical path of the atmospheric waves. By means of similar records made with differently oriented frame aerials it has also been feasible to determine completely the specification of the polarization of the down-coming waves. Such determinations have not so far been found possible with modifications of the group-retardation method.

§ 8. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board and is published by permission of the Department of Scientific and Industrial Research. We wish to express our gratitude to Prof. J. T. Macgregor-Morris for his kind offices in arranging for the site of the transmitter at East London College.

APPENDIX

A LINEAR TIME-BASE OSCILLATOR FOR CATHODE-RAY OSCILLOGRAPHY

The use of the neon tube as a relaxation oscillator for producing a linear and unidirectional time-scale on a cathode-ray oscillograph has been previously described by various writers*. The fundamental circuit from which such oscillators are derived is shown in figure 4.

* G. I. Finch, R. W. Sutton and A. E. Tooke, *Proc. Phys. Soc.* **43**, 502 (1931); and E. L. C. White, *Proc. Camb. Phil. Soc.* **27**, 445 (1931).

Such a circuit has the disadvantage that the time-base departs considerably from linearity unless a current-limiting device, such as a saturated diode, is used in place of the resistance R. Also, the difference between the critical voltages of a neon tube is too small to provide sufficient voltage-swing for the time-base, particularly when medium-voltage oscillographs are being used.

It should be noted that if sufficiently high potential is used for the battery B, figure 4, the variation of the potential across C with time is sensibly linear between the values of the striking and extinguishing potentials. Thus we have found that, if, with an ordinary 200-volt tube, the potential be increased to anything above 400 volts, the potential-change with time is sufficiently linear for practical purposes.

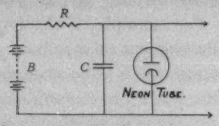


Fig. 4.

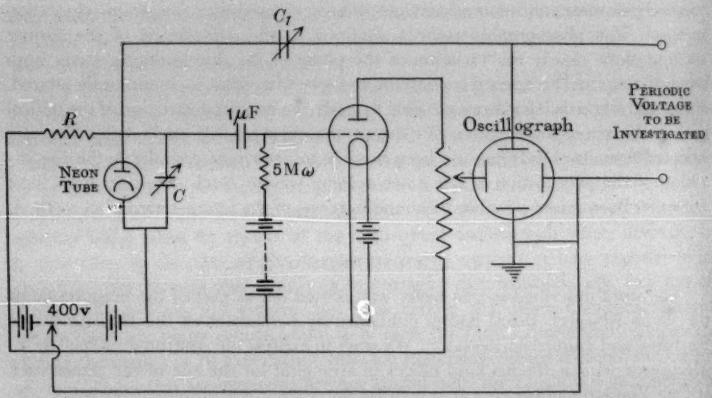


Fig. 5.

In order to overcome the difficulty that, with the simple circuit of figure 4, we are limited to a voltage-swing equal to the difference between the striking and extinction voltages, a valve amplifier may be added.

A suitable circuit incorporating both these modifications is shown in figure 5. We have found this to be very satisfactory and extremely easy to adjust, if attention is paid to the following points:

(*a*) The time-constant of the grid circuit of the amplifier must be of suitable value for the frequencies to be used. The values given are satisfactory down to less than 50 ~.

(*b*) The relation between the grid potential of the triode and the output potential across the anode resistance must be linear over the range of potentials used. An LS 5 valve gives such a relation for grid potentials from -60 to -10 volts, with a 1-megohm resistance, the actual voltage amplification being 5.

The voltage variation applied to the cathode-ray oscillograph may be readily varied to the desired value by a suitable tap on the anode resistance. In cases where it is desired to synchronize the time-base oscillator with any periodic voltage under investigation, the locking condenser C_1 may be used.

The circuit described is not difficult to maintain with satisfactory insulation for, since the linearity of the voltage-variation is not effected by the use of a non-linear resistance, spurious leaks are relatively unimportant. A further advantage is that the time-base frequency is little affected by adjustment of the operating conditions of the oscillograph, as for example, when a change is made from an anode potential of 1500 volts, suitable for visual observation, to one of 3000 volts, for photography.

THE EFFECTIVE MASS OF FLEXIBLE DISCS AND CONICAL DIAPHRAGMS USED FOR SOUND-REPRODUCTION

By N. W. McLACHLAN, D.Sc., M.I.E.E., F.Inst.P.

Received July 25, 1931. *Read October* 16, 1931.

ABSTRACT. An experimental method of measuring mechanical impedance, which is used to ascertain the effective mass of vibrating discs and conical diaphragms, is described. It is shown that the effective mass of a circular aluminium disc vibrating in air is zero at the centre-stationary and centre-moving modes. At a centre-stationary mode the effective mass attains a positive maximum before the zero value and a negative maximum thereafter. From the shape of the curves for a disc it is possible to interpret those obtained for conical diaphragms. In the latter case the curves depend upon the apical angle of the cone. Three types are illustrated: (*a*) a large cone having ψ equal to 160°; (*b*) a loud-speaker cone having ψ equal to 90°, with reinforced edge; (*c*) a loud-speaker cone having ψ equal to 90°, mounted on a rubber annulus. In case (*a*) the disc characteristics are clear, whilst in (*b*) and (*c*) the behaviour is modified owing to the greater degree of conicality. The rubber surround acts as an auxiliary resonant diaphragm, introducing an abrupt change in the effective mass. Finally, the effective mass of a rigid disc vibrating in a finite and in an infinite baffle is considered.

§ 1. INTRODUCTION

The use of conical diaphragms in the design of modern loud-speakers has stimulated scientific interest in their physical behaviour. When a diaphragm is driven by an alternating force, its effective mass varies throughout the acoustic register. During vibration in vacuo the effective mass depends upon the relation between the elastic and inertia forces and also upon internal losses. In air an additional loss is imposed owing to the radiation of sound, whilst an increase in effective mass occurs owing to divergence of sound waves from the diaphragm as source. The latter is known as the "accession to inertia" and has been treated in former publications*. The problem is best approached by consideration of the ideal case of a flexible annularly-driven homogeneous circular disc vibrating in vacuo, there being no loss whatsoever. This has been treated by A. G. Warren†. The curves of figure 1 show the apparent or effective mass M_e from zero frequency upwards. At $f = 0$, M_e is the natural mass of the disc. Owing to the interaction of elastic and inertia forces, M_e increases with the frequency and approaches infinity asymptotically when the disc vibrates with its centre stationary.

M_e
f

Thereafter it increases from $-\infty$ until at the first centre-moving symmetrical mode (one nodal circle) $M_e = 0$, i.e. the sum of the elastic and inertia forces is zero.

* Lord Rayleigh, *Sound*, **2**, 162 (2nd edition, 1894); N. W. McLachlan, *Phil. Mag.* **7**, 1011 (1929); **11**, 1137 (1931).

† *Phil. Mag.* **9**, 881 (1930).

Passing through zero M_e becomes positive until the next centre-stationary mode is reached, when the former cycle is repeated.

An approximate analogy to the first centre-stationary mode can be drawn from the impedance of the parallel condenser inductance circuit of figure 2*a*. The effective inductance is given by $L_e = L/(1 - \omega^2 LC)$. It increases from L at $f = 0$ to $\pm\infty$ at $\omega^2 LC = 1$ or $1/f = 2\pi\sqrt{(LC)}$, where the centre-stationary condition is simulated.

L_e, $L/(1-\omega^2 LC$

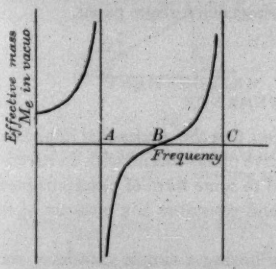


Fig. 1. Curve showing variation with frequency in effective mass of annularly driven loss free circular disc vibrating in vacuo.

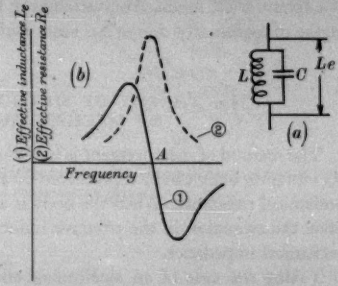


Fig. 2. (*a*) Parallel resistanceless LC circuit.
(*b*) (1) Effective inductance of circuit in 2*a*, (2) effective resistance of circuit in 2*a*, when loss occurs.

The effect of a resistance in series with the coil is to alter the effective inductance curve from the form of figure 1 to that shown in curve 1, figure 2*b*. The effective inductance cannot become infinite owing to the energy required to balance the loss. Similarly in the case of an actual disc vibrating in air, the effective mass cannot become infinite. Moreover, in practice there is central motion at a so-called centre-stationary mode arising from the supply of energy required to sustain vibration and overcome the internal and acoustic losses*. In the electrical case if R is the a.c. resistance in circuit, the effective inductance is zero when $\omega^2 = 1/LC - R^2/L^2$, but the effective resistance and the impedance are maxima. The current is, therefore, a minimum, and this corresponds to minimum amplitude at the centre of the disc. The effective inductance has a maximum and a minimum value approximately equally spaced on either side of the zero point. In the disc case this is illustrated clearly in figure 4. It is evident not only that the resistance causes the effective mass to alter from $\pm\infty$ to zero, but that the resonance frequency is altered slightly. In addition to the various losses there is also the accession to inertia, so that the frequencies of the centre-stationary modes would be expected to occur at values slightly different from those found by calculation from the *in vacuo* theory.

R

* Alternatively, the energy transmitted outwards exceeds that reflected from the edge so that annulment does not occur at the centre; i.e. the centre is not a node but a point of minimum amplitude. This is discussed later in the paper.

By examination of a curve showing the effective mass of a vibrating diaphragm at various frequencies, it is often possible to ascertain the mode of vibration, i.e. whether stationary-centre or moving-centre. We shall consider symmetrical modes of vibration only, since the variation in effective mass of a homogeneous symmetrical diaphragm due to radial modes is substantially zero. It has been found that with actual conical diaphragms having a seam which destroys the symmetry, there is a definite variation in effective mass, but it is small compared with that due to a symmetrical mode. The variation in the magnitude of M_e is a measure of the degree of asymmetry due to the seam and to heterogeneity of the paper.

§ 2. THEORY OF METHOD OF MEASUREMENT OF MECHANICAL IMPEDANCE

The method of measurement is based on the fact that the mechanical impedance of a vibrating body can be represented by an effective mass in series with an effective mechanical resistance. When the body is attached to some form of electromagnetic drive, the alteration in the effective inductance and resistance is a measure of the mechanical impedance.

Taking the case of an electromagnetic drive having a simple cantilever reed or equivalent elastic member, assumed to act in the same manner as a coil spring: when this is loaded by a diaphragm we obtain*:

$$R_m = C^2\omega^2 B/\{\omega^2 B^2 + (k_1 - \omega^2 M_e)^2\} \quad \text{.....(1)},$$

$$L_m = C^2 (k_1 - \omega^2 M_e)/\{\omega^2 B^2 + (k_1 - \omega^2 M_e)^2\} \quad \text{.....(2)},$$

R_m where R_m is the difference between the resistances with reed free and reed fixed;

L_m L_m the difference between the inductances with reed free and reed fixed;

B B {the mechanical resistance due to diaphragm loss and sound-radiation; the in-phase force per unit axial velocity *of the driving mechanism*†;

C C {the electromechanical conversion factor; the e.m.f. induced in winding of driver per unit velocity of reed; the force on the reed per unit current (absolute);

k_1 k_1 the coefficient of restitution of reed, i.e. force per unit deflection;

M_e M_e {the total effective mass on reed. This $= m_0 + M_a + M_1$; = (effective mass of reed) + (effective mass of diaphragm) + (accession to inertia).

The relationship between the various mechanical quantities is shown vectorially in figure 3*a*. B is the effective load or force component in phase with the axial velocity, whilst $(k_1/\omega - \omega M_e)$ is the effective mass-reactance component in quadrature with the load. When $k_1 > \omega^2 M_e$ the effective mass of the system, including

* *Phil. Mag.* **7**, 1017–1020 (1929); **11**, 3–4 (1931).

† The axial velocity of a point on the diaphragm varies according to the mode of vibration.

the reed, is negative owing to the reed compliance, whilst it is positive when $\omega^2 M_e < k_1$, i.e. above resonance. The total mechanical impedance is

$$\{B^2 + (k_1/\omega - \omega M_e)^2\}^{\frac{1}{2}}.$$

The analogy with an electrical circuit is obvious.

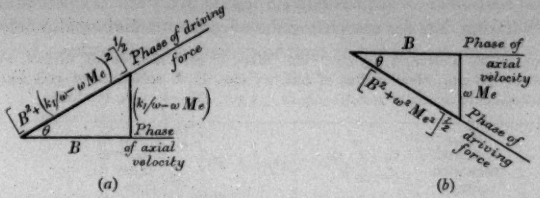


Fig. 3. (*a*) Vector diagram of reed-driven vibrating system. (*b*) Vector diagram of coil-driven vibrating system (no constraint of driver).

Solving (1) and (2) we obtain

$$B = C^2/R_m\,(1 + z^2) \qquad \text{.....(3),}$$

and the total effective mass of the system is M_e, where

$$M_e = \{k_1 - (C^2/L_m).z^2/(1 + z^2)\}/\omega^2 \qquad \text{.....(4),}$$

and

$$z = \omega L_m/R_m = \tan\theta.$$

In the case of a diaphragm driven by a moving coil $k_1 = 0$, and the total effective mass of the structure, including the coil, the accession to inertia and the influence of the surround is given by

$$M_e = -\,(C^2/\omega^2 L_m)\,z^2/(1 + z^2) \qquad \text{.....(5).}$$

The vector diagram for this case is shown in figure 3*b*. The quantity

$$z^2/(1 + z^2), \text{ or } 1 - \cos^2\theta$$

includes flexibility of the diaphragm and various losses. When these are negligible z^2 is large compared with unity so that $z^2/(1 + z^2) \doteqdot 1$. Then the formula reduces to $-\,C^2/\omega^2 L_m = M_e$, which is identical with that used in a former paper devoted to the measurement of accession to inertia*. When the loss component B is comparable with or greater than the mass reactance ωM_e, the quantity $z^2/(1 + z^2)$ is appreciably less than unity—see table 1 near the resonance frequency 120·6 ~.

§ 3. DATA FOR COIL-DRIVEN CIRCULAR ALUMINIUM DISC

A coil 2·5 cm. in mean radius, wound on a paper former having a free length of 2·5 cm., was securely fixed coaxially to an aluminium disc 10 cm. in radius and 0·055 cm. thick. The coil was situated in the radial field of a circular electromagnet

* *Phil. Mag.* 11, 1139 (1931), expression (2).

and the disc was suspended freely by several thin elastic threads, the natural frequency in the absence of the field being about 3 ~. Bridge measurements* of the inductance and resistance of the coil, free and fixed, were taken over a certain frequency band. Some of the observed and deduced data are given in table 1, whilst a curve showing the effective mass is plotted in figure 4.

Table 1: Showing data for computing the effective mass of an aluminium disc.

Radius of disc, 10 cm. Thickness, $5{\cdot}5 \times 10^{-2}$ cm. Mass, 47 gm. Mass of coil, former, adhesive and connecting wires, 7·8 gm. Mean radius of coil, 2·5 cm. D.C. resistance of coil without leads, 0·95 ohm. Accession to inertia at zero frequency, 3·5 gm. $C^2 = 2 \times 10^4$. No baffle.

Frequency (~)	L_m, motional inductance (henry)	R_m, motional resistance (ohms)	$z = \omega L_m/R_m = \tan\theta$	$z^2/(1+z^2)$	M_e, effective mass (gm.)	Remarks
0	—	—	—	—	+ 58·3	Natural mass (47 + 7· plus accession to i ertia (3·5)
60	$-6{\cdot}5 \times 10^{-4}$	10^{-2}	− 24·5	1·0	+ 216	First centre-station mode at 69 ~
75	$+7{\cdot}1 \times 10^{-4}$	3×10^{-2}	+ 11·1	1·0	− 127	
100	$+4{\cdot}2 \times 10^{-4}$	7×10^{-2}	+ 3·8	$9{\cdot}4 \times 10^{-1}$	− 113	
120	$+10^{-2}$	23·3	$+3{\cdot}2 \times 10^{-1}$	9×10^{-2}	$-3{\cdot}2 \times 10^{-1}$	First centre-movi mode. One circle minimum amplitu at 120·6 ~
120·6	0	39·5	0	0	0	
125	$-7{\cdot}2 \times 10^{-3}$	$6{\cdot}3 \times 10^{-1}$	− 9·0	1·0	+ 4·5	
150	$-1{\cdot}45 \times 10^{-3}$	4×10^{-2}	− 34·6	1·0	+ 15·4	
200	$-4{\cdot}5 \times 10^{-4}$	6×10^{-2}	− 9·5	1·0	+ 27·5	

Starting from $f = 0$, where limiting motion is assumed, M_e is the sum of the natural mass (disc plus coil) and M_1 the accession to inertia. Since no baffle was used throughout the experiments, M_1 in the neighbourhood of zero frequency is half its value with an infinite baffle†. The first centre-stationary mode occurs in the neighbourhood of 69 ~ (*A*), and the first centre-moving mode at 120·6 ~ (*B*). Near the centre-stationary mode M_e attains a positive maximum and then falls to a negative minimum as foreshadowed in the argument associated with figure 2*b*. The amplitude of the motion is a minimum when $M_e = 0$, since the effective mechanical resistance and impedance are then maxima.

The arithmetical values at the maxima are much in excess of the natural mass of the disc and coil. Rising from the negative maximum the effective mass becomes zero in the neighbourhood of the first centre-moving symmetrical mode at 120·6 ~. Thereafter it increases, but near 260 ~ there is a minimum value due to some irregularity. The second centre-stationary mode occurs at 410 ~ (*C*). The second centre-moving mode occurs at 484 ~ (*D*). The third centre-moving mode, which should occur in the neighbourhood of 1100 ~, is absent. Considered as a separate

* *Phil. Mag.* **11**, *loc. cit.*

† McLachlan, *J. Inst. E.E.* **69**, 613 (1931). It is assumed, of course, that the disc is actually moving.

vibrator, the disc within the coil has a mode near that of the outer annulus. Moreover, mutual interference suppressed both modes. When allowance is made for the mass of the coil and the portion of the disc within it, the frequencies of the modes determined experimentally are in close agreement with those calculated from Warren's analysis.

From the shape of the curve of figure 4 and the known behaviour of a flat disc it is possible to interpret the more complicated behaviour of conical diaphragms used for loud speakers, by examination of their effective mass curves.

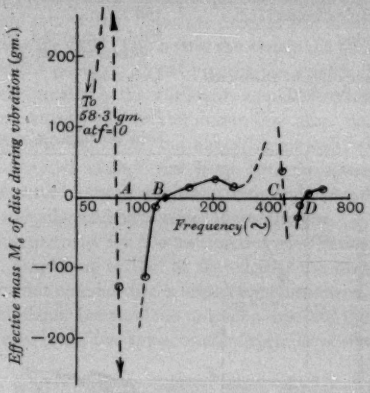


Fig. 4. Curve showing variation with frequency in effective mass of coil-driven aluminium disc 10 cm. radius. *A*, first centre-stationary mode; *B*, first centre-moving mode; *C*, second centre-stationary mode; *D*, second centre-moving mode.

In table 1 C^2, the electromechanical conversion factor, has been assumed constant for simplicity. Actually, there is a variation with frequency due to the iron loss in the circular magnet. The value of C^2 can be found at various frequencies by suspending the coil alone by fine elastic threads and measuring its inductance free and fixed. Then from (5), if M_c is the mass of the coil, $C^2 = -\omega^2 L_m M_e$. M_e

§ 4. DATA FOR LARGE CONICAL DIAPHRAGM

A flat disc is a cone with a plane apical angle of 180°. The radius being kept constant, if the angle is decreased to, say, 160° the disc characteristics will not be entirely lost. As the angle is reduced the properties peculiar to a conical diaphragm will assert themselves. At the extreme end of the scale when the angle is zero we have a cylinder, and its characteristics will be evident in a conical diaphragm, particularly when the apical angle is small. The increase in stiffness due to conicality is remarkable. In the cone treated in this section, the stiffness at the first centre-moving mode ensuing from an angle of 160° is equivalent to that of a disc

of the same material nearly 16 times as thick*. Data pertaining to this cone are given in table 2, and the effective-mass curve is plotted in figure 5. The diaphragm was coil-driven, the conditions being similar to those for the aluminium disc. Referring to figure 5 and starting at 100 ~, we see that the effective mass behaves with increase in frequency as it does in the case of the aluminium disc.

Table 2: Data for coil-driven paper cone.

Natural mass of coil and diaphragm	(7·84 + 40·16), = 48 gm.
Radius at base of cone	16·7 cm.
Plane apical angle of cone	160°
Condition at edge	Free, supported by three thin elastic threads
Natural frequency of cone on threads without magnetic field	Less than 3 ~
Baffle	None
Class of paper	Stiff Whatman 5 × 10^{-2} cm. thick

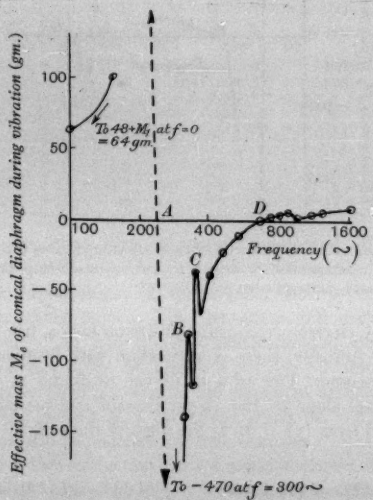


Fig. 5. Curve showing variation with frequency in effective mass of coil-driven cone (ψ=160°) 16·7 cm. in radius. *A*, first centre-stationary mode; *B*, first centre-moving mode; *C*, second centre-stationary mode; *D*, second centre-moving mode.

The first centre-stationary mode occurs about 240 ~ (*A*). Following the curve as it rises from a negative value of 470 gm. at 300 cycles, we are, from the similarity with figure 4, tempted to think that the first centre-moving mode (one nodal circle) occurs at 664 ~ (*D*). Strangely enough, however, tests with lycopodium

* This is treated in the *Philosophical Magazine*, **12**, 771 (1931).

powder revealed two nodal circles at 664 ~ but only one at 350 ~ (*C*). In both cases there were radial nodes also. R_m, the motional resistance, reached a decided maximum at *C*, so that we can regard this as the first centre-moving mode of vibration. A button microphone fitted with a thick wire feeler was drawn radially over the surface from the centre outwards. At 350 ~ there was a position of minimum motion, but no actual position of complete rest.

I have shown elsewhere* that when transmission and radiation losses occur there cannot be a position of complete rest on a vibrating diaphragm. The so-called nodal circle is actually a line of minimum amplitude. At any concentric circle on the diaphragm the energy transmitted outwards exceeds and is out of phase with that reflected inwards from the edge. The nearer the centre the greater the ratio (transmitted energy)/(reflected energy). Consequently, the smaller the radius of the so-called nodal circle the greater is the minimum amplitude of vibration. This effect was obtained with a centrally-driven aluminium disc also. Sand lay placidly on a circle when the amplitude was small, but danced vigorously when it was large.

The expression "nodal circle" has been used in connection with conical diaphragms, but the actual lines of minimum amplitude traced by the lycopodium powder were quite irregular and sometimes discontinuous.

Approximate measurement for the two modes gave ratios of the radii of the circles of minimum amplitude to that at the edge of the diaphragm as shown in table 3. The case of a flat circular disc is added for comparison. It will be seen that there is a close resemblance between the unloaded disc and the loaded cone, which shows that in this respect the two types of diaphragm have something in common.

Table 3: Comparison of radial nodes of cone and disc.

Mode of vibration	Cone (coil-loaded)	Disc (unloaded)†
First centre-moving	0·66	0·68
Second centre-moving	0·45, 0·8	0·39, 0·84

† In the absence of radial nodes.

It is clear that the shape of the diaphragm during vibration is determined partly by (*a*) transmission loss, (*b*) acoustic load, and (*c*) reactive load due to accession to inertia. If the preceding impedance measurements are repeated in vacuo, (*b*) and (*c*) disappear. Moreover, the shape of the diaphragm during vibration in vacuo is different from that in air, more especially at a centre-moving mode where the acoustic load is large. Thus the difference between the effective mass in air and that in vacuo is not the true value of the accession to inertia, although it may be an adequate approximation for certain purposes‡.

* Letter to *The Wireless Engineer and Experimental Wireless*, October, 1931.

‡ *Phil. Mag.* **12**, 814 (1931).

§ 5. DATA FOR LOUD-SPEAKER DIAPHRAGMS

The result of measurements on two conical loud-speaker diaphragms (1) with the edge reinforced by a narrow annulus of presspahn to prevent radial modes, (2) with a rubber surround, are portrayed graphically in figure 6. In case (1) starting from zero frequency, where M_e is the natural mass plus the accession to inertia, the effective mass gradually rises owing to increase in the accession to inertia up to 200 ~. Thereafter the rise is chiefly concerned with the elastic and inertia forces of the diaphragm structure, the action being similar to that of the aluminium disc and the flat conical diaphragm of the preceding section. The maximum at B_1 portends approach to a centre-stationary mode, whilst the minimum and subsequent rise at C_1 indicates the first centre-moving mode.

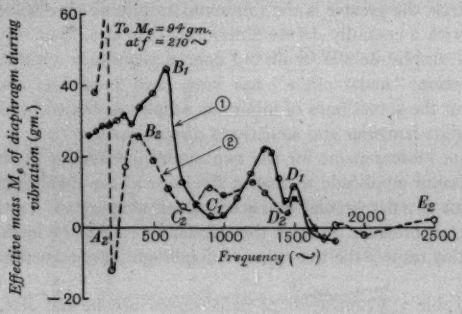


Fig. 6. Curves showing variation with frequency, in effective mass of coil-driven conical paper diaphragms. Radius = 12·2 cm.; $\psi = 90°$. Curve 1. Edge of diaphragm reinforced by narrow presspahn annulus to suppress radial modes. Curve 2. Edge of diaphragm bent over and supported by rubber annulus.

Beyond C_1 the effective mass rises until at about D_1 the second centre-stationary mode occurs. Thereafter M_e steadily falls and becomes negative. The second centre-moving mode occurs about 2000 ~. As with the former cone M_e (not shown graphically) is substantially zero. I have indicated elsewhere* that the stresses in cones and discs are entirely different and, moreover, the behaviour during vibration differs too.

Curve 2 relates to a diaphragm similar to that of curve 1 but mounted on a rubber surround. Resonance of the latter per se† introduces a condition at A_2 of the same nature as a centre-stationary mode (the diaphragm being the driving agent in this case) followed by a centre-moving mode. The remainder of the curve, modified in a degree by the surround, is similar to that of curve 1. The first centre-moving mode is at C_2 and the second about E_2. There are irregularities in each curve, particularly the second, due to asymmetry; for instance, to the seam and heterogeneity of the paper. It is clear, however, that except in the first centre-moving

* See *Phil. Mag.* **12**, 801 (1931). † *Phil. Mag.* **11**, 28 (1931).

mode the diaphragms behave in a somewhat similar manner to the aluminium disc cited in § 3. Moreover, an effective-mass curve is a useful guide to the mechanical behaviour of a vibrating system.

Table 4: Data for conical coil-driven loud-speaker diaphragms.

Radius at periphery of paper cone with reinforced edge (curve 1)	12·2 cm.
Radius at periphery of paper cone with rubber surround (curve 2)	12·2 cm.
Radial width of rubber surround	1·8 cm.
Apical angle of both cones	90°
Natural frequency of reinforced edge cone on elastic threads without magnetic field	Less than 3 ~
Natural frequency of cone as a whole on rubber surround without magnetic field	About 30 ~
Baffle	6 ft.^{2}

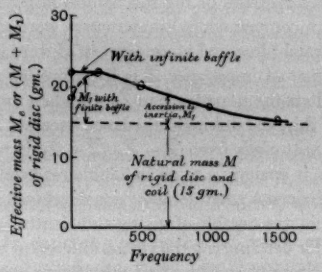


Fig. 7. Curves showing variation with frequency in effective mass of freely supported centrally driven rigid disc, (*a*) with infinite baffle, (*b*) with finite baffle 6 ft. square.

§ 6. EFFECTIVE MASS OF VIBRATING RIGID DISC

Finally it may be useful for purposes of comparison to give the case of a coil-driven rigid disc vibrating in (i) an infinite baffle, and (ii) a finite baffle. The former was treated elsewhere some years ago* and the data obtained from column 2 of table 2 of the former paper are shown graphically by the full-line curve of figure 7. The influence of a finite baffle, say, 6 ft. square, is shown by the dotted curve rising from $M_e = 18{\cdot}5$ gm. at $f = 0$. Here, as stated previously†, M_1 is reduced to one-half its value with an infinite baffle. With a baffle 6 ft. square and a disc 10 cm. in radius, M_1 at $f = 0$ is substantially half its value at 200 ~. So far as rigid discs or diaphragms are concerned, a baffle has a much greater effect on the low-frequency sound-radiation than it has on the accession to inertia.

* *Phil. Mag.* 7, 1011–1038 (1929). † See footnote (†) on p. 92.

DISCUSSION

Dr L. E. C. Hughes: We owe much to Dr McLachlan's exposition of the mode of operation of large-diaphragm loud-speakers, but although his publications have been spread over a period of years there seems to be little desire to use methods other than empirical in the design of commercial models. The method of measuring mechanical impedances described by Dr McLachlan is indirect and involves considerable calculation, which is inconvenient when a large number of measurements have to be undertaken in economic research. A direct method has been described by E. Mallett and R. C. G. Williams*. The present meeting seems an appropriate moment for describing an empirical modification of an electro-dynamic method due to K. Kobayasi† which I put forward for measuring the driving impedance of gramophone reproducers, but which can be modified easily for other applications. The impedance-machine consists of a pot-magnet, in the gap of which moves a former carrying two coils. One of the coils takes the driving-current while the other developes an e.m.f. proportional to the velocity of motion. The ratio of the current to the e.m.f. is a measure of the mechanical impedance $\overline{Z}$ of the system. It was found that, when comparatively heavy masses were attached to the former, the locus of $\overline{Z}$ was a straight line and increments of $\overline{Z}$ were linear with increments of mass. A heavy former of impedance $\overline{Z}$ is therefore used. To this is attached a known mass which alters the impedance to $\overline{Z}_1$. If, when the unknown impedance is attached in place of this, the driving-impedance becomes $\overline{Z}_2$, then the unknown impedance is obtained vectorially from $(\overline{Z} - \overline{Z}_2)/(\overline{Z} - \overline{Z}_1)$ in terms of the calibrating mass, which, of course, is analogous to an inductive reactance. For measuring the currents and e.m.fs. a Pederson potentiometer was developed and was manufactured by Messrs Tinsley. This was capable of giving sufficiently fine adjustments, about 1 in 500 up to 5000 ~, for determining the vector differences with suitable accuracy. By heavily loading the driving circuit so that the driving-current remains constant with small changes of mechanical impedance, the readings of the potentiometer dials may be plotted directly on graph paper with one slide-rule calculation. Such a method has perhaps more appeal to an engineer than to a physicist; unfortunately the method was not used except for demonstrating its validity.

Mr R. S. Whipple enquired whether the author had used Coker's method to locate the nodes in a celluloid diaphragm.

Prof. F. L. Hopwood suggested that cellophane would be more suitable than celluloid for the purpose proposed by Mr Whipple as it is doubly refracting and very homogeneous, and can be obtained in various thicknesses. The variation in accession to inertia with change of frequency depends upon the reciprocating flow of air across the nodal lines of the diaphragm, between places in opposite phases of vibration. To infer that a body always gives its fundamental tone when struck is not justifiable. Two steel balls one inch in diameter when tapped together give an audiofrequency sound, although the fundamental frequency of each is 136,000 ~.

* *J. Inst. E. E.* **68**, 560 (1930). † *Tohoku University Reports* (1929) and *I. E. E. Japan* (1928).

Mr T. SMITH remarked that Coker's method is applicable only under static conditions. It could not be extended to vibrating diaphragms without elaborate equipment designed to restrict observation to a particular phase of the strain cycle.

Mr A. G. WARREN: Reference has been made to other methods of measuring the effective mass of the diaphragm. The mechanical system is so complicated that I cannot but feel distrustful of any method which involves disturbing it by the connexion of any subsidiary measuring device. The author's method avoids this; measurements are made with the diaphragm under normal working conditions.

Mr G. A. V. SOWTER (communicated): I should like to mention that the author devised the present method some years ago when he was unable to test it. In 1927 he lent me a bulky manuscript prepared in 1926 and dealing with the theory of moving-coil apparatus. Two parts of this, dealing with the theory of a coil-driven rigid disc in an infinite baffle, were eventually published*. Another part contained the equations and method of measuring mechanical impedance described in the present paper. Dr McLachlan and myself have taken some 20,000 experimental observations on diaphragms, and several papers dealing with the subject have appeared from time to time in the *Philosophical Magazine* during the present year. The method described by Dr Hughes necessitates structural alteration of the diaphragm which will affect its mechanical properties and therefore its effective mass. Also the measurement of mechanical resistance seems to introduce complications and possibilities of appreciable error. Dr McLachlan's method as applied to moving-coil loud-speakers is unique since the diaphragm is tested under actual working conditions. It is a very simple method since R_m and L_m are determined in routine tests of electrical impedance, and M_s is merely a matter of arithmetic.

AUTHOR'S reply: In criticizing my method of measurement Dr Hughes fails to recognize that in modern science many highly accurate methods are indirect. Objection to an indirect method is akin to refusing payment by cheques or bank-notes in lieu of gold. As Messrs Warren and Sowter point out, the alternative methods would involve structural changes, thereby vitiating the results. The proposed alternatives are not without their defects. Mallett and Williams clearly state the difficulties of their tuning-fork method and its limitation to 500 ~. The mechanical resistance is found by plotting. Is this direct? To justify directness one spends precious minutes juggling with masses in the pious hope of ultimately finding a winner. It also appears that the auxiliary-coil method necessitates graph paper and a slide-rule. Is this what Dr Hughes means by "economic" research? Despite these little irregularities he says that my method "involves considerable calculation." Now the loud-speaker is used as it stands; L_0, R_0, C^2 and $x^2/(1+x^2)$ are extracted from tables or curves; L_1, R_1 are measured simultaneously and L_m, R_m are found immediately by subtraction. The calculation of effective mass and resistance is simple and the total time for all the operations at a given frequency is less than 10 minutes. This method has the advantage that the mass and resistance

* *Proc. R. S.* A. 122, 604 (1929); *Phil. Mag.* 7, 1011 (1929).

components are found separately. In Dr Hughes' method a "lumped impedance" is found from which the two components are derived roughly, by an approximation. This can hardly be regarded as accurate measurement.

My method can be extended to pick-ups and other appliances by aid of a stiff coil of small radius ending in a conical spider with suitable means of attachment. Some form of elastic centering device can be used if desired.

Mr Whipple's suggestion has not been tried owing to lack of apparatus. A stroboscopic effect could be obtained by interrupting the polarized light with a Kerr cell at a frequency slightly different from that of the vibrating diaphragm. This overcomes the difficulty mentioned by Mr T. Smith and would be an interesting experiment if the apparatus were available.

The sound concomitant with Prof. Hopwood's bicycle ball experiment is due to the initial "impulse." This covers the entire frequency spectrum, and the ear picks out the audible portion. It would be of interest and use to know how long a pure audible tone must be sounded before it ceases to be merely an impulse or noise. I imagine the time required would be of the order of 10^{-1} sec.

DEMONSTRATIONS

"The Effect of Mechanical Disturbance on a Neon Lamp." *Demonstration given on October* 16, 1931, *by* T. J. DILLON, M.Sc., and C. M. LOVETT, B.Sc., London (Royal Free Hospital) School of Medicine for Women.

The object of this demonstration was to show that, in certain circumstances, a discharge through a neon lamp could be temporarily stopped on the application of a mechanical disturbance or impulse.

A neon lamp was set up in a Pearson-and-Anson flashing circuit. The circuit conditions were so arranged that the smallest current which would support a steady discharge was flowing through the lamp. In the demonstration circuit a lamp of osglim beehive type, with its series resistance removed, was used in parallel with a condenser of capacity $4\mu F$, and a short-period galvanometer with lamp and scale could be connected in series with the lamp for reading the current. The lamp was mounted, in an ebonite ring, on a stand firmly clamped to the edge of the bench. The mechanical disturbance was produced by a loaded bar pendulum placed so that at the bottom of its swing it hit the stand on which the lamp was supported.

It was first shown that the magnitude of the impulse necessary to stop the discharge depended on the value of the current flowing through the lamp. The sensitivity was shown to alter with the direction of the impulse in a way that suggests that the effect may be due to the sudden movement of the electrodes relative to the glow. For the purpose of demonstrating that an impulse above a set value had been applied to the lamp, a second similar lamp was placed in parallel with the first, this second lamp lighting up when the glow had ceased in the first.

Many series of observations have been taken, with different types of lamp and condensers of varying capacity, showing how the applied momentum necessary to stop the glow increased with the current, until a value was reached above which the discharge could not be stopped by an impulse alone. In the most sensitive state the lamp could be blown out, and would respond to slight mechanical disturbances due to footsteps and to outside traffic.

(*a*) A contrivance for demonstrating the law of errors; (*b*) A new type of surface-tension-meter; and (*c*) A new type of static electrometer*. *Demonstrations given on November* 20, 1931, *by* Prof. KERR GRANT, M.Sc.

(*a*) *A contrivance for demonstrating the law of errors.* The instrument consists of a number of short metal bars loosely mounted on a spindle with a cast-on wheel by means of which the spindle and bars can be set in rotation. Another light bar mounted on a spring permits of the simultaneous arrest of all the bars, which are painted red on one side, black on the other. The experiment is equivalent

* A fuller description of these instruments will appear in the *Journal of Scientific Instruments*.

to tossing a handful of twenty coins, and its repetition enables a frequency-distribution to be determined.

(*b*) *A new type of surface-tension-meter*. The instrument depends upon the force exerted by surface-tension on a lamina or vane partly within and partly without a narrow channel in the liquid. This force is balanced by gravity or torsion in a light balance carrying the vane. Variations of surface-tension, the production of monomolecular layers and other phenomena can be demonstrated optically to large audiences. The sensitivity is of the order of 1 dyne but can be made much higher.

(*c*) *A new type of electrometer*. The moving element is a very light grid stamped out of sheet aluminium and controlled by a fine strip or wire fastened at top and bottom. There are field-plates similarly constructed. Used idiostatically the instrument shown had a range of 200 volts and followed a square law. Used heterostatically with a field $\pm$ 100 volts on the plates it had a sensitivity of 100 divisions (mm. at 1 metre) per volt with a period of about $\frac{1}{4}$ sec. It is portable and requires no levelling.

REVIEWS OF BOOKS

Newton: the Man, by Lieut.-Col. R. DE VILLAMIL, R.E., with a foreword by Professor ALBERT EINSTEIN. Pp. vi + 111 and frontispiece. (London: Gordon D. Knox, 106 Guilford St., W.C. 1) 3*s*. 6*d*. net.

This small but fascinating book is written round Col. Villamil's investigations in the Musgrave Library at Barnsley Park, Gloucestershire, and his most remarkable discovery of the "True and Perfect Inventory of all and Singular the Goods Chattels and Credits of Sir Isaac Newton" at Somerset House. It is divided into three parts: first an extraordinarily interesting and intimate account of Newton's personality and behaviour in everyday life; secondly a copy of the inventory and, lastly, a copy of the catalogue of Newton's library, made *c*. 1760, and discovered by the author at Barnsley Park.

It is to be feared that the portrait is not altogether pleasing. Newton had a cold temperament and, in his later years, was irritable; he was lacking in humour, indifferent to poetry and music, mean in small things and, in his house, had none of the beautiful furniture which characterised his period. On the other hand he was free from conceit, honest and held in the highest esteem by his contemporaries. He died intestate and in consequence an inventory of his belongings was taken by order of the Prerogative Court of Canterbury. His estate was "appraised att the sum of £31,821. 16. 10" but to make an estimate of his savings we must add to this £4000 lost on his holding of South Sea stock, another £4000 given to his great-niece, £500 paid for the translation of the *Opticks* and £200 paid for editorial work on the third edition of the *Principia*. This gives a total approaching £40,000, most of which must have accrued to him from the "profits of the coinage" which formed a large part of his dues as Master of the Mint. In his last years his total income must have exceeded £4000 per annum.

The library comprised 1896 volumes together with "above one hundredweight of pamphets and wast books." The catalogue is remarkable for its large content of theological works and of Greek and Latin classics; English classics are almost entirely absent. Among the scientific books there are no fewer than twenty-two by Boyle but no copy of the *Micrographia*!

Col. Villamil has made a most important addition to *Newtoniana* and the reader is advised to refer to the book for the remarkable history of the library itself and for many interesting sidelights on Newton's life and character.

D. O. W.

James Clerk Maxwell. A Commemoration Volume, 1831–1931. 8vo, pp. 146. (London: Cambridge University Press.) 6*s*. net.

Some of the ten essays by distinguished physicists which make up this volume were delivered as addresses at Cambridge during the Maxwell centenary celebrations in September. Its publication will help the public to understand something of the high appreciation in which Maxwell's scientific work is held, and the affectionate memories those have of him who were his pupils in the Cavendish Laboratory and are still with us. Sir J. J. Thomson's opening essay of 44 pages is the only one which gives biographical details, and it sums up by expressing "profound admiration for the fineness and strength of his character, for his unselfishness and kindness." The rest of the essay and those of Sir J. Larmor and Sir J. Jeans deal with Maxwell's scientific work and its influence on the progress of physics since his day. Each has something to say on his greatest work—that on the electromagnetic field. Jeans deals in addition with his memoir on the *Dynamical Theory of Gases*, and Larmor touches on his relation to thermodynamics. Planck accounts for the slow progress in Germany of Maxwell's views as to the function of the medium

in electromagnetic phenomena and brings out very clearly the reaction of Maxwell and Boltzmann on each other in the development of the kinetic theory of gases. Einstein dwells on the importance of the transformation of our view of reality as an assemblage of particles into the conception of a continuous field, and Sir O. Lodge traces the development of wireless telegraphy from Maxwell's theory.

Sir H. Lamb tells us about Maxwell's first, and Sir A. Fleming about his last, lecture at Cambridge, and Mr W. Garnett about the apparatus in the Cavendish Laboratory in its early days and those who were set to work with it. Sir R. Glazebrook and Sir A. Fleming describe their own researches under Maxwell, and we gather that Maxwell's method was to plan a student's work carefully for him, and to encourage him to tackle in his own way the difficulties he encountered.

A book by so many authors is bound to have some repetitions in it; and this one is not quite free from discrepancies. One is struck by the paucity of the information about Maxwell's earlier professorships, particularly that at King's College, London, which he held from 1860 to 1865. During these years he published his most important papers, and his reputation for fairness and clear insight into fundamentals was such that there was an incessant demand on his services as referee for papers submitted to scientific societies. The contributions he made to science in this way ought not to be overlooked in any account of his life's work.

C. H. L.

The Mysterious Universe, by Sir JAMES JEANS, F.R.S. Pp. viii + 142. (London: Cambridge University Press.) 2*s*.

This unpretentious volume, an expansion of the Rede Lecture delivered before the University of Cambridge in November 1930, needs no introduction to members of the Physical Society. It suffices to say that the first edition has completed its first hundred thousand, and that in this second edition the author has brought the scientific matter up to date and has expunged or rewritten certain passages which, in his opinion, were liable to misinterpretation. And, most astonishing fact of all, this crowning instance of infinite riches in a little room may in its new form—pleasing to see and easy to slip into the pocket—be bought for the price of a "Sapper" novel. *Verbum sapienti*.

A. F.

Science and First Principles, by F. S. C. NORTHROP. Pp. xiv + 299. (London: Cambridge University Press.) 12*s*. 6*d*.

Mr Northrop's aim is "to determine what contemporary scientific discoveries in many different branches of science reveal, and what all this means for philosophy." That quotation will probably convince most scientists that they do not want to read his book, and that it is only another attempt to show that science means something that no scientist can comprehend. I fear that their conclusion will be right; scientific knowledge will not enable the difficulties of this profound and learned treatise to be mastered, nor will it be increased by mastering them.

The truth is, of course, that philosophers mean by science something quite different from what we mean. To them science is, at any given moment, a definite set of propositions, expressed in words, the import of which is almost or quite independent of the methods by which they were produced. Their meaning is to be found by verbal analysis and comparison with other propositions in which the same words are found. Hence it is that Mr Northrop finds continuity of development, and really very little progress, between the Greeks (who invented so many of our words) and modern times. To scientists science is a form of activity, the "conclusions" of which are merely one part of a never-ending process; they are continually in flux and never fully expressible in words. Since the process rests on the discovery of experimental laws, a conception foreign to the Greeks,

nothing important is common to Greek speculation and modern science. There is no essential reason why those who are interested in scientific science should not also be interested in philosophic science; those who have this double interest will doubtless read Mr Northrop's treatise with great pleasure and profit. But any detailed criticism of it lies wholly without the scope of this journal.

N. R. C

An Introduction to Quantum Theory, by G. TEMPLE, Ph.D., D.Sc. Pp. 196. (London: Williams and Norgate, Ltd.) 12*s*. 6*d*.

The student of the new quantum theory who applies himself to the various introductions to this subject may well be surprised that the newcomer changes his appearance somewhat at every new introduction. There is an uncertainty about the character of each new presentation. The treatment in the particular work under review is of an original character and devotes more space than other books of its kind to the methods of matrix and quantum mechanics. The author resembles other writers in that he enters the new territory from a classical base and begins with four very interesting and readable chapters on the principle of duality, the theory of photons, the wave equation and some simple problems. But he soon becomes apologetic for this weakness and proclaims his allegiance to that ascetic school whose walls carry no pictures but which believes that it is impossible to proceed far with any realistic interpretation of physical phenomena. The introduction of this belief into physics is of the nature of the introduction of an important new principle. It merits a careful and not too hurried a treatment. Although the second half of the book is devoted to the methods of the new calculus, the author hurries on to results obtained by it instead of pausing to give some detail about it. This is no adverse criticism of the matter presented, the extent of which is surprising, but one feels that the reading beyond chapter v would have been made much easier if the explanation of the notation had been more extended. The new methods are not so completely new that analogy cannot be traced with those of classical physics, such as that which can be found in extensions of the vector notation. This method helps the new student to make the first plunge and carries him safely to the delights of the deeper waters.

H. T. F.

Der Smekal-Raman-Effekt, by Prof. K. W. F. KOHLRAUSCH. Pp. viii + 392. (Berlin: Julius Springer.) 32 marks, bound 33.80 marks.

The discovery of the Raman effect, early in 1928, opened up a view of research which has almost paralleled the early history of work in X-rays and radioactivity. The rapid development of the subject has been mainly due to the intrinsic interest attaching to the phenomena, and to the extraordinary value of the effect as a tool for fundamental physical and chemical research. Moreover, much of the work can be carried out with relatively simple and standard equipment.

Prof. Kohlrausch lists no fewer than 413 papers on the Raman effect, published between February 1928 and June 1931. The ordering of a proliferation of this magnitude into a consecutive and moderately concise account can have been no light task, even for a writer with the energy and experience of Prof. Kohlrausch, actively engaged as he is in experimental research upon the subject-matter. The task has, however, been very well performed, although the book gives occasional evidences of its necessarily hurried compilation. Prof. Bergen Davis, for instance, generally appears under the alias of "D. Bergen." It would, however, in view of the magnitude of the undertaking, be unjust as well as ungracious to lay stress upon errors of this kind, which are, indeed, almost unavoidable.

The book contains a full account, critical as well as comprehensive, of both the experimental and theoretical sides of the subject, well indexed tables of the known Raman spectra, and a very full bibliography, complete as far as possible to the spring of 1931. There was a real need for a survey of this kind, and Prof. Kohlrausch's book should be assured of a warm welcome. H. R. R.

Constitution of Atomic Nuclei and Radioactivity, by G. GARNOW. Pp. viii + 114. (London: Oxford University Press.) 10*s*. 6*d*.

The investigation of the nucleus is no easy matter, either experimentally or theoretically, and the nuclear electrons are conspicuously refractory to current quantum-mechanical treatment. Nevertheless, notable advances have been made during the last decade in our knowledge of nuclear structure and dynamics. In the book under review Dr Garnow, who has been associated with some of these advances, gives an excellent summary of the present position. The chapter headings are: I, The constituent parts and energy of nuclei; II, Spontaneous disintegration of nuclei; III, Excited states and electromagnetic radiation of nuclei; IV, Artificial transformation of nuclei. The subject-matter is avowedly treated from the theoretical point of view. Experimental details are omitted, but full use is made of the numerical results of the most recent measurements on nuclear phenomena. The book therefore supplements, on the theoretical side, some chapters of the large treatise of Rutherford, Chadwick and Ellis, which was reviewed in these columns last year.

The treatment is throughout exceptionally clear, and non-specialists and macrophysicists will appreciate the marking of the more speculative passages by a distinctive symbol, somewhat resembling the sign commonly used on our English highways to indicate "Dangerous turns ahead." H. R. R.

L'Atome de Bohr, by LÉON BRILLOUIN. Pp. 363. (Paris: Les Presses Universitaires de France.) 100 fr.

The author of this book holds the view that for a considerable period of time teachers of physics will be obliged to base courses of lectures on Bohr's theory of the structure of the atom and of the emission of spectral lines. With this view we are in substantial agreement, and we must therefore welcome the publication of such a well-balanced survey of the Bohr theory and of the mathematical theorems on which it is based. Incidentally, the book follows the courses of lectures which the author delivered at Wisconsin, U.S.A., in 1928, and more recently at the Sorbonne. It is, indeed, no mere restatement of familiar ideas, and is well worth a close study. The treatment of the magnetic behaviour of the atom is particularly good. The description of the Gerlach and Stern experiment is not good, however, and the diagram of their experimental arrangement needs modification. There is no attempt to describe the methods of wave mechanics, but the bearing of the results of the newer theories upon the old is everywhere adequately expressed. L. F. B.

Handbuch der Experimentalphysik, Ergänzungswerk-Band I: *Bandenspektren*, by W. WEIZEL. Pp. xi + 461. (Leipzig: Akademische Verlagsgesellschaft, M.B.H.) 45 marks.

The theoretical interpretation of band spectra of diatomic molecules, begun ten or twelve years ago, has been placed on a new footing and rapidly extended within the last five years. These later theoretical developments have given great impetus to the laboratory investigations of the structures of bands and band systems, the results of which have increased, in number and precision, at a rate which almost bewilders any reader who attempts to keep abreast of them.

The National Research Council Bulletin on *Molecular Spectra in Gases* by Kemble, Birge, Loomis and others, appeared just too soon (at the end of 1926) to include any of the newer theoretical investigations which clarified the subject, but it remains as a valuable record of the knowledge of the spectra of diatomic molecules at a period when no other important book had appeared. In the last year or two, when the theory has reached a more settled state and observations are more completely understood, monographs and larger works have been, and are being, written by several workers on band spectra. Many excellent reviews of the subject and of specific branches of it, such as those relating to heats of dissociation, predissociation, diffuse spectra and continua, active nitrogen, the isotope effect, molecular structure, etc., have appeared from time to time in the *Physikalisches Zeitschrift*, *Ergebnisse der exakten Naturwissenschaften*, *Chemical Reviews*, etc. Considerable sections in the newer text-books and handbooks of theoretical and experimental physics, astrophysics and physical chemistry are devoted to molecular spectra; such sections, in fact, appeared in two former volumes, XXI and XXII, of this *Handbuch der Experimentalphysik*. There have also appeared Ruedy's useful monograph *Bandenspektren auf experimenteller Grundlage* (Vieweg, 1930), Kronig's theoretical work *Band Spectra and Molecular Structure* (C.U.P., 1930) and the earlier parts of Mulliken's very thorough report on "The interpretation of band spectra" in *Reviews of Modern Physics*. Several larger works on molecular spectra have been in preparation, and the first of them actually to appear is the present excellent volume by Dr Weizel.

The first half of the book is almost entirely theoretical; it consists of a preliminary section on the molecular model and the Schrödinger equation, and three chapters dealing with: (I) the theory of the terms of diatomic molecules (141 pages), (II) the structure of band spectra of diatomic molecules (67 pages), and (III) the theory of polyatomic molecules (10 pages). The second half of the book consists of a single chapter of 210 pages dealing with the observed bands and band systems of molecules and groups of molecules, namely (1) H_2, (2) He_2, (3) diatomic metallic molecules, (4) diatomic hydrides, (5) diatomic molecules containing one or two of the elements B, C, N and O, (6) diatomic halogen molecules, (7) diatomic halides, (8) monoxides not included in (5), (9) S_2, Se_2, Te_2, (10) other diatomic molecules, and (11) to (14) certain types of polyatomic molecules. A bibliography appears at the end of each of these sections of chapter IV. Included in this chapter are about 50 energy-level diagrams, Fortrat diagrams, potential curves, etc., and 116 tables of numerical data for individual bands, band systems, and molecular constants. The book closes with a very full index occupying 19 double-columned pages; this is especially necessary, because the large amount of numerical data is interspersed throughout the latter half of the book, rather than collected together into an appendix.

The selection, arrangement and presentation of all the matter, both theoretical and observational, are most excellent. Dr Weizel points out that many of the numerical data tabulated have not been critically tested by him, but he is also careful not to let some observations and analyses be included without a note to the effect that they are uncertain. Whilst the data for several molecules are from papers which appeared as late as the middle of 1931, there are a few instances where better were available long before that date, for instance SO (p. 403). In table 133, the v' numeration for the Cl_2 bands is described as Birge's revision of that given in Elliott's 1930 paper. There appears to be some confusion here; the reviewer's recollection of correspondence with Dr Elliott on this point is that Birge's revision applied to the numeration in Elliott's 1929 paper, and that Elliott's own revision in his 1930 paper was somewhat later than Birge's, and was, indeed, accepted by Prof. Birge. In one or two cases a numerical result, actually obtained by the author of an original paper on the analysis of a band system, is attributed to Birge as the compiler of the very useful table of molecular constants in the *International Critical Tables*, vol. V; slips of this kind, however, are quite unimportant.

For the most part the notation employed agrees with that suggested by Mulliken in

1929–30 and now widely adopted. There are important differences, however; for instance in the choice of the numerical subscripts used to distinguish between the spin components of a multiple electronic state and those of a rotational level, and consequently in the designations of the band branches. Weizel's choice is as theoretically consistent as Mulliken's, and the difference between the two is carefully pointed out; nevertheless, to a reader who has become familiar with one scheme the existence of a second, however good, is disturbing, unless and until it in turn becomes the internationally adopted one. In a book of its size, scope and price, well illustrated though it already is, a few reproductions of spectrograms might reasonably have been expected; the lack of these, however, is heavily outweighed by the lavish scale upon which numerical data are provided and by the all-round excellence of the work.

W. J.

The Practice of Spectrum Analysis with Hilger Instruments, fifth edition, compiled by F. TWYMAN, F.R.S. Pp. 53. (London: Adam Hilger, Ltd.) 3*s*. 6*d*. net.

This booklet is largely a compilation of material contributed by several experienced workers on the spectrographic analysis of various substances, and is intended to serve as a guide to those taking up similar work. This purpose it will admirably fulfil. In the new edition each section of the fourth edition (1929) is brought up to date (August 1931), and new sections dealing with improved methods of quantitative analysis are added. There are three useful appendices: (1) on the determination of wave-lengths in a prism spectrogram by interpolation, (2) on the sensitization of photographic plates for the Schumann region by bathing, and (3) a bibliography of 82 papers and booklets selected from the large number that have appeared on the application of the spectrograph to chemical, metallurgical and mineralogical analyses.

W. J.

Foundations and Methods of Chemical Analysis by the Emission Spectrum, being the authorised translation of *Die chemische Emissionsspektralanalyse*, by Dr WALTHER GERLACH and Dr EUGEN SCHWEITZER. Pp. 123. (London: Adam Hilger, Ltd., 1931.) 12*s*. 6*d*. net.

Many physicists will probably be surprised to read that the eminent investigator whose name they couple with one of the most fundamentally important experiments in present-day atomic physics (the Stern-Gerlach experiment), was, for the seven years ending mid-1929, engaged upon the problem of qualitative and quantitative chemical analysis by means of emission spectra. The valuable results of Prof. Gerlach's work in this field, partly in collaboration with Dr Schweitzer, are recorded in several papers in the *Zs. für anorg. und allg. Chemie* and other German chemical journals, and also form the nucleus of the very attractive book which, thanks to Adam Hilger, Ltd., now appears in our own language. The successful development of quantitative chemical analysis by means of emission spectra has brought many problems concerning metals within scientific control, and has provided new methods of investigation in connexion with many technical processes. The book deals with the development of the subject from its origin to the latest investigations, stress being laid upon the fundamental principles, practicability and trustworthiness of the method. It refers almost entirely to the use of spark spectra, describes many useful methods of procedure both with solid electrodes and with solutions, and deals with many particulars not completely treated in any other book. It is the most satisfying work on this subject that has come to the present reviewer's notice, and is well illustrated with 53 diagrams and reproductions of spectra.

W. J.

Wave-length Tables for Spectrum Analysis, second edition, compiled by F. TWYMAN, F.Inst.P., F.R.S. and D. M. SMITH, A.R.C.S., B.Sc., D.I.C. Pp. xi + 180. (London: Adam Hilger, Ltd.) 14*s.* 6*d.*

The words "spectrum analysis" as used in this title denote the spectrographic detection and estimation of the chemical elements, especially the metals, in mixtures, solutions, alloys, etc., and not the analysis and interpretation of atomic and molecular spectra—the subject to which Sir Arthur Schuster gave the name "spectroscopy." Since 1923, when Mr Twyman compiled the first edition of these very useful tables, much information has been obtained for the application of spectrographic methods to chemical and metallurgical analyses, and Mr Smith's present revision of the work is opportune. All the former tables of persistent lines in spark spectra have now been corrected to the I.Å. scale, and tables of persistent lines by Lundegårdh for flame spectra and by Ryde and Jenkins for arc spectra are now included, as well as the full list of ultimate lines by A. T. Williams, with series and temperature classifications and excitation potentials added. The tables are accompanied by explanatory notes and useful practical information both in the main text and also in an appendix. A brief theoretical account of the various types of spectrum, by Prof. Andrade, is transferred here from another Hilger publication.

The reviewer has very little to note in the nature of criticism. The French *raie* has been correctly translated as *line* everywhere except on p. 110 and pp. 115–18, where the unsatisfactory rendering *ray* appears. It is surprising that no mention is made of Kayser's very useful *Tabelle der Schwingungszahlen* (Hirzel, Leipzig, 1925) for the reduction of λ_{air} between 2000 and 10,000 Å. to ν_{vacuum}. No spectroscopist who has used it would, in spite of its many typographical errors, willingly return to the two-stage process mentioned in part I, namely reduction from λ_{air} to λ_{vacuum} by means of table 2 and subsequent conversion to ν_{vacuum} by means of a table of reciprocals.

W. J.

Photochemical Processes, A general discussion held by the Faraday Society. Pp. 216. (London: Faraday Society.) 10*s.* 6*d.*

As recently as 1925 photochemical processes were discussed by the Faraday Society. The fact that again in 1931 the same society should meet to consider the same subject was welcomed by Prof. Mecke, who opened the discussion, because of the growing interest in this branch of reaction chemistry. After the first meeting the views became general that for the primary photochemical reaction each absorbed quantum puts one and only one molecule into a reactive state; and that photochemical efficiencies not equal to unity must, therefore, be influenced by the secondary reactions that follow. These secondary reactions have received increasing attention since 1925.

Three out of the four lines of inquiry at this recent meeting were purely physico-chemical in scope. In part I were discussed, under the title "Molecular spectra in relation to photochemical change," the activation of the almost sluggish non-reacting molecules by the absorption of a light quantum $h\nu$. The reactions of the activated molecules in gaseous systems were considered in part II, and in solid and liquid systems in part III. Prof. Max Bodenstein introduced part II, and Prof. A. Berthoud led lucidly in part III. Of the great interest to physicists, chemists, and biologists who analyse photo-effects in living cells, of the new data presented and of the views put forward in these three parts there can be no doubt.

In part IV, under the title "Photosynthesis," is included a medley of papers. Two of these bear wholly on the title, in that in them are considered the synthesis of organic substances *in vitro* under the influence of light energy. The contribution by Prof. Baly, a well-known pioneer in this field, is sure to be widely read. Prof. Otto Warburg, a distinguished bio-chemist, gives very briefly his views concerning the energy-absorption

by green plant cells during the photo-reduction of carbon dioxide. He then deals with an interesting effect of light on the respiratory mechanism of cells. This is most certainly not photosynthetic in nature. The last paper in this part, on the "Measurement of ultra-violet radiation," seems to be misplaced under the title of the part. We do not know what aim those who planned the latter had in mind. Possibly it was intended to show the importance of the purely physico-chemical work in the analysis of the photo-biochemical reaction, taking place in living green plant cells, on which the food supplies of most living organisms ultimately depend. If this was the case, it is to be hoped that such a happy gesture to the green leaf will again be made when the society next discusses photochemical processes; but that then the primary activation of plant physiologists in this country will be followed by the secondary reaction of their taking part in the discussions.

M. T.

The Physics of High Pressure, by Prof. P. W. BRIDGMAN, Ph.D. Pp. vii + 398. (London: G. Bell and Sons, Ltd.) 22*s*. 6*d*.

This is a monograph by a pre-eminent authority on a subject which he has made peculiarly his own. It deals fully with the technique of the production and measurement of high pressures, up to the order of 20,000 atmospheres, and with the remarkable variety of phenomena which can be observed in substances exposed to such pressures. The scope of the book may be gauged from this (incomplete) list: elasticity of solids at high pressures, with special sorts of rupture observed thereat; equations of state; melting phenomena; polymorphic transitions; effects of high pressure on electric, thermo-electric, optical and thermal properties and on viscosity; and even the effect of pressure in facilitating the germination of clover seeds! Many of these discussions are here collected together for the first time, and the resulting volume should be of absorbing interest to engineers and physical chemists, as well as to physicists. Certainly no one engaged in high-pressure work can afford to be without this volume, and no experimental physicist, whatever his main professional interest, can fail to be fascinated by it. The editor and publishers are to be congratulated upon securing the book for their series of "International Text-books of Exact Science."

H. R. R.

Die Elliptischen Funktionen von Jacobi, by L. M. MILNE THOMSON. Pp. xiv + 69. (Berlin: Julius Springer.) Geb. 10.50 marks.

The functions sn u, cn u, dn u are tabulated separately for values of the argument at intervals of 0·01, and for values of k^2 at intervals of 0·1; in every case, except of course for $k^2 = 1$, the ranges covered exceed a quarter-period. The first differences are conveniently inserted between the consecutive entries. These tables, as is explained in the preface, are entirely new, and for many purposes they are much more convenient than those hitherto available, which tabulate u against the amplitude ϕ. They are followed by eight-figure tables of the complete integrals K, K', E, E', and of q and q_1, for values of k^2 at intervals of 0·01. It would be useful if a table of $E(u)$ could be added in any future re-issue. There is a short introduction with a convenient collection of formulae. The arrangement and printing are admirably clear, and this handy little volume will be a valuable addition to the reference library.

G. S. L.

Numerical Examples in Physics, by W. N. BOND. Pp. 128. (London: E. Arnold and Co.) 4*s*.

There is little glory and much toil in the writing of text-books; and this is peculiarly true of the compiling of books of examples. All the more seemly is it to thank Dr Bond

for facing the ungrateful task and for providing us with a really well chosen and comprehensive selection of some 450 examples. A few of these are of intermediate standard; the great majority will give good practice to pass and to honours students. Not all the questions are numerical—a number provide subjects for brief essays. It would be very helpful, especially to the student working unaided, if the book contained an appendix giving some sources of information on the subject-matter of these essays, and we hope that Dr Bond will supply this information in the second edition which will, we trust, be necessary.

The book may be unreservedly commended. A. F.

Problems in Physics, second edition, by WILLIAM D. HENDERSON, Ph.D. Pp. ix + 245. (London: McGraw Hill Publishing Co., Ltd.) 11*s*. 3*d*.

This book contains a collection of 825 problems in mechanics and physics originally designed to accompany the courses in physics arranged for students of engineering in the University of Michigan. About two-thirds of the problems deal with mechanics and electricity; the remainder with surface tension, diffusion, heat, light, sound and radiation. Each section is prefaced by a short statement of the fundamental principles and usually also by one or two worked examples. The standard corresponds approximately to that of the intermediate classes of the English universities; exceptionally there are sections on technical electricity (e.g., a.c. phenomena and electric generators) which would not fall within intermediate physics syllabuses in this country. The fact that a second edition has been called for shows that there is a demand for the book in the United States. English text-books usually contain an adequate collection of numerical problems and the reviewer imagines that few teachers over here would require their students to purchase a book costing 11*s*. for their exercise work. The problems themselves are straightforward and practical rather than ingenious. The text is good and the author neither omits nor muddles his units. The only bad error appears on p. 138 where Dr Henderson has permitted himself to write "The electromotive force (e.m.f.) of an electric generator (battery or dynamo) is its capacity for generating electric pressure or potential...e.m.f. is measured in terms of work, ergs." The book closes with 8 pages of data and an index. It can be recommended for the use of teachers too busy to devise problems themselves.

D. O. W.

Electricity and Magnetism, by VINCENT C. POOR, Ph.D. Pp. ix + 183. (London: Chapman and Hall.) 11*s*.

This work, by the Associate Professor of Mathematics, University of Michigan, belongs to the class of text-books of mathematical physics. The treatment is based mainly on the methods of vector analysis, and the book is put forward not as a comprehensive treatise but rather as "a path through" the subject of electrical theory. The author also expresses the view that a course such as this should be taken not only by the student of physics but also by the good electrical engineer before he leaves college.

After a concise introduction to vector analysis, theorems being stated but not proved, come four chapters treating of the classical side of electricity and magnetism: one on electrostatics, one on magnetism and dielectric polarization, one on current electricity, and one on the dynamics of the electric current. In the last-mentioned chapter the author expounds d'Alembert's principle, Hamilton's principle and Lagrange's equations, and treats the electric current as a cyclic mechanical system. The final chapter v is in two parts. The first, entitled "Electron theory," includes a treatment of the electromagnetic field from the Lorentz point of view, a derivation of ether stresses in matrix form, and a brief treatment of radiation pressure. The second part of chapter v is entitled "Special

relativity and the electrodynamic theory," and in it the invariance of Maxwell's equations is shown and the expressions for the longitudinal and the transverse mass of the Lorentz electron are derived. Exercises are set at the end of the earlier chapters. The book is not exactly easy reading, but should prove of much value to the advanced student under the guidance of a competent teacher.

D. O.

Magnetic, Meteorological and Seismographic Observations made at the Government Laboratories, Bombay and Alibag, in the year 1927, under the direction of S. K. Banerji, D.Sc. Pp. iii + 132. (Calcutta: Indian Government.) 17*s.* 6*d.*

This volume is the 49th issue and brings the record of meteorological and magnetic observations up to 82 years. The mean temperature during the year was 79·9° F., 0·3° F. below the normal, and the rainfall 74·4 in., 3·8 in. above the normal. The horizontal magnetic field at Alibag was 0·371 and slowly increased; deviation of the compass 1·9° west, an increase of 2·7° west on 1926. The dip was 25° 26′, an increase of 3′. There were 5 days of very great and 8 of great magnetic disturbance, and one seismic disturbance of considerable amplitude on December 28 at 19 h. 2 m. 11 s. Greenwich mean time.

C. H. L.

The Observatories Year Book, 1929 (M.O. 330), published by the authority of the Meteorological Committee. Pp. 441. (London: H.M. Stationery Office.) £3. 3*s.* 0*d.*

This is the eighth issue of the Year Book. It contains daily records of the meteorological elements at the principal first-order stations—Aberdeen, Eskdalemuir, Valencia and Kew—of atmospheric electricity and terrestrial magnetism at Lerwick and Eskdalemuir, and of atmospheric electricity, atmospheric pollution, seismology and aerological data obtained with sounding-balloons at Kew. Each station reports that 1929 was a normal year.

C. H. L.

Meteorological Office 331c. *Geophysical Memoirs* No. 53, *Characteristics of Rainfall Distribution in Homogeneous Air Currents and at Surfaces of Discontinuity*, by A. H. R. Goldie, M.A., F.R.S.E. Pp. 19. (London: H.M. Stationery Office.) 1*s.* 3*d.*

The author shows that a depression does not produce rain independently of the time of day or of the region over which it is passing. The time-effect appears greater in warm or cold fronts than in homogeneous equatorial or polar currents. In warm fronts it seems to depend on the isolation and radiation of the cloudy warm air ascending at the front, and in cold fronts on that of the warm air ahead of the front.

C. H. L.

Journal of Research of the Bradford Technical College, Vol. 1 (1930). Pp. xii + 295. (Bradford Education Committee.)

In this volume are collected the records of thirty-three researches, the work of students and members of the staff of Bradford Technical College. From an institution whose work must be closely allied to that of a surrounding industrial area one would expect a preponderance of technical researches: yet although the majority of papers in this collection are of that kind, subjects of "pure" scientific interest are to be found in several notable

contributions. An investigation by T. W. Price of the decomposition of carbamyl chlorides substituted by hydroxyl compounds and a general solution of $\nabla^2\psi = w$ in n-dimensional Euclidean space by A. J. Carr should satisfy the most unsullied of purists. The subjects, indeed, range widely from matters of purely industrial interest, such as the properties and peculiarities of textile fibres and the design of machine components, to the detailed analysis of power-transmission by belts; from the analysis by chemical means of the constituents of "union materials" in which different types of fibre are inextricably mingled, to the measurement of dielectric losses in coils; from the cataloguing of algae in a bog near Bradford to the measurement of the optical properties of gaseous carbon disulphide.

Among so varied a collection considerable differences of quality are to be expected: nevertheless most of the papers have already been printed in recognized scientific and industrial journals, a guarantee at least of usefulness. In point of fact, several papers are not only of considerable scientific value, but excellently presented. They are here beautifully reprinted, and form a collection on which the contributors may be congratulated and which could occupy a useful place in the collection of any investigator.

J. P. A.

Proceedings of the Institution of Mechanical Engineers, Vol. II, June to December, 1930. Pp. viii + 614.

This volume contains two or three papers which should be of considerable interest to physicists; of these perhaps the most important is one on "The coefficients of heat-transfer from tube to water," which is a report of an investigation carried out by Messrs Eagle and Ferguson under the direction of a committee of prominent engineers, experienced in condenser design, formed by the British Electrical and Allied Industries Research Association. The committee have looked into some 200 papers on the problem of heat-transfer and have superintended a very admirable and thorough piece of research. The paper is of the nature of an interim report; the work done so far only deals with the transfer of heat from a hot tube to water circulating through it, and the work will be followed by an investigation into the transmission of heat from the tube to the medium surrounding it. This promises to be the most thorough research of its kind which has yet been done on the problem of heat-transfer. Mr V. E. Pullin, Director of Radiological Research, Woolwich, writes an interesting paper on the subject of "X-rays in engineering practice." The paper is largely confined to radiography in its application to the examination of castings and forgings for flaws. One realizes the remarkable development which has taken place in the technique of this branch of science when one considers that by applying a voltage of 240,000 to an X-ray tube it is possible to penetrate a steel forging 4 in. thick. Prof. J. J. Guest contributes a paper on "The effects of rapidly acting stress." In his analysis of his results he concludes "that the stress which is important in design... is very little affected by the suddenness or the brief duration of the loading," and his work would suggest that there is room for more research on the problem. Dr Gough in his paper on "The effect of low temperature on the shock-resisting properties of new wrought-iron chain" concludes that the shock-absorbing value of a chain at low temperatures depends largely on the welding of the links, as one might expect; best-quality chain iron does not develop any brittleness in the absence of notches, even at temperatures as low as $-78°$ C.

Mr L. St L. Pendred in his presidential address makes some important "random reflections." He deplores the necessity for extreme specialization in engineering, and appeals to mechanical engineers to adventure more into the domain of physics.

The 17th Thomas Hawksley Lecture was this year delivered by Prof. J. W. Gregory, F.R.S., on "The machinery of the earth."

G. A. W.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 2 *March* 1, 1932 NO. 242

THE SPECIFIC VOLUMES OF SOME GASEOUS REFRIGERANTS

BY J. H. AWBERY, B.A., B.Sc., F.INST.P., AND EZER GRIFFITHS, D.Sc., F.R.S.

Physics Department, National Physical Laboratory

ERRATUM

Proc. Phys. Soc. **44**, 106 (1932). Review of *The Constitution of Atomic Nuclei and Radio-activity*. The author is G. Gamow and the publisher Humphrey Milford.

§ 1. INTRODUCTION

THE specific volume, i.e. the volume of 1 gm. of vapour at a given temperature and the corresponding vapour pressure, is a quantity which assumes importance in considering the utility of a substance as a refrigerant, since it determines the size of compressor necessary for a given duty of the refrigerating plant.

§ 2. EXPERIMENTAL METHOD

The method we have adopted for this determination is in principle a very simple one. The liquid is contained in a vessel which communicates through a valve with a second vessel which can be exhausted, and the whole apparatus is immersed in a bath controlled thermostatically. Thus, if the valve is opened when temperature equilibrium has been reached, vapour from the liquid expands and fills the second vessel at the appropriate vapour pressure. After closing the valve the second vessel may be exhausted again and the process commenced anew. Thus the measurement consists essentially in filling this vessel with vapour a counted number of times, and determining the amount of liquid used up in the process; this latter quantity is found by weighing the first vessel (the liquid container) before and after the experiments.

The actual apparatus is shown in figure 1, where the vessel containing the

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Received July 19, 1931. *Read and discussed December* 4, 1931

ABSTRACT. The volume per gram of various vapours at temperatures between 0 and 40° C. and under the corresponding vapour pressure has been measured. The method consists in permitting the liquid to evaporate repeatedly into an exhausted vessel so that automatically the vapour is under saturation pressure. The volume of the vessel is known, and the weight necessary to fill it is found from the decrease in weight of the liquid.

The substances studied were sulphur dioxide, dichloroethylene, ether, pentane and trichloroethylene.

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The actual apparatus is shown in figure 1, where the vessel containing the

liquid is seen on the left. It is made of brass and entirely sealed up, and contains two re-entrant tubes, one for a thermometer and one for a heater. The heater is provided for accelerating the process of re-establishing temperature equilibrium with the bath, after the cooling which accompanies the evaporation when the valve is opened. It is essential to stir the liquid, and the problem of doing so without the possibility of loss or leakage was overcome by operating the stirrer electromagnetically. A coil was slipped over the neck of the vessel, and current could be sent through it by closing a circuit with a tapping key. The stirrer consisted of a brass plate with a number of holes in it, attached to a brass rod carrying a piece of soft iron at the top. Thus when the circuit was completed the stirrer was lifted, and by repeated tappings the stirrer could be oscillated up and down.

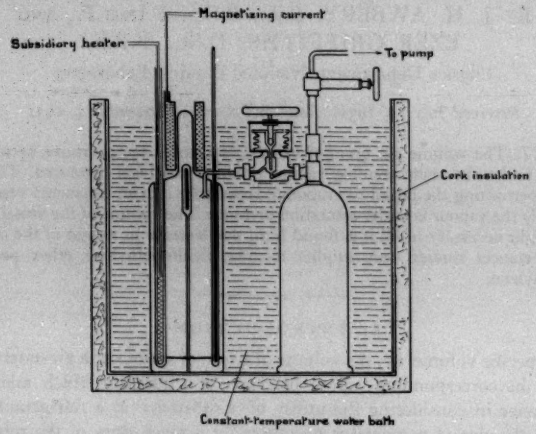

Fig. 1. Section through apparatus.

The construction of the valve between the two vessels may be seen in the diagram, in which its size is somewhat exaggerated in order to show the details. Since sulphur dioxide was one of the substances to be studied, it was necessary to avoid all the usual packing materials; this was carried out by surrounding the upper part of the valve with flexible metal tubing having corrugated walls, soldered to the base and to the upper plate seen in the figure. By this construction it will be seen that no gas can escape through the valve.

The vessel for receiving the vapour was of glass, so that the presence of condensed liquid could be detected if the temperature changed at any time. It was provided with a valve through which connexion could be made to the pump used in exhausting the vessel. This valve was a needle valve with an exceptionally long and fine screw thread on the stem. The volume of this vessel was 1270 cm.^{3}, determined by weighing it full of water.

§ 3. RESULTS

Two corrections are required for each observation. After being exhausted the second vessel still contains a small quantity of vapour, corresponding to the pressure down to which the pump exhausts, so that the effective volume is reduced for the next filling. The details of a typical experiment will show how this correction is made, and will illustrate the order of magnitude of the quantities involved. The second correction is for the weight of vapour which remains in the first vessel, to fill the space originally occupied by the liquid which has been evaporated.

A typical experiment: temperature 0° *C., sulphur dioxide.* Twelve expansions were carried out and the decrease in weight of the first vessel was 73·0 gm. The volume of the expansion vessel was 1270 cm.^{3} When the latter was first exhausted, the residual gas was air, which by Dalton's law does not affect the quantity of SO_2 required to fill the vessel at any given pressure. The mean of the residual pressures in the remaining expansions was 8 mm. of mercury, the barometer being about 760 mm. Thus the effective volume was 752/760 of the true volume, or the weight may be increased in the ratio 760/752. Thus the weight corrected for this effect is 73·8 gm. For the second correction, we know that the density of the liquid is of the order 1·4, so that 73·8/1·4 or 53 cm.^{3} have been removed. Since the density of the vapour is approximately 73·8/12 × 1270 gm. per cm.^{3} (from the experiment itself), the mass of vapour left in the vessel and included in the weighing is about 53 × 73·8/12 × 1270 or 0·3 gm., and this should be added to the observed loss in weight. Thus the weight finally corrected is 74·1 gm. and the specific volume is 12 × 1270/74·1, or 206 cm.^{3}/gm. at 0° C.

The remaining results for SO_2 are set out in table 1, and shown in figure 2.

Table 1. Results for SO_2.

Temperature (° C.)	Specific volume (cm.^{3}/gm.)	Density of saturated vapour (gm./cm.^{3})
0	206	0·00485
0	218	0·00459
13·4	134	0·00749
23·6	88	0·01133

The SO_2 used had a moisture content of 0·3 per cent. in the liquid phase and 0·1 per cent. in the gaseous.

Dichloroethylene. This material, which corresponds chemically to three different substances, was purchased from a firm of manufacturing chemists and appeared to consist of a mixture of organic compounds. It had no definite boiling-point, but after two previous fractionations it was found that:

6 per cent. distilled below 49° C.
22 ,, ,, between 49° and 54° C.
8 ,, ,, between 54° and 55° C.
13 ,, ,, between 55° and 56° C.

16 per cent. distilled between 56° and 57° C.
22 „ „ between 57° and 58° C.
11 „ „ between 58° and 58·6° C.
2 „ „ above 58·6° C.

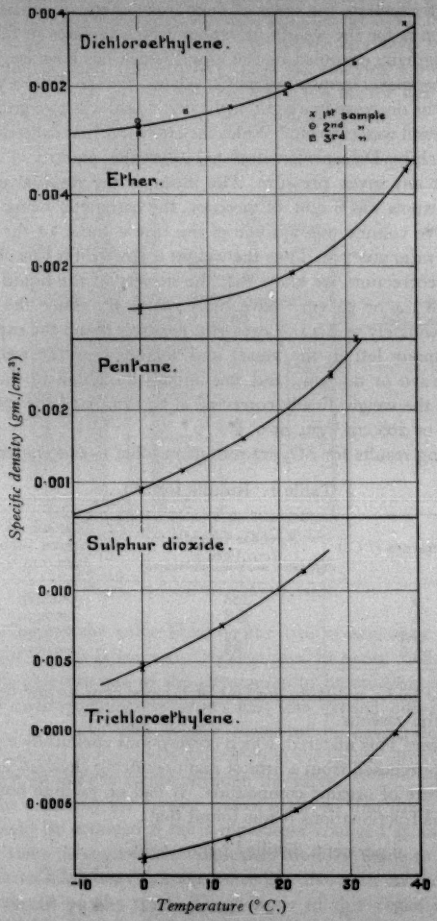

Fig. 2. Results.

Samples taken from different bottles of similar material did not give identical results. They are given in figure 2 and table 2.

Table 2. Results for dichloroethylene.

Temperature (° C.)	Bottle	Specific volume (cm.³/gm.)	Density of saturated vapour (gm./cm.³)
0	A	1350	0·00074
0	A	1390	0·00072
0	B	1110	0·00090
0	B	960	0·00104
7·0	C	758	0·00132
13·4	A	694	0·00144
21·4	A	552	0·00181
21·8	B	490	0·00204
31·5	C	379	0·00264
38·8	A	266	0·00376

Ether. It was thought to be of interest to determine the specific volume of commercial "sulphuric ether." The results are given in figure 2 and table 3.

Table 3. Results for commercial "sulphuric ether."

Temperature (° C.)	Specific volume (cm.³/gm.)	Density of saturated vapour (gm./cm.³)
0	1350	0·00074
0	1010	0·00099
22·1	540	0·00185
38·7	211	0·00473

Pentane. Pentane from a commercial source of supply was used, and appeared to be a mixture of hydrocarbons containing both iso-pentane and normal pentane. The results are given in figure 2 and table 4.

Table 4. Results for pentane

Temperature (° C.)	Specific volume (cm.³/gm.)	Density of saturated vapour (gm./cm.³)
0	1140	0·00088
0	1110	0·00090
6·1	870	0·00115
15·0	630	0·00159
25·7	403	0·00248
25·8	397	0·00252
31·4	334	0·00299
31·4	333	0·00300
31·4	332	0·00301

Trichloroethylene. The results with this material are shown in figure 2 and table 5.

Table 5. Results for trichloroethylene.

Temperature (° C.)	Specific volume ($cm.^3/gm.$)	Density of saturated vapour ($gm./cm.^3$)
0	8850	0·000113
0	7630	0·000131
12·1	3940	0·000254
22·1	2220	0·000450
36·7	1010	0·000990

§4. ACKNOWLEDGMENTS

This work was carried out for the Food Investigation Board, to whom we are indebted for permission to publish these results. Our thanks are due to Mr T. I. Jones, B.Sc., A.R.C.S., D.I.C., for bringing to our notice the principle employed, and to the Superintendent of the Physics Department, for facilities for the work. Mr A. Snow, Observer, assisted in the design of the apparatus, whilst we have to thank Mr A. R. Challoner, Senior Observer, for his patient assistance in taking readings.

DISCUSSION

For discussion see page 126.

THE LATENT HEAT OF SOME REFRIGERANTS

By EZER GRIFFITHS, D.Sc., F.R.S., and
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Physics Department, National Physical Laboratory

Received July 19, 1931. *Read and discussed December* 4, 1931

ABSTRACT. The latent heats of evaporation of ethyl chloride, methyl chloride, sulphur dioxide, pentane and dichloroethylene have been measured over temperature ranges lying between − 18° C. and + 25° C. The method used is an electrical one in which a known amount of energy is supplied at constant temperature, and the mass evaporated is obtained by weighing.

§ 1. INTRODUCTION

In refrigeration certain fluids are commonly used for the cycle, and suitable types of machine have been evolved for the compression of the vapours of these fluids. In large-scale work, ammonia and carbon dioxide are the two refrigerants in general use. For land work ammonia is largely favoured because the pressures involved in the refrigerating cycle never greatly exceed 12 atmospheres. In the event of leaks, the gas not only has the disadvantage of being noxious to human beings, but also is said to reduce the commercial value of any foodstuffs with which it comes into contact. The first disadvantage is so great in the confined space of a ship that carbon dioxide, despite the higher pressures necessary (up to 70 atmospheres), is very generally used, and particularly so when the refrigerating plant is located in the main engine-room. On account of their wide use, the properties of these two fluids have been extensively studied* and their physical constants may now be considered as well determined.

There have, however, in recent years, been suggestions from various quarters that other organic and inorganic fluids might prove to be more suitable refrigerants for special requirements. The data available on the physical properties of these suggested fluids are very meagre, and some years ago we searched the available literature and compiled tables based on the best estimates we could then make for a number of properties, including the latent heat†. In the absence of direct determinations this had frequently to be calculated thermodynamically from the vapour pressure and the volume of the vapour, the latter having sometimes to be estimated by Boyle's law from the standard density, or from the assumption that a gram-molecule occupies 22·4 litres at normal temperature and pressure. It was from the

* *Proc. I. Mech. E.* 639 (1914) (CO_2); *Phil. Trans.* A, 215, 215 (1915) (CO_2); *Bureau of Stds.* Bulletin No. 14 (1918) (NH_3).

† *Proc. British Cold Storage and Ice Association*, 21, 63 (1925).

first clear that these tentative values would require revision and we have now measured the latent heat and the specific volume of the saturated vapours of some of the substances in question. The present paper is devoted to the latent-heat determinations.

§ 2. DESCRIPTION OF APPARATUS

The apparatus is shown in figure 1. In principle, the method is to evaporate the liquid in a calorimeter enclosed in a constant-temperature air bath, by supplying heat to it electrically. By this device heat-loss corrections are eliminated, since if the rate of electrical supply is suitably adjusted the liquid boils at the constant temperature, and all the energy dissipated is used in supplying the latent heat necessary for the evaporation. To look at the matter in a slightly different way,

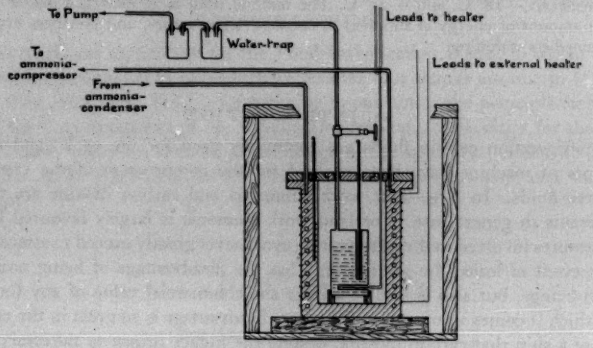


Fig. 1. Section of Apparatus.

it is clear that if the valve were opened, with no electric current flowing, the escaping vapour would abstract heat from the liquid and so lower the temperature of the calorimeter. The electrical supply is adjusted so that this fall of temperature is exactly compensated and there is a balance between the (electrical) supply and the energy carried off as latent heat.

The calorimeter proper consists of a steel vessel about 16 cm. high and 9 cm. in diameter, so that it holds nearly a litre of liquid. Two closed-end steel tubes project into the calorimeter (see figure 1). Of these, the vertical one was used for a thermometer and the horizontal one for the heater, which consisted of platinum wire wound on mica. The resistance of the heater was about 20 ohms. A narrow-bore steel tube projecting from the top of the calorimeter leads to a valve which is situated above the lid of the constant-temperature enclosure. All the joints in the calorimeter are welded.

The calorimeter was placed within the constant-temperature enclosure, and insulated from it by means of three boxwood pegs. The enclosure itself was made by casting aluminium around a spiral of iron piping, so as to make an aluminium

vessel about 25 cm. in diameter and about 30 cm. high, with the piping embedded in its walls. Thus by the use of the spiral as the evaporator of a refrigerating plant, the whole of the metal was kept at a uniform low temperature which was measured by means of a thermometer in a thermometer well cast in the walls of enclosure. The lid of the enclosure was also of aluminium, and was made in two halves with three holes to take the thermometer, current and potential leads, and the tube carrying the valve. A heating coil was also wound around the outside of the aluminium enclosure, to give a finer control than would be obtained with the refrigerating plant, and also for maintaining the enclosure at temperatures above atmospheric when required. The whole enclosure was further surrounded by cork in a box, deep enough to enclose also the valve from the calorimeter.

For liquids of high boiling-point, whose rate of evaporation at atmospheric pressure was very slow, a filter pump was provided so that the pressure in the apparatus could be reduced to that corresponding to the vapour pressure of the liquid at the temperature of experiment.

§ 3. METHOD OF CONDUCTING AN EXPERIMENT

The enclosure and all the apparatus having been allowed to reach a steady temperature, the valve was opened and the suction pump was started if necessary. In the earlier experiments, the current also was switched on at the same instant, but later it was found more convenient to defer this for one minute. Readings of the current in and the e.m.f. across the heater were taken with an ammeter and voltmeter. The temperature in the calorimeter was carefully watched, and the current was adjusted, when necessary, at times which were noted accurately, so as to keep the mean calorimeter temperature equal to that of the enclosure. It was also important to ensure that the final temperature of the calorimeter was equal to its initial temperature, since otherwise a part of the energy would have been expended in raising the temperature of the calorimeter and its contents.

At the close of the experiment, this condition being fulfilled, the current was switched off and the valve was closed. The quantity of liquid evaporated was deduced by weighing the calorimeter before and after the experiment.

§ 4. SOURCES OF ERROR

Two possible sources of error might be pointed out: (i) an unbalanced water-equivalent correction, and (ii) want of uniformity of temperature throughout the calorimeter.

As regards (i), calculation shows that if the final temperature of the calorimeter differed by as much as 0·1° C. from the initial temperature the error would be of the order of $\frac{1}{3}$ per cent. There was no difficulty in bringing the final temperature to within 0·1° C. of the initial value.

As regards (ii), the thick metal walls and the tubing of the calorimeter helped to equalise the temperature by conduction. Furthermore, the heating of the bottom layer of liquid would tend to set up convection currents.

In constructing a new calorimeter of this type it might be advantageous to incorporate an electromagnetically operated stirrer such as that described in the authors' paper on "The specific volumes of some gaseous refrigerants."*

§5. RESULTS

The liquids studied, with the results obtained, are set out below. Samples obtainable commercially were studied, as these were the fluids of interest in refrigeration.

Ethyl chloride. This is a liquid which has already been used to a moderate extent in refrigerating plants. Its thermal properties were carefully studied by Jenkin and Shorthose† and it was therefore a suitable liquid to check the accuracy of the apparatus. The liquid was purchased in 2 lb. tins from a firm of manufacturing chemists. It was analysed for the moisture content, which was 0·4 per cent. in the liquid and 0·6 per cent. in the vapour. Four experiments were carried out, one with the liquid as purchased, and three with a re-distilled sample. They showed no

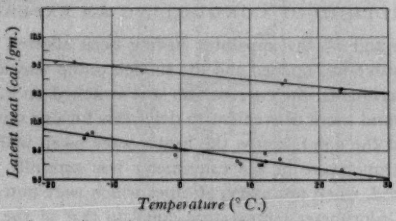


Fig. 2. Latent heat of evaporation of pentane (upper graph) and sulphur dioxide (lower graph). Redistilled material ×.

difference outside the limits of experimental error, the mean values being 92·3 cal./gm. at 14·5° C. and 90·5 at 23·0° C. against Jenkin and Shorthose's values of 92·2 and 89·3 at the corresponding temperatures.

Methyl chloride. As with ethyl chloride, measurements were taken on this material merely as a check on the apparatus. The material was used as purchased, its moisture content being of the order of 0·05 per cent. The result found at 10·8° C. was 93·3 cal./gm. as against Shorthose's value of 94·2.

Sulphur dioxide. This material was widely used for large plants at one period in the history of refrigeration, but nowadays its field of application is restricted to plant of the domestic type. The results obtained are set out in table 1* and are shown in figure 2.

Over the range investigated the results are sufficiently well represented by a straight line, as shown in figure 2, its equation being $L = 95{\cdot}5 - 0{\cdot}34\,t$, where L is the latent heat in cal./gm. and t is the centigrade temperature. All the above results refer to sulphur dioxide as taken from the cylinders in which it is sold

* See p. 115.

† Food Investigation Board, *Special Report*, No. 14.

commercially. One experiment at 11·7° C. was carried out with sulphur dioxide which had been distilled over sulphuric acid. It is marked in figure 2, and clearly gives a result similar to those from the raw product.

Table 1. Latent heat of sulphur dioxide.

Temperature (° C.)	Latent heat (cal./gm.)
− 14·0	99·1
− 13·6	100·6
− 12·8	101·0
− 0·9	96·5
− 0·8	93·4
+ 8·1	91·2
+ 8·6	90·2
+ 11·2	90·0
+ 12·1	94·0
+ 12·2	91·3
+ 14·3	92·1
+ 23·2	88·2
+ 25·0	87·3

Pentane. There are two forms of C_5H_{12}, the iso- and the normal form, which have boiling-points of about 36·2° and 27·9° C. respectively. It might be mentioned that the values recorded in the literature for the boiling-points are rather variable. The material used was that obtainable commercially. It was fractionated and was found to contain fractions boiling at temperatures from 32·5° to 35·0° C. with no marked preponderance of any particular fraction. The density at 4° C. was 0·6453 gm./ml.

The results are shown in figure 2, from which it appears that the straight line $L = 92{\cdot}2 - 0{\cdot}238t$ represents the latent heat of this material between − 20° and + 30° C.

Dichloroethylene. This material is only just beginning to be used at all extensively in refrigeration. Chemically, there are three substances which can be included under this name, and we can trace no previous determinations of the latent heat of any of them. The substance purchased appears to have been a complex mixture, for after two previous fractionations it showed the following composition:

6 per cent. distilled below 49·0° C.
22 ,, ,, between 49° and 54° C.
8 ,, ,, between 54° and 55° C.
13 ,, ,, between 55° and 56° C.
16 ,, ,, between 56° and 57° C.
22 ,, ,, between 57° and 58° C.
11 ,, ,, between 58° and 58·6° C.
2 ,, ,, above 58·6° C.

The density of the fraction boiling between 49° and 54° C. was 1·266 and that of the sixth fraction (57–58° C.) was 1·278, both taken at 22·5° C.

It has a very high specific volume, and would only be suitable for a turbo-compressor installation designed for cooling large quantities of air through a moderate temperature range. Only four experiments were carried out with the sample that was obtainable, the results being as set out in the first two columns of table 2.

These are best represented by a straight line, giving the smoothed values shown in the third and fourth columns.

Table 2. Latent heat of dichloroethylene.

Observed values		Smoothed values	
Temperature (° C.)	Latent heat (cal./gm.)	Temperature (° C.)	Latent heat (cal./gm.)
11·2	79·1	10	79·7
12·0	80·1	15	79·2
22·7	78·8	20	78·7
24·2	78·0	25	78·2
		30	77·7

§6. ACKNOWLEDGMENTS

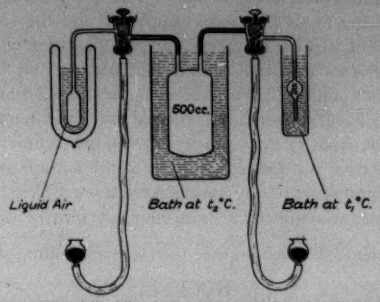
We are indebted to the Engineering Committee of the Food Investigation Board for permission to publish this account of the work, and to the Superintendent of the Physics Department for his interest and for providing facilities to carry out the work. Mr A. R. Challoner, Senior Observer, assisted in the construction of the apparatus and with the observations, and we wish to express our appreciation of his help in the investigation.

DISCUSSION ON THE PRECEDING TWO PAPERS

Mr T. I. Jones. The authors have successfully applied an idea in the development of which I had a small share about eight years ago. The principle underlying the method was first conceived by Professor A. O. Rankine and was utilized by myself, preliminarily to a measurement of the viscosity of the vapour of carbon bisulphide, for investigating the relation existing between the pressure, volume and temperature of that vapour in the region of pressure and temperature prevailing during the viscosity-measurements. Some years later I found that such a method fulfilled a long-felt want, and I therefore avail myself of this opportunity of making it better known.

The original method differed from the one described in the paper in that the volume of liquid distilled was measured in a calibrated tube, and the suction pump

was replaced by a Dewar flask containing liquid air. The latter provision made possible an indefinite number of repetitions of the experiment with the same small original quantity of liquid. Mercury-seal taps lubricated with graphite were used,



Vapour-density apparatus.

and a pellet of mercury was introduced into the tubes to safeguard the taps on the excess-pressure sides. The method possesses the singular advantage that it can measure the density of a saturated vapour.

Mr A. G. TARRANT said that the papers might give rise to the impression that while sulphur dioxide and ammonia were noxious to human beings, dichloroethylene was not. He personally had worked with trichloroethylene and considered it more dangerous than the first-mentioned gases, in consequence of its being odourless.

Mr J. H. COSTE (communicated): As regards the paper on specific volumes, I do not feel quite certain as to the authors' statement that the residual air after the first exhaustion does not affect the quantity of SO_2 required to fill the vessel at any given pressure. At 0° and 760 mm. the density of SO_2 is 0·0029266, which is only about 1 in 1000 above that corresponding to its molecular weight on the assumption that it is as perfect a gas as oxygen. As the exhaustion was fairly good the error, if any, on twelve fillings would be small. I presume that the re-establishment of temperature-equilibrium was adopted as the criterion of equilibrium between the contents of the end vessels, but if determinations of the pressure of the vapour could have been made these would have added to the value of this very useful piece of work.

The work on latent heats is useful, not only in connection with the immediate technical problems of the authors, but as furnishing data for reference on a subject which has not been much investigated. From this point of view it was to be regretted that liquids of technical quality only were investigated, but the reason for this is obvious. A very similar method has recently been used for a variety of organic liquids, mostly of higher boiling-point than those which are used for

refrigerants, by J. H. Mathews and P. R. Fehlandt*, who determined the time for evaporation of (usually) 10 gm. by means of a constant current flowing through a heating-coil.

As to the substances themselves, it may not be inappropriate to point out that methyl chloride, owing to its poisonous properties and the lack of a distinctive and unpleasant smell by which leaks may be detected, has been found to be unsuitable for multiple refrigerators with one central compression unit in large residential buildings. Many cases of illness have occurred in the United States from this cause.

There are three isomeric pentanes—normal, iso- and tetra-methyl methane, boiling respectively at about 37°, 30°, and 9·5° C., but the technical product probably contains only the normal and the iso- isomers as the authors seem to have found. There are three dichloroethylenes, an unsymmetrical one $CH_2 : CCl_2$, which is not of technical interest, and two symmetrical ones, which are formed together by the action of chlorine on acetylene. These comprise the *cis* form, boiling at 60·25° C. $\begin{matrix}\mathrm{HCCl}\\ \ddot{\mathrm{H}}\ddot{\mathrm{C}}\mathrm{Cl}\end{matrix}$, and the *trans*, boiling at 48·35° C. $\begin{matrix}\mathrm{HCCl}\\ \mathrm{Cl}\ddot{\mathrm{C}}\mathrm{H}\end{matrix}$†. The dielectric constants differ in a remarkable degree. The authors were evidently working with the usual technical mixture of these, which is used largely as a solvent and to some extent in surgery.

Dr A. Ferguson. These very interesting papers emphasise the need for the accurate determination of the physical properties of carefully purified liquids and of the variation of these properties over a wide range of temperature. The dependable data are surprisingly meagre, and one reason for this is not far to seek. The preparation and purification of the liquids are matters for the chemist, and the determination of their physical properties is more immediately the business of the physicist. If the work is carried out by a chemist, we hear complaints concerning the accuracy of his thermometry and the like; while the physicist who adventures on the task is likely to be reproached for his lack of care in testing the purity of his specimens.

Professor Sydney Young has shown how the problem may be successfully attacked, and his paper is the classic of its kind. Physicists and physical chemists are to be congratulated in that the authors are turning their attention to these problems. I trust that they will be able to extend the scope of their work both in respect of temperature-range and of the nature of the substances studied—it would be difficult to overestimate its value. The investigation would involve the measurement of the orthobaric densities of liquids and of their saturated vapours and the corresponding vapour-pressures over the whole range of their normal existence, culminating in the determination of their critical constants. This task has been accomplished by Young for some thirty pure organic substances; the results are to be found in the paper already mentioned‡.

* *J. Amer. Chem. Soc.* **53**, 3212–3217 (1931).

† J. Errera and M. Lepingle, *Bull. Acad. Roy. de Belg.*, Classe de Science, **5**, 150 (1925).

‡ *Proc. R. Dub. Soc.* June (1910).

Such results can be accumulated but slowly, and it would be very helpful if the authors could be persuaded to obtain for us reliable values of the critical constants, especially of the critical density and temperature.

Appeal may then be made to a known law connecting reduced density and reduced temperature which, for unassociated substances at least, will give us approximate values of the orthobaric densities of the liquid and of its vapour at any required temperature. Some time ago I deduced such a relation from certain surface-tension formulae. Thus, it is well known that, over the whole temperature-range of existence of an unassociated liquid, we have for the relation between surface tension γ and density

$$\gamma = C(\rho_e - \rho_v)^4,$$

where C is a constant and ρ_e, ρ_v are the orthobaric values of the liquid and vapour densities respectively. C, ρ_e, ρ_v

If we eliminate γ between this and the power law for the variation of γ with temperature

$$\gamma = A(1 - m)^n,$$

where A is a constant, m is the reduced temperature and n may be taken as equal to 1·2, we arrive at an equation for $\rho_e - \rho_v$. But the law of rectilinear diameters gives A, m, n

$$\rho_e + \rho_v = a - bm,$$

where a, b are constants. If now we determine the various constants involved from the considerations that at the absolute critical temperature θ_c we have $\rho_e = \rho_v = \rho_c$, where ρ_c is the critical density, and that when the temperature $\theta = 0$, ρ_v is negligible and $\rho_e = 4\rho_c$, we easily find by addition that a, b θ_c, ρ_c, θ

$$\rho_e = 2\rho_c[(1 - m)^{0·3} + 1 - 0·5m],$$

and subtraction gives a similar equation for ρ_v. If we regard this as a relation between the reduced values of the orthobaric density and the temperature, we see that it contains no constant characteristic of the substance in question.

Young's experimental data serve to test the equation. Writing it as

$$(\rho/2\rho_c) - (1 - 0·5m) = (1 - m)^{0·3},$$

we see that a plot of the logarithm of the left-hand side against the logarithm of $(1 - m)$ should yield a straight line through the origin, of slope equal to 0·3.

The substances so far investigated give very definitely straight lines. The slope of each line is very near to 0·3. No line goes through the origin. This indicates that the equation should be amended to

$$\rho = 2\rho_c[A(1 - m)^{0·3} + (1 - 0·5m)],$$

where A is now a constant characteristic of the substance. A does not, however, vary very greatly from substance to substance and may to a first approximation be taken as equal to 0·9. The formula has been tested for a number of substances, and represents the march of density with temperature very accurately. In one or A

two instances it is necessary to take account of the deviation of the index from 0·3 if the highest accuracy is necessary. Some examples of the equation follow:

Benzene: $\rho = 2\rho_c\,[0{\cdot}8790\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$

Methyl formate: $\rho = 2\rho_c\,[0{\cdot}9192\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$

Chlorobenzene: $\rho = 2\rho_c\,[0{\cdot}8995\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$

Pentane: $\rho = 2\rho_c\,[0{\cdot}8760\,(1-m)^{0{\cdot}29} + 1 - 0{\cdot}5\,m]$

Fluorobenzene: $\rho = 2\rho_c\,[0{\cdot}8831\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$.

A comparison of observed and calculated values is given for benzene in the table.

Table: Values for benzene

Temperature (° C.)	ρ (observed)	ρ (calculated)	Temperature (° C.)	ρ (observed)	ρ (calculated)
0	0·8992	0·9001	160	0·7185	0·7185
20	0·8792	0·8790	180	0·6904	0·6906
40	0·8585	0·8576	200	0·6602	0·6605
60	0·8371	0·8357	220	0·6266	0·6255
80	0·8153	0·8145	240	0·5878	0·5851
100	0·7926	0·7927	260	0·5391	0·5328
120	0·7690	0·7692	280	0·4615	0·4514
140	0·7442	0·7440	(Critical) 288·5	0·3045	0·3045

The figures for benzene show the closeness of the agreement between the observed and calculated values. It will be seen that there is a slight deviation in the neighbourhood of the critical point, which may possibly be connected with corresponding deviations from the law of rectilinear diameters. As may be expected, the equation subsumes a number of short-range and other empirical formulae which have from time to time been put forward. These and other allied matters I hope to be privileged to discuss before the Society in the future.

It may be *à propos* to point out here a mode of attack on the problem of the relation between latent heat and other physical quantities which has not been explored as fully as it might be. If we assume a characteristic equation $f(p, v, \theta) = 0$ which gives curves of the van der Waals type, and if then we draw across any given isothermal, below the critical point, the horizontal line which represents the course of change from the saturated vapour to the liquid phase, the well-known rule for the equality of areas gives

$$p\,(v_2 - v_1) = \int_{v_1}^{v_2} p\,dv,$$

which may be integrated by substitution of the value of p given by the characteristic equation. We have, further, two other equations, since v_1 and v_2 are abscissae of points on the isothermal under discussion, and these three equations, together with the Clapeyron equation, form a basis for the discussion of the relation of L to other physical quantities. The matter is not free from pitfalls*.

* See Kuenen, *Die Zustandsgleichung der Gase u. Flüssigkeiten*, p. 90.

AUTHORS' reply. We wish to thank all the contributors to the discussion for their helpful suggestions. We are particularly indebted to Mr Coste and Dr Ferguson for their remarks, which will be of value in further work on the subject. The present stage of the enquiry is concerned with the development of methods of measurement, and the apparatus has been tried out on fluids of technical interest.

The differences in form between the apparatus shown in Mr Jones's diagram and the one used by us is an example of how proportions and details must be modified to suit the general magnitude of the quantity studied, and to fit in with the chemical characteristics of the materials present. We had to deal with fluids of high vapour-pressures, so the reservoirs and stopcocks had to withstand considerable internal pressures. The apparatus of Mathews and Fehlandt referred to by Mr Coste is intended for liquids with boiling-points above normal room-temperatures. It resembles that devised by Ramsey and Marshall in 1896*. It could not be readily adapted to deal with the liquids we were studying.

Mr Tarrant raises a point that industry will do well to remember: of two equally poisonous substances, the more dangerous is the one which gives no warning of its presence.

Dr Ferguson's equations for saturation densities are most interesting. He has apparently found a law of considerable generality, and we shall look forward to a detailed account of it in due course.

His remarks on the use to which Maxwell's "equality of area" law might be put are also of interest. The same idea had occurred to us in connexion with other work, but we had hoped to be able to use it in the converse manner. Very accurate empirical vapour-pressure equations are available, and it seemed that it might be possible to proceed from one of them to deduce the general nature of the class of characteristic equations which could lead to this particular vapour-pressure law. If this could be done, we should be a considerable step forward on the road to determining a really reliable equation of state. Unfortunately the mathematical difficulties are very great, and pressure of other work has caused us to abandon this particular enquiry.

* *Phil. Mag.* **41**, 38 (1896).

THE BASIC LAW OF THE WET-AND-DRY-BULB HYGROMETER AT TEMPERATURES FROM 40° TO 100° C.

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ABSTRACT. The relation between relative humidity and wet-and-dry-bulb readings, at dry-bulb temperatures between 40° and 100° C., has been studied experimentally. The relative humidity was measured both by absorbing and weighing the moisture in a known volume of air, and by the dew-point method. The results are finally expressed by means of a skeleton table for the wet-and-dry-bulb hygrometer in the temperature range referred to.

§ 1. INTRODUCTION

Practical experience with various hygrometers has shown that the wet-and-dry-bulb type is particularly well adapted for certain classes of measurements, such as, for example, the determination of humidity in a region of high temperatures where observations have to be taken from some point outside the heated zone. The wet-and-dry-bulb is also a convenient form of hygrometer for adaptation to the control of humidity, and consequently we find that the majority of the appliances developed for the operation of kilns for the drying of timber are based upon the wet-and-dry-bulb principle.

Whilst much attention has been given to the development of the mechanical side of this form of hygrometer, comparatively little has been done to establish the validity of the basic formulae by means of which the observations are reduced to relative humidities.

The best known tables for use with the ventilated type of psychrometer are those published by the Prussian Meteorological Institute*, which extend to a maximum temperature of 40° C. only for the dry bulb.

Of tables proceeding to 100° C., only two are known to us. One, prepared by the staff of Messrs A. B. Cleworth, is published by Francis and Taylor, and the other, in the form of a chart, was prepared by Tiemann and is to be found in his book on the *Kiln Drying of Timber*†. The Cleworth book of tables gives no information as to its mode of preparation, nor any indication as to the degree of ventilation of the wet bulb to which the tables apply. Tiemann's chart is stated to have been constructed by extrapolation from determinations at lower temperatures

* *Aspirations Psychrometer-Tafeln.* (Vieweg und Sohn, Braunschweig.)

† Published by the Lippincott Co.

and to have been checked by experiment at a few points, but no details of such tests are available.

It was therefore deemed advisable to carry out a systematic study of the fully ventilated instrument in the temperature range 40–100° C., so that tables could be prepared depending directly on the results of experiment. For the purpose of this investigation, the actual humidity was measured by two independent methods, the dew-point method and the gravimetric method.

§ 2. APPARATUS

The apparatus was arranged to deliver a stream of air initially heated, and then mixed with steam to raise its humidity to an appropriate value, into a vessel containing the wet-and-dry-bulb thermometers and the dew-point apparatus together with means for aspiring samples of the air at the same point for gravimetric determinations.

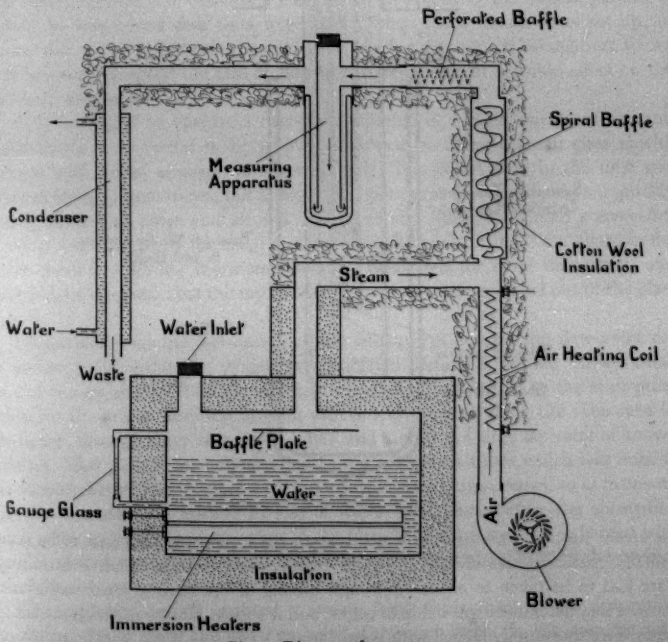

Fig. 1. Diagram of apparatus.

A section of the apparatus is shown in figure 1, the details of the measuring instruments being omitted. The boiler had a capacity of 30 gallons; it was lagged

with magnesia and was provided with four immersion heaters rated at 7 kW. in all, the rate of production of steam being controlled by the energy input. Since the heat losses from the tank were small, the delivery of steam was under accurate control through the electrical input. Air from the room was taken in by the blower, of the Sturtevant type, after which it was heated on its way to the junction with

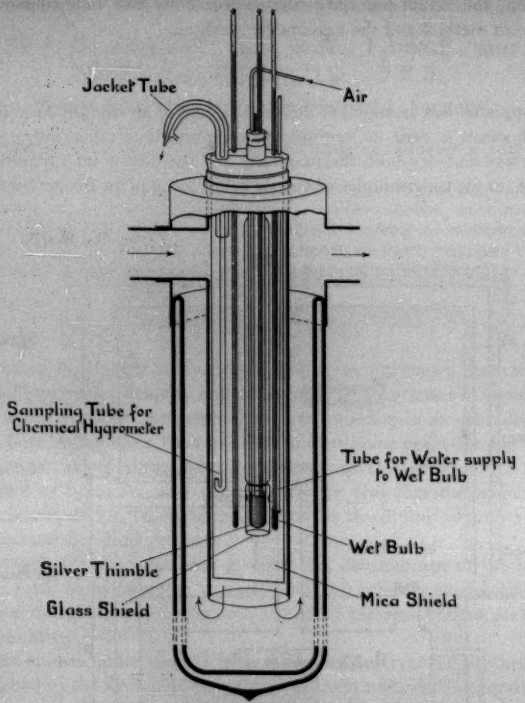

Fig. 2. Measuring apparatus.

the steam supply. This heating was essential, since two variables had to be controlled separately, namely the dry-bulb temperature and the relative humidity. Care had to be taken to ensure thorough mixing of the stream, since experience showed that patchiness was liable to occur, and it should also be mentioned that all pipes were thoroughly lagged with cotton-wool.

The air stream next passed into the measuring apparatus shown in detail in figure 2. To obtain thermal insulation, and yet permit observation of a dew-point thimble, it was found convenient to use a vacuum flask about 40 cm. deep and

11 cm. in internal diameter; centrally within it was a glass tube 6 cm. in diameter and divided down the middle by a thin strip of mica sheet. The tube carrying the air stream to the measuring apparatus entered this glass tube through a brass fitting on the vacuum vessel. The delivery of the air was down the central tube from this point, the air passing up around the outside of the glass so as to jacket the latter and prevent heat losses from the contents; after which it was led away to waste as shown in figure 1.

The glass tube is the central point to which the rest of the apparatus is auxiliary. It contained the wet-and-dry-bulb thermometers under study, as well as a dew-point thimble and a sampling tube for withdrawing air to an absorption hygrometer. Since the dew-point instrument withdraws moisture from the air, whilst the wet-bulb thermometer is liable to raise its humidity, these two instruments were placed on opposite sides of the mica sheet.

The wet-bulb thermometer, at these temperatures, dries rather rapidly, and it was found necessary to provide a continuous slow feed to the muslin jacket. It should be mentioned that tests were made, when the apparatus was set up, to ensure that the air speed past the wet bulb exceeded 3 m./sec., it having been found that beyond this speed the rate of flow of the air is without sensible effect on the wet-bulb depression.

It is not usual to operate a dew-point thimble in a rapid stream of air, and observations were found to be greatly facilitated by fixing a small glass shield, pierced with holes, around the silver thimble. The liquid used in the tube was ether at lower temperatures, but at higher temperatures this evaporated too quickly and a mixture of ether and alcohol or alcohol and water was found convenient. Cooling was effected by the usual method of bubbling air through the liquid in the silver thimble. All the thermometers were calibrated for fixed immersion, and their height was such that the top of the mercury column projected out of the glass vessels.

Observations on the wet-and-dry-bulb instrument and on the dew-point are of necessity discontinuous, whilst the absorption method is not so. Thus a small part of the air stream entering the measuring vessel was led off to the absorption tubes for the whole duration of each experiment, passing along the tube seen in the figure, through three absorption tubes, and thence to a large aspirator of known volume. That part of the tube leading to the absorption tubes which was outside the vacuum vessel was jacketed with hot air from the vacuum vessel, so as to ensure that no condensation, leading to loss of water, took place before the absorption tubes were reached. The latter were in the form of U-tubes and were charged with pumice soaked with concentrated sulphuric acid, prepared by heating the pumice to bright redness and dropping it into the acid.

The aspirator, which ensured a steady flow of air through the absorption apparatus, was of metal and had a capacity of about 20 litres. It was operated with water, which was permissible since the air had at this stage passed the absorption tubes, and water vapour would not diffuse back against the air stream. In any case, such a possible source of error would have been revealed, since in each case

care was taken to note that the third of the absorption tubes did not change appreciably in weight.

§ 3. OBSERVATIONS

Before commencing an experiment, the warm air was allowed to flow for some time before the steam was turned on, since otherwise the cold surfaces of the glass vessels tended to deposit moisture when the mixed air was admitted. After the steam flow had been switched on conditions were allowed to become steady, and after this preliminary period the aspirator tap was opened and observations on the dew-point and on the wet-and-dry-bulb thermometers began. In most experiments the time for the aspirator to empty was of the order of half an hour, and during this period about 10 observations of the wet-and-dry-bulb instruments, and about 45 or 50 of the dew-point, were taken, these being shared between two observers. When the water meniscus in the aspirator reached a definite mark in a glass tube projecting from its base the flow was stopped, the absorption tubes were plugged up and the pressure and temperature in the aspirator were noted. The absorption tubes were weighed in the usual manner after being allowed to cool.

A typical experiment will serve to show the order of the quantities involved and the corrections applied.

10 observations of dry bulb: Mean = 85·7$_0$° C., Max. = 85·8, Min. = 85·4° C.
10 observations of wet bulb: Mean = 60·3$_6$° C., Max. = 60·6, Min. = 60·2° C.
62 observations of dew-point: Mean = 58·2$_4$° C., Max. = 58·6, Min. = 57·7° C.

The variations in the dry and wet bulbs were parallel to each other, owing to a slow drift in the room temperature. The gain in weight of the absorption tubes was 3·314 gm., including 0·008 in the third tube—a value rather above the average. At the conclusion of the experiment the temperature of the air in the aspirator was 14·7° C. (corrected) and the pressure there was 16 cm. of water below atmospheric, the barometer standing at 755·7 mm. of mercury.

After correction for emergent stem and calibration errors, the various temperatures become:

Dry bulb 85·6$_8$° C.
Wet bulb 60·2$_6$° C.
Dew-point 58·2$_0$° C.

To calculate the relative humidity from the dew-point observation, we have: saturation vapour pressure at 58·2$_0$° C. = 141·3 mm., and at 85·6$_8$° C. = 445·8 mm., whence the relative humidity is 141·3/445·8 = 31·7 per cent. Throughout, the vapour pressure of water as a function of temperature has been taken from the *Wärmetabellen* published by the German Reichsanstalt.

From the gravimetric observations we have the known volume of the aspirator as 20,225 cm.^{3}, determined on several occasions by weighing the water required to fill it. Thus the volume of air drawn through the absorption tubes is 20,225 cm.^{3},

after re-saturation and at a total pressure of (755·7 − 160/13·6) mm. of mercury and a temperature of 14·7° C. Thus the actual volume at the dry-bulb temperature (85·7° C.) and at barometric pressure is

$$20{,}225 \times \frac{273{\cdot}1 + 85{\cdot}7}{273{\cdot}1 + 14{\cdot}7} \times \frac{(756 - 160/13{\cdot}6 - 13)}{756 - 141},$$

since the partial pressure of the water vapour when the air is in the aspirator is saturation pressure at 14·7° C. (i.e. 13 mm. mercury) and the partial pressure of the water vapour at the measuring instruments is that corresponding to the dew-point (i.e. 141 mm. mercury).

This reduces to 29,970 cm.^{3} of dry (and therefore, by Dalton's law, of moist) air, at the dry-bulb temperature of 85·7° C. and a barometric pressure of 756 mm. of mercury. This gave up 3·314 gm. of water, so that initially the moisture content in gm./m.^{3} must have been 110·6.

Now the saturation moisture content in gm. per cu. metre at 85·6$_8$° C. is 356·$_0$ from determination at this Laboratory (see the accompanying paper), so that the relative humidity from the gravimetric method is

$$\frac{110{\cdot}_6}{356{\cdot}_0} \text{ or } 31{\cdot}1 \text{ per cent.}$$

Thus the two methods, in this particular case, agree to 0·6 per cent. relative humidity.

§ 4. ANALYSIS OF RESULTS

The results of all the experiments carried out are shown in table 1.

Table 1.

Temperature of		Relative humidity		
Dry bulb	Wet bulb	By dew-point	By gravimetric method	Mean
100·6	75·0	35·3	35·6	35·4
100·4	82·7	50·0	51·6	50·8
100·0	64·6	22·2	22·2	22·2
99·8	60·2	17·6	16·7	17·2
99·7	70·1	29·5	29·7	29·6
97·4	82·0	54·2	—	54·2
96·2	78·2	48·5	49·6	49·0
96·0	60·2	20·9	20·0	20·4
95·5	74·4	41·8	43·4	42·6
95·4	80·7	56·2	59·5	57·8
95·0	64·6	27·1	26·8	27·0
95·0	81·0	57·2	59·8	58·5
94·6	70·0	35·5	34·3	34·9
93·8	76·3	49·3	50·0	49·6
92·4	70·5	38·7	32·9	35·8
90·2	64·8	32·7	33·5	33·1
90·0	70·3	42·8	41·4	42·1
89·8	59·2	24·1	—	24·1
89·7	74·1	52·4	53·1	52·8
88·5	71·1	48·3	48·0	48·2
86·2	43·0	8·8	9·0	8·9

Table 1 (*contd.*).

Temperature of		Relative humidity		
Dry bulb	Wet bulb	By dew-point	By gravimetric method	Mean
86·1	74·0	60·7	61·2	61·0
85·7	70·0	50·8	50·8	50·8
85·7	60·3	31·7	31·1	31·4
85·6	56·5	25·3	—	25·3
85·0	70·2	52·4	51·9	52·2
84·0	51·7	20·7	19·9	20·3
84·0	56·0	27·5	25·0	26·2
84·0	64·3	42·1	41·1	41·6
82·1	68·0	54·6	54·0	54·3
80·8	67·3	56·3	57·4	56·8
80·4	59·9	37·8	37·9	37·8
80·4	56·0	30·2	32·7	31·4
79·2	51·2	24·0	24·1	24·0
78·2	69·1	65·8	—	65·8
76·8	66·9	65·1	63·7	64·4
76·3	54·6	35·7	33·8	34·8
76·1	60·0	48·3	46·4	47·4
76·0	46·4	20·5	19·9	20·2
74·7	49·4	27·7	29·1	28·4
71·0	60·0	60·0	58·3	59·2
70·8	55·7	47·7	46·8	47·2
70·3	61·9	68·8	65·8	67·3
69·8	46·1	28·0	27·5	27·8
69·6	49·5	34·9	33·1	34·0
69·5	41·0	19·1	18·6	18·8
67·8	57·7	62·8	59·7	61·2
65·4	41·5	24·9	23·7	24·3
65·3	55·9	62·6	61·5	62·0
64·8	42·2	26·5	25·5	26·0
64·8	48·9	42·6	42·0	42·3
64·7	42·8	28·2	28·1	28·2
63·4	35·7	16·5	16·3	16·4
60·2	46·0	44·5	44·1	44·3
60·1	39·6	27·6	27·5	27·6
60·1	31·8	12·4	12·8	12·6
60·0	49·4	56·3	55·8	56·0
59·8	53·5	72·2	71·6	71·9
59·3	35·9	21·0	22·0	21·5
59·2	38·9	28·2	29·8	29·0
54·8	41·0	43·9	43·3	43·6
54·6	44·6	55·3	—	55·3
54·6	43·6	54·2	53·6	53·9
54·4	48·6	72·7	67·6	70·2
54·2	34·6	25·9	28·3	27·1
50·9	44·4	71·0	69·2	70·1
50·4	29·8	21·0	20·6	20·8
50·3	44·4	72·4	67·6	70·0
50·2	39·8	51·7	50·0	50·8
48·8	34·5	36·8	36·3	36·6
45·1	24·1	14·8	—	14·8
44·5	34·8	52·0	51·6	51·8
44·4	31·0	35·0	35·6	35·3
44·0	39·7	77·5	—	77·5
40·4	35·2	70·2	70·2	70·2
40·3	30·6	47·3	47·5	47·4

Relative humidity (per cent.)

As a first step in the reduction of these data to a system, the values of the relative humidity were plotted against the depression of the wet bulb for approximately constant dry-bulb temperature. Such a curve is shown in figure 3, where it will be seen, for example, that a change in the dry-bulb temperature of about 34·6° C. (i.e. from top to 3rd curve) changes the relative humidity by 17 per cent. at a depression of 20° C., and by 18 per cent. at a depression of 30° C. Such curves, 14 in number, were used as a basis to correct each observation to the relative humidity that would have been shown had the dry bulb been at the nearest of the round values 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100° C. without alteration of the wet-bulb depression. It is clear both from the above example and from the

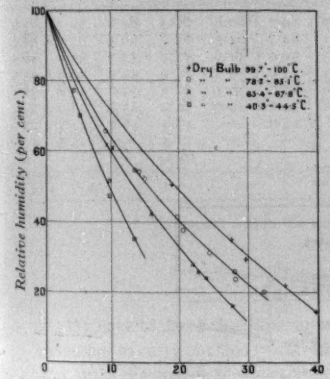

Fig. 3.

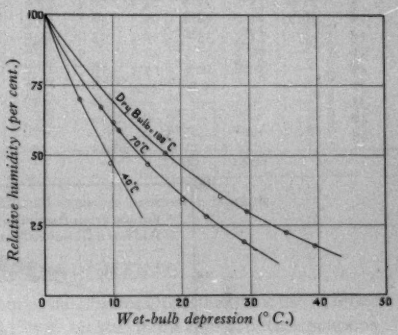

Fig. 4.

well-defined nature of the curve in figure 3 that this alteration will in all cases be small, since no wet-bulb reading had to be altered by more than 2·5° C. Thus, the first six values of relative humidity in table 1 were changed from 35·4, 50·8, 22·2, 17·2, 29·6 and 54·2 to 35·1, 50·6, 22·2, 17·4, 29·7 and 53·1 per cent. respectively.

These adjusted values were next plotted in exactly the same manner as before, and gave points lying much more closely on smooth curves. An example is shown in figure 4. From curves drawn through these points, values were read off at depressions of 5, 10, 15, 20, 25, 30, 35, 40, 45° C., for dry-bulb readings of 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100° C., and gave table 2 below. Points which were doubtful owing to extrapolation are enclosed in brackets.

At this stage of the reduction, it is known that the values in any one vertical column give a smooth curve when plotted against wet-bulb depression, but it is also necessary to ensure that the numbers in a horizontal row give smoothed values against dry-bulb temperature (which would correspond to a slight shift, or a slight distortion, as the case may be, in the curves of which figure 4 is an example).

The result of plotting the numbers in table 2 on this basis is shown in figure 5, to which, however, additional points have been added below 40° C., dry bulb. The latter points are taken from the *Aspirations Psychrometer-Tafeln* referred to in the introduction. The smoothed curves drawn in figure 5 do not depart appreciably

Table 2.

Wet-bulb depression (° C.)	Dry-bulb temperature (° C.)												
	40	45	50	55	60	65	70	75	80	85	90	95	100
5	71·6	73·1	73·8	74·8	76·9	77·8	78·5	80·7	80·8	81·5	82·0	82·3	83·6
10	46·8	50·2	52·8	55·3	58·2	59·7	61·2	64·2	64·5	65·7	66·5	67·5	68·9
15	—	31·5	35·8	39·7	43·0	45·0	47·3	50·0	51·0	52·5	53·7	55·2	56·5
20	—	16·2	21·7	26·6	30·0	32·6	35·5	37·9	39·6	41·4	42·9	44·8	45·9
25	—	(0·0)	10·0	15·5	18·7	22·2	25·6	27·8	30·1	32·0	33·6	35·8	36·4
30	—	—	(0·0)	(5·9)	8·6	13·2	17·1	19·2	22·0	24·2	25·7	28·0	29·2
35	—	—	—	—	—	(5·9)	—	12·0	—	17·4	18·9	21·3	22·6
40	—	—	—	—	—	—	—	—	—	11·5	—	15·5	17·0
45	—	—	—	—	—	—	—	—	—	(6·2)	—	—	—

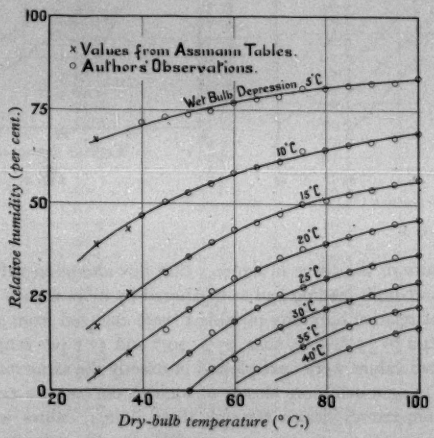

Fig. 5.

from the points of table 2, and it has been assumed that values taken from the smoothed curves will still give curves approximately smooth when plotted against wet-bulb depression. Points taken from the smoothed curves of figure 5 have the advantage that they join smoothly on to the well-established tables prepared for use at lower temperatures, and they are in consequence tabulated, as our final smoothed values, in table 3.

Although the values of relative humidity are given to tenths of 1 per cent., it is not to be assumed that they are reliable to better than 1 per cent. The additional figure is given to assist those who wish to plot the results or to prepare more detailed tables, using table 3, which is only a skeleton, as the basis. In any application of the results the values, whether interpolated or not, should be rounded off to the nearest 1 per cent.

Table 3. Final smoothed values*.

Wet-bulb depression (° C.)	Dry-bulb temperature (° C.)							
	30	40	50	60	70	80	90	100
5	66·4	70·7	74·3	77·0	79·3	81·0	82·4	83·3
10	38·6	46·9	52·9	57·6	61·4	64·5	66·9	68·7
15	18·5	27·5	35·7	42·6	47·7	51·5	53·9	55·9
20	(4·2)	13·3	21·9	29·6	35·0	39·8	43·2	45·8
25	—	—	10·0	18·7	25·2	30·1	33·8	36·5
30	—	—	—	9·9	16·9	22·2	26·0	29·2
35	—	—	—	(2·0)	9·0	15·0	19·4	22·8
40	—	—	—	—	(3·6)	(9·1)	13·5	17·2
45	—	—	—	—	—	(4·0)	(8·7)	—

* Values not within the experimental range are enclosed in brackets.

§ 5. ACKNOWLEDGMENTS

We have to thank the Superintendent of the Physics Department for the facilities necessary to carry out this work. Mr A. Snow assisted in the design of the apparatus and with the initial observations, whilst Mr M. J. Hickman, Junior Observer, assisted with the greater number and suggested useful modifications in the apparatus.

DISCUSSION

Dr J. H. Brinkworth remarked that the authors had found that when the speed of the air past the wet bulb exceeds 3 m./sec. the depression is independent of the rate of flow. He enquired what was the effect of variation in the rate of flow below this speed? He thanked the authors for emphasising the importance of the use of dummy tubes in the direct weighing experiments.

Mr R. S. Whipple said that hygrometric measurements were being made at higher temperatures than was formerly the case and he knew from experience that the data provided by the authors were urgently needed.

Major W. S. Tucker referred to the problem of hygrometric measurements on high-flying airplanes, where very low temperatures had to be dealt with. If the authors could extend their work to low-temperature conditions the results would be valuable in connexion with another investigation at present in hand.

Mr A. Blackie. In the past one has been apt to regard results obtained from wet-and-dry-bulb hygrometers with a considerable degree of doubt. These experiments put the subject on a much firmer basis.

Authors' reply. (In reply to Dr Brinkworth): With the particular form of apparatus used it would be difficult to determine the quantitative relationship between the air speed and the wet-bulb depression. The wet bulb would need to be placed in a wind channel to obtain uniform air-flow past it. In the experiments a high rate of air-flow was maintained and by varying the conditions it was ascertained that small increases or decreases in the air-velocities used had no measurable effect on the results obtained, so it was presumed that the velocities were adequate for the purpose we had in view. (In reply to Major Tucker): It is proposed to extend the investigation to low temperatures, and particularly to the range which is of interest in refrigeration work.

THE WATER-CONTENT OF SATURATED AIR AT TEMPERATURES UP TO 100° C.

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ABSTRACT. The water-content of air which is just saturated has been measured at temperatures from 29° to 94° C. by absorbing the moisture and weighing it. To obtain saturation, air was bubbled through water in a thermostatically controlled bath.

In some experiments the air was initially below saturation, whilst in others it contained more moisture than would be necessary for saturation at the bath temperature. It was verified that the results agreed in the two cases.

§ 1. INTRODUCTION

The quantity of water vapour which can be retained by air at any temperature t from $-40°$ to $+40°$ C. has been tabulated in the *Aspirations Psychrometer-Tafeln* prepared by the Prussian Meteorological Institute. No reference is given as to the origin of these data, but they are possibly a recalculation of Regnault's results which are given in their original form in Kaye and Laby's tables*. Regnault's data are the only comprehensive measurements of which we are aware.

Assuming Dalton's law, which may be expressed roughly in the form that "two gases may be mixed without affecting each other" (so that if the two occupy a volume v, and either alone in the same volume would have the pressure p_1 or p_2, then the total pressure is exactly $p_1 + p_2$), it would follow that the mass of water vapour which just saturates 1 cm.^{3} of air at $t°$ C. is the same as that which would just occupy 1 cm.^{3} at the same temperature in the absence of air. This latter quantity is not easy to measure directly, but is easily calculated from Clapeyron's equation $dp/dt = t^{-1} L/(v_2 - v_1)$, where p is the vapour pressure of water, t the absolute temperature, v_2 the reciprocal of the quantity referred to and v_1 is the volume of 1 gm. of liquid water at the same temperature, whilst L is the latent heat in mechanical units.

Values produced in this way up to and beyond 100° C. are given by the German Reichsanstalt in their *Wärmetabellen*. It seems desirable, however, in view of the fact that these are not experimental results, and that they involve differentiating the vapour-pressure curve, as well as of the fact that Dalton's law cannot be assumed without proof, to carry out direct determinations, at any rate up to 100° C.

* Sixth edition, p. 41.

§ 2. EXPERIMENTAL ARRANGEMENTS

The method adopted was to saturate a current of air and then to absorb the moisture from a measured volume in sulphuric acid, and to determine its quantity by weighing. Very careful precautions must be taken to avoid even the slightest cooling of the air after it has been saturated, since this would involve a deposition of moisture and consequently a low result.

The apparatus employed is shown in section in figure 1. In one method of experiment, the air was first heated and then mixed with steam, so that the resulting mixture was at a temperature above that for the intended experiment and contained water in excess of the amount that would ultimately saturate it in the experiment. It then entered the apparatus (at the left-hand side of figure 1) and passed through

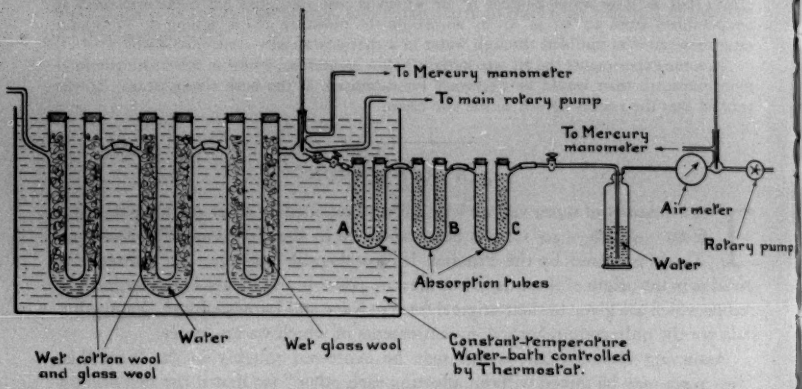

Fig. 1. General diagrammatic view of apparatus.

three U-tubes which contained wet glass wool and were maintained at a constant temperature by immersion in a thermostatically controlled water-bath. From these U-tubes a sample of the air was by-passed to the absorption tubes containing the sulphuric acid, the first of these also being in the controlled water-bath, so that the greater part of the contained moisture was removed before any change of temperature could occur.

After leaving the absorption tubes, the air was metered by a gas-meter of the wet type, passing first through wash-bottles which served also to indicate the rate of flow when the apparatus was being adjusted before an experiment. The temperature and pressure at the meter exit were recorded, as well as those immediately before the absorption tubes were entered. The Wright meter employed was calibrated from time to time throughout the work by comparison with a known volume.

The absorption tubes may be mentioned here. They contained broken pumice,

saturated with the sulphuric acid, so as to present a large surface. To impregnate the pumice with acid, the former was raised to a white heat and dropped into the acid. In carrying out an experiment, the air was allowed to run for some time, with the by-pass tap closed, until conditions became steady. The meter was then read, the tap was opened, the pump was started in the by-pass circuit, and absorption was allowed to take place. At the end of the experiment, the absorption tubes were plugged as rapidly as possible and allowed to cool for weighing. It should be noted also that each of the absorption tubes was provided with a dummy of approximately the same dimensions, and weighted with sand. This was desirable owing to the rather small weight of the moisture (as low as 1·5 gm. in one experiment) compared with the total weight measured (possibly 150 gm.) and to the known vagaries of glass in picking up moisture from the air on its surface in quantities sufficient to affect the weight.

A second method was employed which contrasts with the one just described, in that the air initially supplied to the apparatus was drier than that ultimately required, instead of containing excess moisture. As in the other method, the air was passed through the three **U**-tubes containing wet glass wool, and from these to the absorption tubes. The glass-wool tubes in this case, however, were partly filled with water, through which the air was forced to bubble, their function now being to supply moisture, and not to remove it.

Results. The calculation of a typical experiment is set out in full below, in order to show the corrections applied and the methods of reduction employed.

§ 3. SECOND EXPERIMENT OF APRIL 16, 1930

The barometer (corrected) was 762·7 mm. and the reading of the manometer at the outlet of the gas-meter was 18·0 mm., so that the air left the meter at a pressure of 744·7 mm. Its volume at this stage was 1·008 ft.^{3} as read by the meter, or $1{\cdot}008 \times 0{\cdot}02831$ m.^{3}, account being taken of the calibration error. Now the manometer at the entrance to the absorption tubes read 9·0 mm., so that the pressure at this point was 753·7 mm. These pressures are total pressures, and must in each case be reduced by the saturation vapour-pressure of water at the temperature prevailing at the corresponding position, since the air is saturated at both positions. These temperatures are 70·5° C. and 12·5° C. respectively, the corresponding vapour-pressures being 238·8 mm. and 10·9 mm. Thus the volume of the air when it entered the absorption tubes was

$$\{(273{\cdot}1 + 70{\cdot}5)/(273{\cdot}1 + 12{\cdot}5)\}\{(744{\cdot}7 - 10{\cdot}9)/(753{\cdot}7 - 238{\cdot}8)\} \times 1{\cdot}008 \times 0{\cdot}02831,$$

i.e., $0{\cdot}0489_3$ m.^{3}.

The differences in weight between the absorption tubes and their dummies were as below:

	Before experiment	After experiment
Tube A	0·755	10·564
Tube B	0·222	0·326
Tube C	0·406	0·407

These show gains of weight equal to 9·809, 0·104, 0·001, a total of 9·914 gm. Consequently the result of the experiment is to show a moisture content, for saturation, of $9{\cdot}914/0{\cdot}0489_3$ or 202·7 gm./m.^{3}.

It is to be noted that the pressure-measurements which have been used are necessary only for finding the actual volume of the air, at the point where it is saturated and at the temperature under investigation, and not for correcting the saturation moisture-content to a standard pressure. If Dalton's law were absolutely true, the moisture-content would be identical at all pressures from zero upwards, and even though it is not strictly true, the variation over the range of barometric pressures may be neglected. In a few cases experiments were carried out, at approximately the same temperature, by both the methods described above. Typical pairs are shown in table 1.

Table 1. Comparison of methods 1 and 2.

Temperature (° C.)	Moisture-content (gm./m.^{3})	
	Method 1	Method 2
60·3	131·9	132·4
70·4	203·5	—
70·5	—	202·7
75·0	245·3	—
75·1	—	244·1
85·0	343·6	343·4

Similarly, pairs of observations which differed only in the rate of air-flow were found to show no systematic variation due to this cause.

The results are shown in table 2 and plotted in figures 2 and 3, in which the continuous lines represent the results given in the *Wärmetabellen* of the Reichsanstalt,

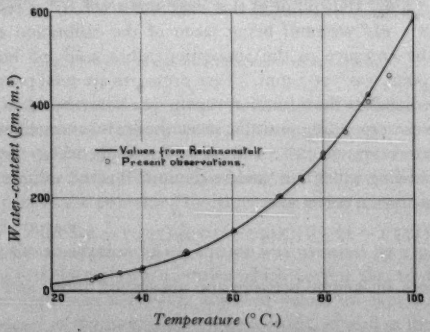

Fig. 2. Moisture-content of saturated air.

and the points are shown individually. If the absolute content be plotted in grams of moisture per cubic metre of saturated air, as in figure 2, the results of the two investigations agree up to about 85° C., after which temperature the present results

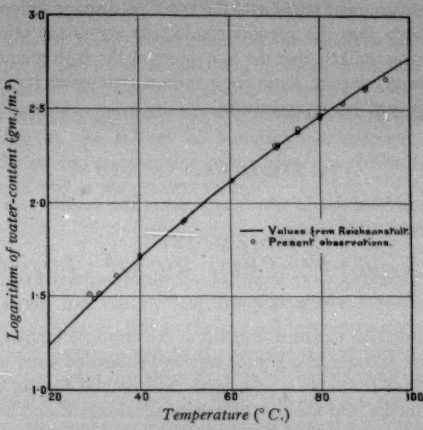

Fig. 3. Moisture-content of saturated air.

Table 2.

Temperature (° C.)	Moisture-content Q (gm./m.³)	log Q	Weight of moisture collected (gm.)
29·0	32·74	1·5151	2·46
30·0	31·13	1·4932	3·07
31·0	32·95	1·5179	1·50
35·0	41·34	1·6164	3·12
40·0	52·97	1·7240	6·55
40·1	51·68	1·7133	6·59
40·1$_5$	52·89	1·7234	2·60
49·6	80·38	1·9052	4·22
49·9	82·62	1·9171	4·63
50·1	81·74	1·9124	11·91
60·3	131·90	2·1202	16·92
60·3	132·43	2·1220	11·10
70·0$_5$	205·28	2·3124	9·96
70·4	203·49	2·3085	11·49
70·5	202·74	2·3069	9·91
70·6$_5$	206·27	2·3144	6·59
75·0	245·27	2·3896	13·72
75·1	244·05	2·3875	14·02
80·0	289·68	2·4619	17·36
80·0	298·56	2·4750	9·89
85·0	343·56	2·5360	12·72
85·0	343·42	2·5358	9·95
90·1	408·40	2·6111	14·50
90·2	422·82	2·6262	16·28
94·4	464·17	2·6667	25·90

fall on a lower curve than the German ones. On a logarithmic scale, figure 3, in which a given displacement represents the same percentage difference at all temperatures, it appears that the present results cut across the others at an angle. This makes it appear unlikely that the low values at the high temperatures are due to lack of saturation, since if there were such a systematic error, it would exist (and even be larger) at low temperatures.

§ 4. SMOOTHED VALUES

No formula has been found which represents the results very well. The best two are of the forms

$$\log U = A + B/t + C \log t + D\,(t_c - t)^{\frac{1}{3}} + E\,(t_c - t)$$

and

$$p/U = a + bt + ct^2 + dt^3,$$

where U is the saturation moisture-content at the absolute temperature t, t_c is the critical temperature of water, and p is the vapour-pressure of water at temperature t.

The first of these formulae is of a form used by the Bureau of Standards in their work on ammonia, and has the theoretical advantage that it behaves correctly in the neighbourhood of the critical temperature.

The second formula is a generalization of one used in the Assmann psychrometer tables, viz., $p = Kut$, where K is a constant. The best values resulting from this work will be taken as the mean of the smoothed values derived from the two formulae, the constants in them having been determined by the method of least squares. These adopted values are set out in table 3.

Table 3. Smoothed values adopted to represent the results of the experiments.

Temperature (° C.)	Moisture-content of saturated air (gm./cm.)
25	24·2
30	31·7
35	41·1
40	52·8
45	67·2
50	84·7
55	105·9
60	131·3
65	161·7
70	197·6
75	239·9
80	289·2
85	346·7
90	412·8
95	489·0
100	(576·0)

The accuracy is probably about 2 per cent. throughout, although the same number of decimal figures is retained in every case for convenience when differences are in question.

§ 5. ACKNOWLEDGMENTS

I have to thank Dr Kaye, Superintendent, for facilities for carrying out this work. The apparatus used was adapted and modified from that designed by Dr Griffiths and myself for studying the wet-and-dry-bulb hygrometer at higher temperatures, and I feel deep gratitude to Dr Griffiths for his helpful advice in many matters pertaining to the subject of humidity-measurement. Mr Hickmann assisted in the design and construction of the apparatus, whilst his patient care in taking observations was of the greatest value.

DISCUSSION

Mr J. H. Coste (communicated): Mr Awbery is probably acquainted with the paper by Vernon and Whitby* on the quantitative humidification of air, in which air and steam were cooled to the desired temperature and then allowed to pass through a long length (about 75 ft.) of glass tubing in the thermostat. Five determinations of the water vapour in air at 25° C. agreed in all cases within 0·5 per cent. with Regnault's value of 22·8 mg./litre, although it is not clear whether they actually represent the water *in* or that *associated with* the volume of dry air which was finally measured. If the latter, their results should be reduced by about 3 per cent. The agreement of the results obtained by the author's methods (1) and (2) seems to remove all doubt as to the close approximation to saturation obtained in the very simple saturator.

Dr A. R. Lee (communicated): In view of the fact that one of the methods employed by the author introduced bubbling, I should like to draw his attention to a paper by Vernon and Whitby† on the quantitative humidification of air. These workers concluded that the most efficient method of obtaining saturation was one similar in principle to that first used by the author. They also showed, however, that it was impossible to obtain saturation merely by bubbling the air through water, and reference is made to other workers who have noted the defects of such a method. Failure is presumably due to the relatively low vapour-pressure within a bubble caused by the concavity of the liquid surface, and also to the condensation caused by the release of pressure when the bubble bursts. The good agreement which the author obtains between his two methods suggests that the wet glass wool is able to compensate for such deficiencies. It seems worth while, however, to call attention to this point, since the errors due to the simple bubbling method are very frequently overlooked.

Mr A. Blackie. I would like to ask whether the bubbling-bottle shown before the air-meter was found, in conjunction with the meter itself, to saturate the air at the temperature in the meter? Personally I have found the greatest difficulty in saturating air by bubbling. I would also draw attention to the really large effects that the condensation film of moisture on glass or rubber tubing has on the moisture

* *Trans. Farad. Soc.* **27**, 121 (1931). † *Ibid.* **27**, 248 (1931).

contained in the air passing through it when there are changes in the original hygrometric state of the air. This should not have troubled the author, however, as he maintained constant conditions for considerable periods.

AUTHOR'S reply. I thank both Mr Coste and Dr Lee for their remarks on the problem of saturating air. The paper to which they call attention had escaped my notice, and I am glad to learn of its existence. I was well aware that it was practically impossible to saturate air by bubbling it through water, just as it is difficult to obtain saturation in the converse case when air is passed through a spray of water. I suspect that diffusion inside the bubble, or in the space between the droplets, is an important factor, and that subdivision of the bubbles is therefore important, so that lengthening of the time of contact would be valuable. Doubtless the glass wool tends to be of assistance in this way. Mr Blackie's warning as to the change of weight of glass vessels over periods of several hours cannot be too strongly emphasised. I have used the precaution, in various pieces of work, of having a dummy tube of the same size and subjected to the same conditions, and have found it very satisfactory for work of moderate accuracy. The device is by no means new, but its originator is unknown to me.

SOME PROPERTIES OF THE SOUND EMITTED BY AIRSCREWS

By C. F. B. KEMP, A.R.C.S., B.Sc., D.I.C.

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ABSTRACT. A condenser-microphone amplifier system has been employed to determine the intensities and directional properties of the first six harmonics in the sound of rotation of an airscrew operating at zero rate of advance and actuated by a silenced engine. The sound-energy associated with the frequencies considered has been found to be 18 watts, the fundamental (first harmonic) being responsible for 50 per cent. of this, while the first three harmonics together contribute 90 per cent. Maximum intensity occurs from 15° to 30° behind the plane of rotation, and in this region the sound-output is particularly steady. Large intensity-fluctuations occur along the axis of the slip-stream. The intensity follows the inverse-square law of distance at points further than 200 ft. from the airscrew centre, a 2·4-power law holding approximately for nearer points. Cathode-ray oscillograms demonstrate the existence of frequencies of the order of 600 ~ which are probably not components of the sound of rotation. The hypotheses of Lynam and Webb, as modified by Paris, are briefly discussed in relation to the results of experiment.

§ 1. INTRODUCTORY

The low-frequency sound generated by a rotating airscrew is known to comprise a harmonic train of frequencies commencing with one equal to the number of blades × airscrew revolutions per sec.[1, 2, 3]; and some experiments made by Fage[4] on model airscrews 3 ft. in diameter indicate that the harmonics become progressively smaller in intensity. The sound is also known to have directional properties, this aspect of the subject being first treated by E. J. Lynam and H. A. Webb[5] and later by M. D. Hart[3]. Recently E. T. Paris[6] has made an experimental study of the intensity-distribution of the fundamental component around a stationary aeroplane and has discovered that the alternative theories of Lynam and Webb may be suitably combined to give a result agreeing well with his observations. An analysis of the sound emitted by aircraft in flight has been attempted by J. Obata and Y. Yosida[7], whose results, so far as airscrew sound is concerned, are discussed at a later stage.

In this paper further full-scale experiments to determine the properties of airscrew sound are described, information being given regarding (i) the rate of intensity-decay at moderate distances from the airscrews; (ii) the directional properties of the first six harmonics in the "sound of rotation"; (iii) the absolute intensities of these components; and (iv) the existence of high-pitched sounds.

Earlier investigators obtained certain results which were not well understood and the bearing of recently acquired knowledge on these is briefly discussed.

§ 2. FACTORS AFFECTING THE CHOICE OF AN EXPERIMENTAL METHOD

Ideally, observations should be made on an airscrew operating under normal flight-conditions by causing an aircraft to maintain a constant height and known rectilinear course with reference to the sound-receiving apparatus. The interpretation of any results so obtained is rendered difficult for the following reasons.

The exhaust sound, which is comparable in intensity with the airscrew sound, has its own appropriate series of harmonics and usually has directional properties of a complicated nature. If the aircraft is one in which the airscrew is driven directly on the crankshaft, the frequencies present in the airscrew sound of rotation must necessarily coincide with certain frequencies in the exhaust sound, so that separation of the two effects is impossible. An aircraft equipped with a geared airscrew is therefore preferable for purposes of experiment. To illustrate the general nature of the aircraft sound in this case, the engine and airscrew finally used by the author will be taken as an example. The engine was a Rolls Royce Condor developing 600 h.p. at 1800 r.p.m.; it had twelve cylinders arranged in two banks of six, each bank having its own exhaust manifold. The two-bladed airscrew was geared down in the ratio 0·477 : 1. The subject of exhaust sound has been treated in considerable detail by M. D. Hart[3] and he has shown that for cylinder-in-line engines the sound consists of a harmonic series of tones having a fundamental f_E given by:

$$f_E = \text{crankshaft r.p.m.} \times \tfrac{1}{60} \times \tfrac{1}{2} \times \text{number of cylinders per bank}$$
$$= 90 \sim \text{for the above engine.}$$

Each component frequency has its own directional effect, the nature of which is dependent upon the relation between the wave-length and the spacing of the exhaust manifold outlets. The frequency of the fundamental airscrew note is f_A where

$$f_A = \text{crankshaft r.p.m.} \times \tfrac{1}{60} \times \text{no of blades} \times \text{gear ratio}$$
$$= 28{\cdot}6 \sim \text{for the above airscrew.}$$

At 1800 r.p.m. the following frequencies are therefore to be expected:

Airscrew	28·6	57·2	85·8	114·4	143·0	171·6	200·2 etc.
Exhaust			90			180	etc.

These two trains of frequencies being non-simply related, continuous oscillograph records of the sound show no wave-form repetitions, this condition being aggravated by the continual change in aspect of the aircraft by reason of which the relative importance of the various exhaust harmonics at the distant recording point also changes. Interpretation of the record could be effected by enharmonic analysis, but in the absence of any rapid method of accomplishing this, the practicability of the idea seems doubtful in view of the great number of analyses required to obtain the relationship between the sound spectra and the aspect of the airscrew.

Selective sound-receivers, tunable to any desired frequency in the sound, would

yield results easier of interpretation, provided that the tuning arrangements were such as to produce a band-pass-filter response, the width of the band being great enough to allow for change in frequency due to the Doppler effect. Some difficulty in interpretation arises, however, in those frequency ranges where the airscrew sound has a frequency component close to one forming part of the exhaust sound (e.g. 85·8 ~ and 90 ~ in the instance quoted above), for, in general, the frequency-separation is less than the frequency-change due to the Doppler effect.

Whatever system of sound-measurement is employed, the time track of the aircraft must be accurately determined, and further, local meteorological conditions must be such that the longitudinal axis of the aircraft coincides with the direction of flight.

For a first study of airscrew sound it was deemed advisable to use an airscrew rotating at a fixed point on the ground, and to neglect, temporarily, any effect due to ground reflection. Expense did not permit of the airscrew being actuated electrically, with the consequent elimination of exhaust noise, and recourse was had to a silenced Rolls Royce Condor engine of the type mentioned above.

§ 3. AIRSCREW EQUIPMENT

The engine and airscrew were mounted on the fuselage of a Berkeley aircraft, the wings, tail plane and rudder having been removed. By placing a heavy trestle of adjustable height under the rear of the fuselage, the plane of rotation of the airscrew could be made vertical, and to prevent the equipment tipping forward, the back was securely lashed to screw pickets in the ground. As a further precaution, all the engine controls were removed from the cockpit to a remote control box, it being thought that some danger existed of the structure ultimately breaking up as a result of running the engine at normal flying speed under these conditions. The fuselage was placed with the two exhaust outlets above a silencing pit, 10 ft. deep, filled with steel shavings and covered with a firm layer of earth at the top. The gases from each exhaust manifold were conducted into the pit through a flexible iron pipe 4 in. wide attached to the normal exhaust outlet. From the pit, the gases were conveyed underground through 50 ft. of iron pipe 6 in. in diameter to a second pit, also filled with steel shavings but having no earth cover, so that escape to atmosphere was possible. Arrangements for cutting out the silencer were made by providing, at the junction of each exhaust outlet with its flexible connexion, an alternative path leading direct to atmosphere. These paths could be opened or closed by spring-controlled baffles, and besides enabling the degree of silencing to be assessed, functioned as a safety device by preventing the accumulation of unburned mixture within the pit when the engine was being started. The performance of the silencing system was found to be improved by wrapping several layers of asbestos blanket round the pipes leading to the pit.

The directional effect of any component in the airscrew sound of rotation may be completely represented by a polar surface with the airscrew boss as origin, the

length of any radius vector being proportional either to the pressure-amplitude or to the intensity in that direction. In the absence of any sound-reflection this surface may, for reasons of symmetry, be regarded as a surface of revolution about the airscrew axis, the generator being the polar curve taken in any plane perpendicular to the plane of rotation of the airscrew and including the axis of the airscrew. With the equipment described, it is therefore sufficient to take observations at points in a horizontal plane through the airscrew axis, if ground-reflection effects at a distant point are to be neglected. The conditions of experiment are simplified if it can be assumed that a polar curve so obtained is symmetrical about the airscrew axis. That this assumption is justifiable was shown by E. T. Paris who, in some early experiments on the fundamental airscrew note, took observations at a fixed point, while an aircraft situated in the middle of the landing ground was turned through 360° in regular angular increments, the airscrew boss being brought always to the same place.

Accordingly, the equipment was installed so that ahead, astern and to port there were no reflecting surfaces other than the ground. To starboard there were no buildings nearer than 150 yds., so that measurements made to port at distances of the order of 100 yds. could be regarded as free from serious errors due to reflection. Even so, the results obtained must be considered strictly as applying to an airscrew operating at zero rate of advance, and at nearly zero altitude, and actuated by an engine which has been silenced as well as circumstances permit.

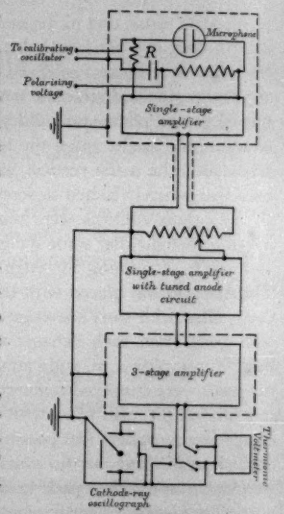


Fig. 1. Schematic of recording apparatus.

§ 4. SOUND-RECORDING APPARATUS

The sound was received by a condenser microphone, the variations in charging-current producing potential changes which were amplified to operate a cathode-ray oscillograph or thermionic voltmeter of wide range, figure 1.

The microphone, a Western Electric model, was calibrated by the resonance tube and Rayleigh-disc method which, for the low frequencies involved, yields the same result as a "free air" calibration[8]. This calibration was performed by the Post Office Engineering Research Department. Up to a frequency of 200 ~ (above which no airscrew sound-measurements were made) the sensitivity was found to be uniformly 2 mV/dyne-cm.^{2}, with reference to the actual r.m.s. pressure on the diaphragm. Amplitude-distortion was assumed to be absent, as it is understood that no condenser microphone of this type has yet been found to suffer from the defect. The first stage of the amplifier was supplied with the

microphone by Standard Telephones & Cables Co. Ltd.; it was housed in a small screened box with the microphone clipped into one side. When observations were being taken, the box was moved around the airscrew and placed upon the ground with the microphone diaphragm horizontal, so that nodal measurements were obtained. This procedure also minimized diffraction effects, and effects due to direct reflection from the ground. It is true that the polar curves were thus taken in a plane not passing through the airscrew axis, but since the points of observation were mostly on a semi-circle of radius 300 ft., the angle included between the horizontal and a line joining the airscrew-centre to the microphone was less than 2°. Isolation of any desired frequency was effected by means of a tuned anode circuit with the resonance curve arranged to show practically uniform response over a small frequency-band determined by the inevitable fluctuations in engine-revolutions and inaccuracies in the revolution meter, while there was only a small response to frequencies about 30 ~ removed from the one considered. To obtain the resonance curves, the microphone was replaced by a condenser of equal capacity (about 0·0003 μF), a measured alternating current from a precision oscillator was passed through the 2 ohm calibrating resistance R, and the output was read on a thermionic voltmeter. The amplifier as a whole was designed so that the overloading of any stage necessarily caused the succeeding one to be overloaded, the effect being indicated by a change in reading of the milliammeter in the plate circuit of the last valve.

For visual observations on pressure amplitude a triple range thermionic voltmeter (0·5 to 120 r.m.s. volts) was used, and for the recording of wave-form a Wood cathode ray oscillograph (the tuning stage of the amplifier was omitted in this case).

§ 5. PRECAUTIONS OBSERVED DURING THE TAKING OF RECORDS

Before the commencement of a series of observations it was ascertained that (i) background noises caused negligible response of the apparatus; and (ii) a light surface wind was blowing. If the atmosphere is still, undesirable refraction effects are introduced by the lapse rate in the stagnant air just above the ground.

No readings were taken when any other aircraft approached the aerodrome.

§ 6. INTENSITY/DISTANCE RELATION

Measurements of pressure-amplitude are most conveniently made at distances of the order of a few hundred feet if the airscrew is operating under the conditions specified. For greater distances it is seldom possible to mark out a semi-circular distribution of observation points without introducing into the sound-field some obstacle which would adversely affect the free passage of the waves; furthermore the pressure-amplitudes are of such a magnitude that, at a few hundred feet, a normal background disturbance produces a negligible effect on the recording instrument. It is imperative, however, that measurements be made at distances great enough for the inverse-square law of intensity to be obeyed, if a knowledge

of the intensity at more distant points is required. An experiment was conducted, therefore, to determine the intensity/distance relation at distances from the airscrew between 50 ft. and 500 ft. The determination was made in one direction only, namely 25° behind the plane of rotation, since preliminary experiments had shown that the sound-output was particularly steady and comparatively great in this direction. The following method of taking readings was adopted. A forward observer signalled when the required engine revolutions had been reached and thereafter this speed was maintained for half a minute, a variation of ± 10 r.p.m. being permitted. During this period the output-meter was read at frequent equal intervals and an average value was found. The results are given in table 1, and are consistent with the inverse-square law $I \propto R^{-2}$ when 200 ft. $< R <$ 500 ft. and with $I \propto R^{-2\cdot4}$ approximately when 50 ft. $< R <$ 200 ft.

Table 1. Intensity/distance relation.
(Engine speed = 1700 ± 10 r.p.m.)

Distance R from airscrew boss (ft.)	Pressure-amplitude × arbitrary constant
490	1·0
400	1·25
300	1·55
200	2·3
100	6·0
50	10·85

It is improbable that the inverse-square law holds at very great distances, dissipation and refraction effects causing the intensity to decay at a greater rate.

§ 7. INTENSITY AND DISTRIBUTION OF HARMONICS 1–6

On a semi-circle of 300 ft. radius with the airscrew boss as centre, thirteen microphone positions were marked out at intervals of 15°, the first ($\psi = 0°$) being directly ahead of the airscrew and the last ($\psi = 180°$) directly astern. The output-meter readings fell into three classes: (i) constant; (ii) slowly fluctuating between well-defined limits; (iii) as (ii) but with occasional excursions well beyond these limits. In constructing the polar curves of figure 2 these excursions were neglected. The nomenclature adopted identifies the terms "fundamental" and "first harmonic," "octave" and "second harmonic," etc. It appears that (*a*) In general, the higher the order of the harmonic the less the pressure-amplitude, especially for values of $\psi > 75°$. This is in accordance with Fage's model experiments and with information derived from cathode-ray-oscillograph records, § 8. (*b*) Maximum pressure-amplitude occurs between $\psi = 105°$ and $\psi = 120°$, i.e. from 15° to 30° behind the plane of rotation of the airscrew, except in the case of the sixth harmonic which is maximal at $\psi = 150°$. (*c*) Sound-energy is also projected forwards within limits which may be taken approximately as $\psi = 0°$ and $\psi = 60°$. The spur in the polar curve of the first harmonic was also found by Paris, who used a Tucker hot-wire

microphone. At very low frequencies the energy of this part of the sound is small compared with the energy thrown laterally and backward, but becomes of relatively greater importance at the higher frequencies considered, so that the energy in the fourth, fifth and sixth harmonics appears to be distributed fairly equally, though not symmetrically, in front of and behind the plane of rotation.

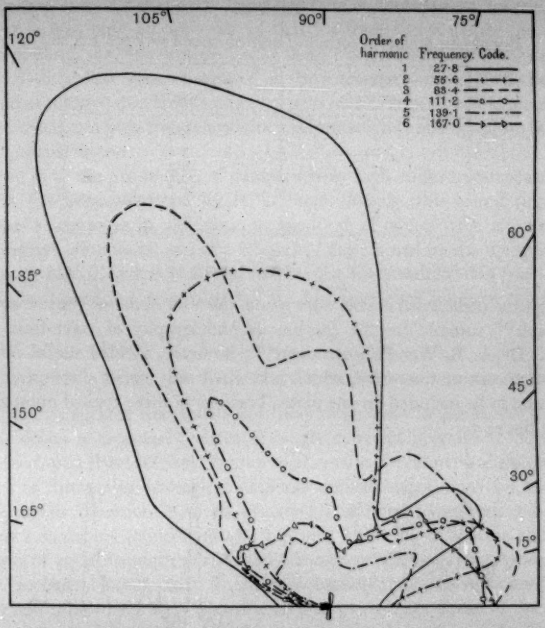


Fig. 2. Polar curves for distribution of airscrew sound. Scale: 1 cm. = 0·84 dyne/cm.^{2} (r.m.s.). (Airscrew pitch at pressure face, 3200 mm.; blade-diameter 4500 mm.; engine speed 1750 r.p.m.)

The lengths of the radii vectores give the actual pressures on the microphone diaphragm, so that the pressures existing in the unimpeded wave-front are half these. An estimate may now be made of the total sound-energy associated with each harmonic, the polar surfaces being generated by the rotation of the two-dimensional polar diagrams of figure 2 about the axis of rotation of the airscrew. (See table 2.)

A study of the pressure-fluctuations observed during each recording period of 30 sec. showed that (1) Along the slip-stream the sound output is exceedingly unsteady, the pressure frequently rising to four or five times the normal value. The effect is quite local, disappearing 15° off the axis. (2) The output is steadiest

between $\psi = 75°$ and $\psi = 165°$. In and near to the direction of maximum output, the amplitude-fluctuation is not more than 3 per cent. while in other directions, within the range stated, the fluctuation is about 5 per cent. for the very low frequencies and not more than 10 per cent. for the higher ones. (3) In any chosen direction the lower the frequency the steadier the output.

Table 2.

Energy in airscrew sound. (Engine speed = 1750 r.p.m.)

Order of harmonic	Sound-energy (watts)
1	8·9
2	3·7
3	3·4
4	1·2
5	0·3
6	0·6

§ 8. CATHODE-RAY-OSCILLOGRAPH RECORDS

At the time these experiments were made, the Von Ardenne type of cathode-ray oscillograph[9], suitable for the continuous photography of wave-form, was not available. Dr A. B. Wood's instrument[10], however, yielded useful information in spite of the short time-base, which permitted only about three periods of the fundamental to be included on one plate. Tracings of three typical photographs are shown in figure 3.

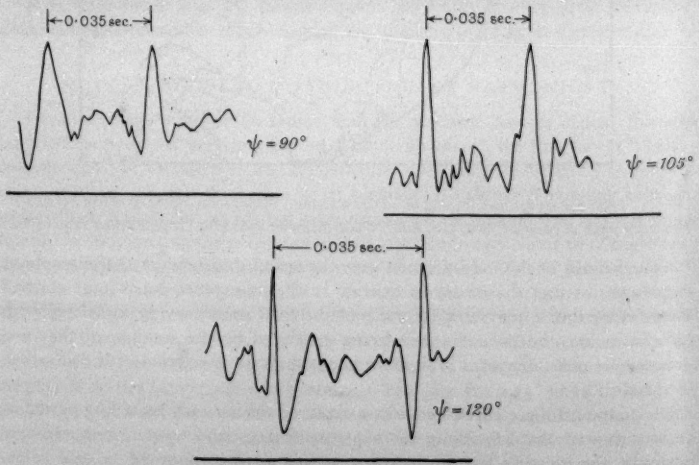


Fig. 3. Tracings of oscillograph records.

Owing to imperfect exhaust-silencing and the presence of engine clatter, wave-form repetitions do not occur with sufficient accuracy for a Fourier analysis to be justified.

The general form of the oscillograms, which were taken near the direction of maximum output, indicate that the sound is of an impulsive character and therefore rich in harmonics; in fact it is more impulsive than at first appears, for the amplification at 30 ~ was only 40 per cent. of that for frequencies within the constant amplification band. On the original plates there was evidence of the existence of a frequency of about 600 ~ superimposed on the main wave-trace. If the harmonics in the sound of rotation decrease in intensity with increasing order, it may reasonably be doubted whether a high frequency which is immediately obvious from inspection forms part of this sound.

A theory as to the production of comparatively high audio-frequency sound by an airscrew has been advanced by H. Glauert. Briefly, this sound (termed the "drag noise") originates in the aperiodic shedding of eddies from all parts of the blades. It is non-directional and of a frequency determined by the tip-speed of the blades. It appears from the oscillograms that the amplitude of this 600 ~ note is small compared with the amplitudes of the low-frequency notes in the sound of rotation.

§ 9. APPLICATION OF THE THEORY OF LYNAM AND WEBB

Lanchester suggested that the sound from an airscrew is due to the movement in circular orbits of pressure-centres of nearly constant magnitude. Lynam and Webb[5] developed this idea, and from a mathematical treatment of the properties of systems of fluctuating acoustical doublets derived expressions for the sound-amplitude in any direction from an airscrew. Their first hypothesis was that the airscrew as a source of sound might be replaced by a ring of sources ahead and a ring of sinks of equal strength at an equal distance astern, both rings being co-axial with the airscrew and parallel to it. In their second hypothesis the sinks were removed to infinity and the sources were placed a very short distance ahead of the airscrew. Both these systems yield zero amplitude along the axis, but whereas the former also gives zero amplitude in the plane of rotation with maxima symmetrically disposed on either side, the latter gives maximum amplitude in the plane of rotation. It was hinted that the truth might be found in a combination of these systems, and this aspect of the subject has recently been considered by E. T. Paris[6].

Assuming the sources of the second system to lie in the airscrew disc and to be $\pi/2$ out of phase with respect to the corresponding sources and sinks of the first system, he obtained the following expression for the sound-amplitude:

$$A \propto J_{pm}(pm\omega a^{-1} R \sin\psi)\{\mu - 2\sin(pm\omega a^{-1} Y \cos\psi)\},$$

where J_{pm} is Bessel's function of order pm;

p the order of the harmonic considered;

m the number of blades;

ω the angular velocity of the blades;

R the radius of the orbits, assumed to be $\frac{3}{4}$ of a blade-length;

a the velocity of sound;

ψ the aspect angle, as defined in figure 2;

μ the ratio of the strength of the sources of the second system to the strength of the sources and sinks of the first system; and

Y the distance of the sources and sinks of the first system from the plane of the airscrew.

It was shown that this expression yielded a polar curve very similar to the experimentally determined polar curve of amplitudes for the first harmonic when $\mu = 2{\cdot}6$ and $Y = 3 \times$ blade-length. It indicated, however, less output behind the

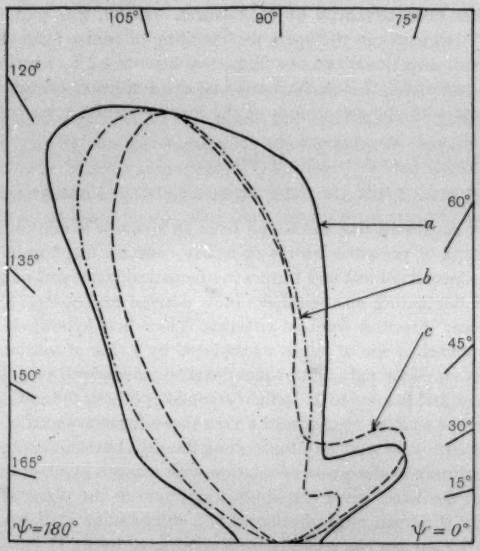


Fig. 4. Pressure-amplitude of first harmonic according to the theory of Lynam and Webb as modified by Paris.
Curve a, experimental. Curve b, $\mu = 2{\cdot}6$; $Y = 3 \times$ blade-length.
Curve c, $\mu = 3$; $Y = 4 \times$ blade-length.

plane of rotation than was found by experiment. So far as the author's measurements of the first harmonic are concerned, the agreement is best for $\mu = 3$ and $Y = 4 \times$ blade-length, the theoretical output behind the plane of rotation again being less than the measured output, figure 4.

Attempts to assign suitable values to μ and Y for the second and higher harmonics have not been successful. For the second harmonic of a two-bladed airscrew, the value of $J_4(4\omega a^{-1} R \sin\psi)$ for $\psi < 30°$ is inconveniently small compared with the value for $\psi = \pi/2$. The polar curve of figure 2 shows a forward, subsidiary, maximum near $\psi = 15°$, and to preserve the relative amplitudes at $\psi = 15°$ and $\psi = 105°$ (the principal maximum) the value of $\sin(4\omega a^{-1} Y \cos\psi)$ must in practice be large and negative in the former direction, but large and positive at the latter. Since the values of the Bessel function are symmetrical about $\psi = \pi/2$, this implies that the amplitude at $\psi = 75°$ must be much greater than at $\psi = 105°$, which is definitely not so. The same objection applies to values of $pm > 4$, and in any case the appreciable output along the axis at $\psi = 0$ could not be accounted for.

§ 10. TWO NOTES ON THE WORK OF PREVIOUS INVESTIGATORS

(i) *Obata and Yosida*. This work is notable as forming the first published account of experiments to obtain continuous oscillographic records of the sound from aircraft in flight and to correlate the distance and the aspect of the machine with the harmonic analysis. Most of the records refer to an aircraft fitted with a geared airscrew (gear-ratio 29/44) and the analysis should therefore have yielded useful information concerning both airscrew sound and exhaust sound. Unfortunately, the method of analysis adopted is open to serious criticism, for although the numerous lengths of record reproduced in the paper do not exhibit recurring wave-forms, yet a classical Fourier analysis has been applied to certain portions which seem to have been selected in a rather arbitrary manner. In every instance the sound is represented by a single train of harmonics, sometimes commencing with the airscrew fundamental frequency, but more often with the fundamental of the exhaust. The sound spectra given cannot, therefore, be a true indication of the composition of the sound, and without a complete recalculation the effects due to the airscrew cannot be separated from those due to the exhaust. The authors were not concerned with the verification of any theories of sound-emission and admit the weakness in the analytical method. In an approximate manner the analyses show the distribution of energy in the sound spectrum, and in many cases are considered to have a bearing on aural sensations.

(ii) *McKinnon Wood*[1]. In 1919 the Royal Aircraft Establishment made some determinations of the range of audibility of various airscrews actuated electrically near ground level. At the same time observations were made on the frequency of the predominant note in the sound, a pianoforte and pitch pipe being used for comparison purposes. Two installations were employed and different results were obtained from each. When the airscrew was rotated on the spinning tower at a point fixed in relation to the observer, the frequency of the note heard was usually an integral multiple of the frequency given by the (number of blades × r.p.s.). When the airscrew was given a translational motion in a circle 150 ft. in radius by means of the whirling arm, the note heard was not related to the theoretical funda-

mental frequency in a simple manner; further, for a given airscrew, the note perceived was different on different occasions. The directional nature of the sound, not then appreciated, may explain this experience. If the airscrew is advancing in a circular path and an observer is stationed at a fixed point outside this path, the region of maximum intensity in the sound-field associated with the sound of rotation passes the observer twice during each revolution of the arm, i.e. at intervals of 5 sec., the airscrew being assumed to have a forward speed of 90 ft./sec. The directional effect of the intensity is such that the harmonics of low order are relatively loud for about a quarter of this period and comparatively inaudible for the remainder of the period. Under these conditions an estimate of pitch would appear to be very difficult indeed, especially in the presence of a medley of high-pitched sound. Unless the observer were a long way from the path of the airscrew, the loudness would be much greater when the airscrew passed on the near side of the mounting than when it passed on the remote side and this would render the determination of pitch still more difficult. If an airscrew produces a relatively high-pitched sound which is non-directional in character, then this would be heard continuously and would almost certainly receive the prior attention of the observer on account of its continuity and the increased sensitivity of the ear at moderately high audio-frequencies. If the frequency of this sound is not dependent on the angular velocity of the airscrew, the observed note will probably not be recognized as a harmonic of the fundamental sound of rotation.

§ 11. ACKNOWLEDGMENTS

Most of the observations described in this paper have already appeared in various reports submitted to the Aircraft Noise Sub-Committee of the Aeronautical Research Committee. The experiments were conducted under the direction of Dr W. S. Tucker, O.B.E., and the fuselage and airscrew provided by the Director of Scientific Research, Air Ministry. The author is indebted to Dr E. T. Paris for various suggestions and advice tendered during the course of the experiments.

REFERENCES

(1) R. McK. Wood. *Aero. Research Committee Reports and Memoranda*, No. 694 (1920).
(2) E. Waetzmann. *Zeit. f. tech. Phys.* **6**, 167 (1921).
(3) M. D. Hart. *Aero. Research Committee R. and M.* No. 1310 (1930).
(4) A. Fage. *Proc. R.S.* A, **107**, 456–458 (1925).
(5) E. J. Lynam and H. A. Webb. *Aero. Research Committee R. and M.* No. 624 (1919).
(6) E. T. Paris. *Phil. Mag.* **13**, 99 (1932).
(7) J. Obata and Y. Yosida. *Aero. Research Inst. Tokyo Imperial University*. Report No. 59 (1930).
(8) W. West. *J. Inst. E.E.* **67**, 1137 (1929).
(9) M. v. Ardenne. *Experimental Wireless*, **7**, 66 (1930).
(10) A. B. Wood. *J. Inst. E.E.* **63**, 1046 (1925).

DISCUSSION

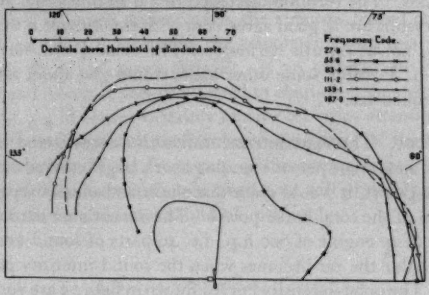
Major W. S. TUCKER. The paper describes a difficult piece of team-work done entirely under Mr Kemp's control. The airscrew is a very interesting example of a directive source of sound and I believe this is the first time that the Physical Society has had a paper of this type. I hope that it will be followed up by other contributions of a similar character and will afford some encouragement to others to help in establishing a good theoretical basis for the phenomena which the author has described.

Dr E. T. PARIS. The formula discussed in § 9 assumes that R is $\frac{3}{4}$ of a blade-length, and the results are in good agreement with experiment as regards the fundamental tone, but not as regards harmonics. The value $\frac{3}{4}$ was, however, somewhat arbitrarily chosen. Possibly some other value would give more satisfactory results at the higher frequencies.

Dr A. B. WOOD. The experimental difficulties encountered in such work are considerable. To an average person standing near a large tethered aeroplane developing its full horse-power, it would seem that the sound-output represents an appreciable proportion of the total horse-power! The author's measurement of 18 watts sound-power from an engine of 600 h.p., i.e. 40 parts of sound-energy per million, shows how unreliable the ear becomes when the sound-intensity passes the "threshold of feeling." The polar-intensity curves shown in figure 2 are very interesting, the striking feature being the backward inclination of the maximum intensity. The impression one obtains from such curves is that the sound-distribution is influenced by the slip-stream. If such is the case, the polar diagram of intensity should change as the rate of revolution of the airscrew is varied. Perhaps the author could supply information on this point.

Dr A. H. DAVIS. The author has obtained information concerning the fundamental and lower harmonics of airscrew noise which should provide an experimental basis for a reliable theory of airscrew sounds. I was interested to see his curves for the wave-form of the sound emitted by airscrews, for they agree in a general way with the periodic impulsive nature of the wave-form that Mr Lock and I have observed near a model airscrew. On the basis of our experience with this airscrew, which was driven by a comparatively silent electric motor, I suggest that there may be irregularities inherent in the disturbance set up by an airscrew itself, which would explain the irregularities and fluctuations that the author attributed in his case to interference from engine and exhaust noise. We found the wave-form unsteady in detail, although fairly constant in its general characteristics. I would mention that in a preliminary analysis of a noise of a motor-driven airscrew (diameter 3 ft., r.p.m. 1500) Mr Dadson and I found appreciable energy distributed generally and irregularly over the higher-frequency range.

The author mentions that many of his observations have been communicated to the Aircraft Noise Sub-Committee of the Aeronautical Research Committee, but, of course, their value is not confined to the aspects with which that committee is concerned. I would, however, comment that the low-frequency sounds measured are sounds to which the ear is not specially acute. I have made a rough estimate of the loudness that the various components plotted in figure 2 of the paper would have if they existed independently. The results are indicated in the accompanying diagram. It will be seen that the fundamental is the component which is least important to the ear, at any rate at the range 300 ft. at which the author worked. Also the sixth harmonic is more important than the fifth, so that still higher harmonics may, perhaps, be of importance as far as noisiness is concerned.



Estimated polar curves for distribution of components of airscrew sound expressed in terms of the loudness of each component.

Dr Paris, to whose as yet unpublished work reference is made, has presented a valuable theoretical equation to express the polar distribution of the fundamental sound around an airscrew. I should be much interested to hear of theoretical calculations which could predict the characteristic wave-form of airscrew noise at any point.

Author's reply. (In reply to Dr Paris): I have not tried the effect of making R different for different harmonics; the value of $\frac{3}{4}$ of a blade-length was selected because the centre of pressure of the blade is situated approximately at this radius. (In reply to Dr Wood): Since the paper was written I have investigated the effect of engine speed on the shape of the polar curve for the fundamental note, and have found no appreciable change to occur. The observations were confined to values of ψ between 90° and 150°, the range of engine speeds being from 1500 r.p.m. to 1800 r.p.m. only. Lack of suitable calibrating apparatus prevented me from working below the former speed, which corresponds to a blade frequency of about 24 ~. (In reply to Dr Davis): The suggestion that there may be inherent irregularities in the sound from an airscrew is particularly interesting. I have found that string-galvanometer records of the sound from a single-engined aircraft in flight usually

show a succession of perfectly repeating wave-forms, indicating steady sound-sources. At times, however, no such perfect repetition is evident—an experience shared by Obata and Yosida*. Meteorological conditions are thought to be responsible for this difference, and if Dr Davis' experiment with the model airscrew was made in the open air, the same disturbing factors may have been operative. I agree that harmonics of higher order than those considered in the paper may be physiologically important. I was more concerned with the purely physical aspect of the problem, and did not proceed to higher frequencies on account of the increasing difficulty of frequency-separation with the simple tuned-anode device described.

* *Loc. cit.*

SOME THERMOMAGNETIC EFFECTS IN NICKEL AND IRON

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ABSTRACT. The paper relates to the production of an e.m.f. in nickel and iron wires by the simultaneous application of a longitudinal magnetizing field and a temperature gradient. This e.m.f. has been measured for various fields and temperatures.

§ 1. INTRODUCTION

This paper presents a brief description of the results of some work now in progress. A complete account will be published later, with further results included.

If a magnetic field is applied to a ferromagnetic substance parallel to a temperature gradient, an electromotive force is produced in that direction. The

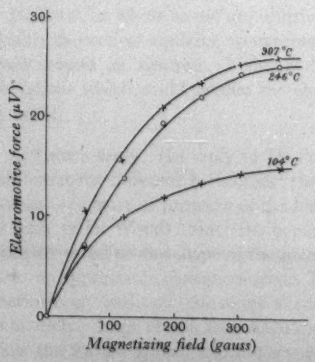


Fig. 1. Results for nickel wire. (Diameter 0·15 mm.; temperature at cold end 14·2° C.; temperature at hot end as indicated against curves.)

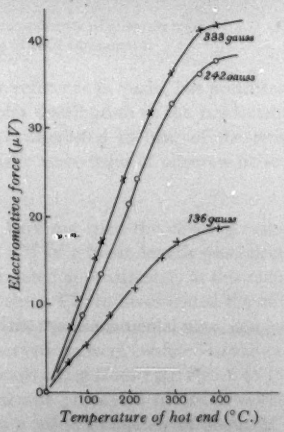


Fig. 2. Results for nickel wire. (Diameter 0·15 mm.; temperature of cold end 13·8° C.; magnetizing field as indicated against curves.)

experiments are essentially measurements of this e.m.f., with different fields and temperatures, for wires of nickel and iron. The method used is similar to that employed by Gerlach* who has, however, published results only for nickel.

§ 2. EXPERIMENTAL RESULTS

Figure 1 shows the electromotive force E as a function of the magnetizing field for various temperatures T of the hot end of a nickel wire. Figure 2 shows the value of E as a function of T for various constant values of H. These curves will be seen to be almost the same as those published by Gerlach. A very slight hysteresis was also observed, but its magnitude was too small to allow any useful measurements.

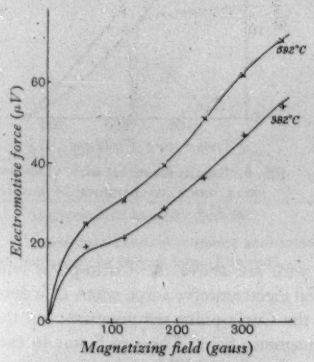


Fig. 3. Results for iron wire. (Diameter 0·57 mm.; temperature of cold end 13·8° C.; temperature of hot end as indicated against curves.)

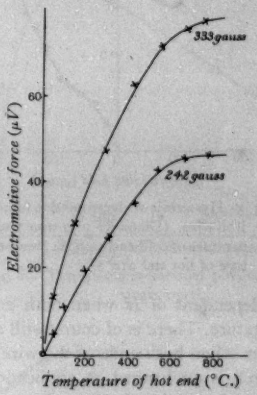


Fig. 4. Results for iron wire. (Diameter 0·57 mm.)

Figures 3 and 4 show E as a function of H for various constant values of T, and as a function of T for various constant values of H, in the case of an iron wire. The effect in iron is greater than that in nickel, but similar in nature. The magnetic field, however, does not appear to have been strong enough to produce saturation in iron as it did in nickel.

Figure 5 shows the remarkable hysteresis effect in iron at constant temperature with a complete cycle of the magnetizing field. There is an anomalous increase in the value of E just as the value of H is allowed to fall back from its maximum. This anomaly led us at first to suspect that some fault had developed in the apparatus; but it was repeated several times, and no fault could be detected.

* *Proc. Phys. Soc.* **42**, 418 (1930).

Figure 6 duplicates the phenomena Gerlach observed in the neighbourhood of the Curie point. The curves refer to our experiments on nickel, and show that E is

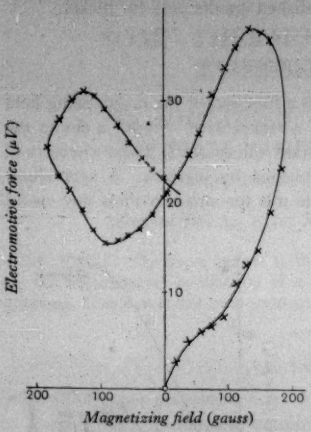


Fig. 5. Hysteresis of electromotive force in iron wire. (Diameter 0·57 mm.; temperature of cold end 13·8° C.; temperature of hot end 210° C.)

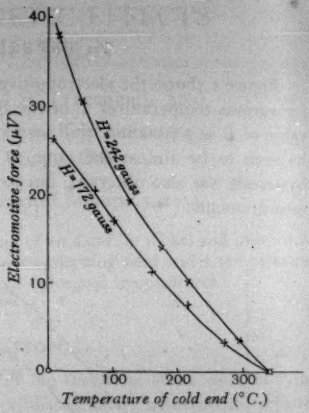


Fig. 6. Results for nickel wire. (Diameter 0·15 mm.; temperature of hot end 390° C.)

independent of H when both ends of the wire are above the Curie-point temperature. There is of course still a finite initial electromotive force when H is zero, even when both ends of the wire are above the Curie-point temperature; but this e.m.f. is unchanged on application of the magnetic field. The e.m.f. was in each case observed to be opposite in direction to the normal Thomson e.m.f. in zero magnetic field.

DISCUSSION

Dr L. F. Bates. The authors have drawn our attention to some rather interesting thermoelectric effects. The phenomena illustrated by figure 5 of their paper are particularly striking, and they direct attention to one or two aspects of these problems which appear to have escaped notice previously. As their paper represents a preliminary survey we cannot legitimately complain that data concerning the purity of their materials are not given. It is, of course, obvious that some chemical data concerning the iron used will be necessary at a later stage in the investigation. However, even at this stage, a knowledge of the hysteresis curve for the iron specimen would have been of assistance. It is the lack of hysteresis data which leads me to think that the authors have not paid sufficient attention to the work of Ellwood* on the heat changes accompanying the magnetization of iron.

* W. B. Ellwood, *Phys. Rev.* **36**, 1066 (1930).

Ellwood took a series of 104 steel rods, and arranged them in concentric cylinders so that the bunch of steel rods formed an ellipsoid of revolution with a long major axis. Between the steel rods an equal number of copper rods of the same diameter and length were mounted. These iron and copper rods were not in contact, but adjacent copper and iron rods were joined alternately by short pieces of constantan and copper wire, so that a system of 102 thermocouples in series was formed. The whole arrangement was mounted in a special calorimeter inside a solenoid. On changing the magnetic field of the solenoid, heat changes due to magnetization were produced in the iron rods but not in the copper rods. The change in temperature of the iron was therefore recorded by a galvanometer connected in series with the thermocouple system.

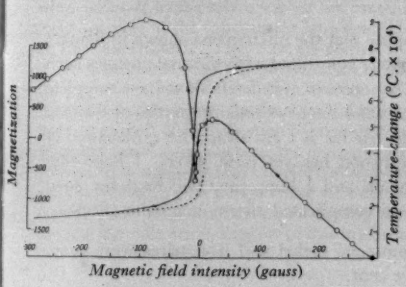


Fig. *A*. The magneto-thermal relations associated with the largest hysteresis loop (Ellwood).

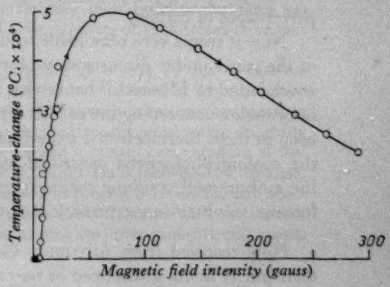


Fig. *B*. The magneto-thermal phenomena of the virgin curve.

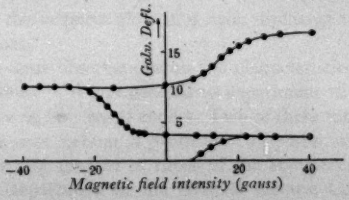


Fig. *C*. Honda's curve for carbon steel (0·2 per cent. C).

Figure *A* shows the curves which Ellwood obtained in the case of steel containing 1·08 per cent. of carbon. In this figure we have the hysteresis curve for this steel and the curve showing the observed temperature-changes as the magnetizing field changed from + 290 to − 290 gauss. As the field is reduced from 290 to 20 gauss a steady rise in temperature is observed. On reduction of the field below + 20 gauss a cooling effect is observed, and this cooling persists until a field of − 8 gauss is attained. Between − 8 and − 90 gauss a sudden rise in temperature is observed, and finally between − 90 and − 290 a gradual cooling effect. The coercive force was approximately 8·4 gauss.

The temperature-changes recorded for the same specimen when it is first thoroughly demagnetized and later magnetized in a field which is gradually increased to 290 gauss are shown in figure *B*. Now, such curves have been obtained so far for two steels containing 1·08 and 1·35 per cent. of carbon respectively, and we do not know whether they are to be observed with other specimens. It is true that Honda* has found no evidence of these cooling-effects with the carbon, tungsten and Japanese magnet steels which he has used. His results for a carbon steel containing 0·2 per cent. of carbon are reproduced in figure *C*. It should be mentioned, however. that Honda's apparatus was very much less sensitive than Ellwood's, so that, as far as one can tell from the available data, we are only justified in stating that such cooling phenomena are observed with two steels containing the stated percentages of carbon.

Now it seems very reasonable to suppose that the phenomena shown in figure 5 of the paper under discussion are intimately connected with thermal changes such as are found by Ellwood. That is why, in my opinion, it is desirable to have complete information concerning the carbon content and the hysteresis properties of the iron used in these thermoelectric experiments. As far as I am aware, no explanation of the cooling phenomena described by Ellwood has yet been given. I hope that the authors will continue their experiments, and I hope they will have the good fortune to obtain an explanation of these complicated thermomagnetic effects.

Note received from AUTHORS. Commercial nickel and iron wires were used, their purity being guaranteed as 99·5 per cent.

* K. Honda, J. Okubo and T. Hirone, *Sci. Rep. Tokio University*, **18**, 409 (1929).

ON PERIODIC MOVEMENTS OF THE NEGATIVE GLOW IN DISCHARGE TUBES

By W. A. LEYSHON, Ph.D.

Received October 8, 1931 *Read and discussed January* 15, 1932

ABSTRACT. An account is given of experiments designed to discover the cause of the jumping-glow phenomenon in discharge tubes. It is shown that the effect is produced when traces of hydrocarbon vapour are present in the tube in addition to the filling gas (neon, in most of the experiments here described). Some effects of such traces of vapour on the characteristics of the tube are given.

It is suggested that the phenomenon of the jumping glow may be due to internal flashing at that part of the surface of the cathode which is not covered by negative glow. The flash may be caused by the electrical breakdown of a partially insulating hydrocarbon layer, as a result of the collection of positive ions, or by a surface chemical action occurring when the reaction-products of the discharge have reached a certain concentration at that surface. It is supposed that the electron emissivity of the surface is increased by the flash, and that the main glow jumps to the activated surface. The process is reversible and hence may be periodic. Observations connecting the periodic time of jumps with the current through the tube are in support of this explanation, as also are other experimental results described in the paper.

§ 1. INTRODUCTION

Periodic movements of the negative glow in a neon discharge tube were first observed in December 1929.

As a result of some previous observations on the characteristics of discharge tubes under flashing conditions[1], it was decided to experiment with neon tubes of the Osglim I type but having very small anodes. Two of these tubes, filled with 98 per cent. neon and 2 per cent. helium at pressure of 14·7 mm. of mercury and free from the impurities usually present in tubes of the commercial type, were supplied by the Research Department of the General Electric Company. (The electrodes, as in the ordinary Osglim lamps, were of iron.) On attempting to obtain their volt ampère characteristics, it was found that for certain adjustments of current and voltage the negative glow began to execute periodic movements over the surface of the cathodes. This periodic movement was demonstrated to the Physical Society and a short account was given[2] of the observations which had been made on the two lamps. Later, four other lamps of somewhat similar construction were obtained, two of them filled with neon at 7·7 mm. pressure, and these also showed periodic movements of the negative glow under suitable conditions. Some of these were demonstrated at the Royal Institution soirée on January 23, 1931.

A number of commercial tubes were tested, but in one tube only was the effect observed, and in that only when the wire and not when the plate was used as cathode.

Figure 1 shows the circuit used: in the first experiments made the diode was replaced by a resistance of the order of 100,000 ohms. Figure 2 illustrates the regularity with which these movements were carried out. Figure 3 illustrates some of the effects observed.

In these early observations two distinct types of movement were recorded. In the first, which occurred with larger values of current and voltage, the glow was reddish and the movements were slow, apparently unaffected by transverse magnetic

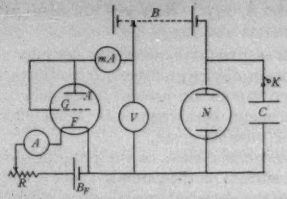


Fig. 1. *N*, neon lamp; *B*, battery; *mA*, milliammeter; *A*, *G*, *F*, triode used as diode; *A*, anode; *F*, filament; *G*, grid; B_F, filament battery; *A*, ammeter; *R*, filament resistance; *K*, key; *V*, electrostatic voltmeter; *C*, condenser.

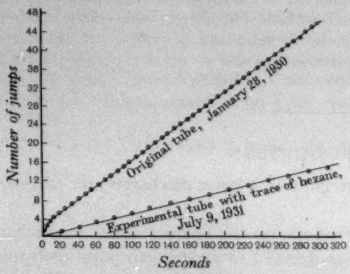


Fig. 2.

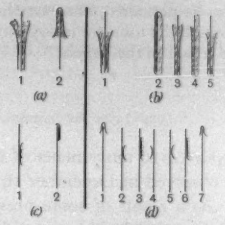


Fig. 3.

Fig. 3. (*a*) and (*c*) pressure 7·7 mm.; (*b*) pressure 14 mm.; (*d*) commercial Osglim I type, small electrode as cathode; (*c*) 1 and (*d*) 3 and 6 indicate very rapid flickers. In (*b*) the distance between the diagrams illustrates relative time-intervals. The cathode is viewed edgewise in all diagrams.

fields; increase of applied potential increased the frequency of movement; heating the tube* by radiation increased the frequency; the changes in current through the tube accompanying the jumps were small. This type only was seen in the tubes containing neon at 7·7 mm. pressure.

The second type of movement occurred almost at the extinction point of the lamp. The glow was much whiter and smaller; it moved very rapidly; the movement was appreciably affected in frequency by a transverse magnetic field; increasing the applied potential slightly slowed up the movement; heating the tube* slowed up the movement; the current changes during the cycle were comparatively large.

* Heating the tube probably liberated adsorbed material from the glass walls.

After some days of use the lamp which had shown the second type of oscillation did so no longer. Both types of movement altered to some extent with time and in such a way that the periodic time tended to become longer as the tube was used; also the adjustments necessary to obtain the jumps became more critical. If the tubes were left unused they did not appear to alter much with time.

Another set of tubes supplied by the General Electric Company with a different electrode design showed no jumping movement at all. One of these tubes contained helium.

§ 2. SOME FURTHER OBSERVATIONS ON THE ORIGINAL TUBES

Some observations were made on the current changes taking place through one tube during the movements of the glow. The circuit is shown in figure 4. A Gambrell short-period galvanometer with a sensitivity of 0·25 μA per division was used. It was balanced so that only the variations in the current through the neon tube were indicated.

It was observed under some circuit conditions that just before a jump took place small periodic variations of the current occurred, and increased in amplitude until finally the glow jumped; at this stage the largest change in current was observed. These observations were repeated later, a cathode-ray oscillograph being used to indicate changes in voltage across the tube. The circuit is shown in figure 5.

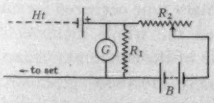


Fig. 4.

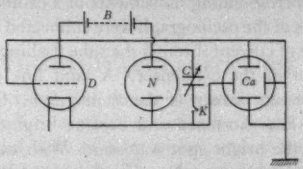


Fig. 5.

Fig. 4. *Ht*, high-tension battery of set; R_1, R_2, resistances; *G*, galvanometer; *B*, battery.

Fig. 5. *N*, lamp; *B*, battery; *Ca*, cathode-ray oscillograph; *D*, diode; *C*, condenser; *K*, key.

Again, small periodic movements of the spot on the fluorescent screen were sometimes observed before the jump took place; when this jump actually occurred, a larger displacement of the spot was seen.

On other occasions the oscillograph spot gave one rapid movement as if a small flash had occurred in the lamp. Sometimes after a jump had occurred the spot would move slowly towards the zero position corresponding to smaller current through the lamp, and then jerk back suddenly as the return jump occurred.

Similar observations were made with the oscillograph on a neon-filled experimental tube, with aluminium electrodes, described in § 4.

Phenomena observed with condensers in parallel with the discharge tube. When, in the circuit of figure 1, a condenser was put in parallel with a tube which showed the jumping-glow effect, with suitable values of current and voltage the tube flashed once or twice, then a steady glow was seen which moved to a new position on the cathode and disappeared; the whole process was then repeated. With a suitable value of condenser capacity and very careful adjustment of diode-filament current it was possible to obtain a condition in which the glow moved very rapidly, with the second type of movement seen previously in one of the original tubes with no condenser in parallel. The rapidity of movement of the glow was affected as in the previous case by the presence of a transverse magnetic field. With larger values of the condenser and a greater current through the diode it was possible to get the following sequence: flash (whitish glow all over cathode), darkness, steady reddish glow which slipped, darkness; and then the cycle repeated. The current was in each case near the critical current for flashing. With a still greater capacity and an apparently steady glow, a further increase in capacity altered the appearance of the glow and a quick ripple appeared at the top. With suitable adjustments this flicker became so great in extent that the top reached the end of the cathode and then the glow contracted and jumped to the end of the cathode. After a time it dropped again, the flicker appeared and the whole cycle was repeated. This effect was seen later in other neon-filled tubes without external capacity, and also in a hydrogen-filled tube. The value of the parallel capacity necessary to produce these effects depended on the tube used and the current at which jumps tended to occur. In one tube in which these results were obtained with capacities of the order of 0·002 μF, the corresponding movements of a cathode-ray-oscillograph spot were noted. The plates of the oscillograph were connected in parallel with the diode. For a particular value of current through the tube flashing of the ordinary type occurred for medium values of the condenser. A faint line then covered the whole diameter of the fluorescent screen of the oscillograph. On reduction of the condenser capacity the faint line shortened and became brighter, and usually shortened gradually until only the bright spot was seen. With careful adjustment, however, the flickering effect could be produced, and then the faint line with a superposed bright line was seen on the screen of the oscillograph. If a jump occurred, and the glow contracted to the top of the cathode, the line on the screen disappeared and the spot appeared steady. No doubt this effect is connected with the high note often heard on reduction of the current, just before flashing occurred. It may also have some relation to the saw-tooth oscillations described by Finch, Sutton and Tooke[3]. The tube containing helium showed the flicker effect with a condenser in parallel, but no jump.

One of the original tubes was opened to the air, fitted with a tube and tap and connected to a Hyvac pump. When the pressure had been sufficiently reduced the tap was closed, and the tube was included in the circuit of figure 1. An exceedingly slow periodic movement of the negative glow was observed on one occasion only: considerable hysteresis in the extent of the glow was, however, noticed on first increasing and then decreasing the current through the tube.

Taylor[4] observed a periodic movement of the negative glow in a beehive Osglim lamp filled with air; he ascribed the phenomenon to a gas-film at the cathode surface[5].

§ 3. EXPERIMENTS WITH HYDROGEN

In order to be able to examine the jumping effect in more detail, it seemed desirable to have an experimental tube into which different gases could be introduced. The tube mentioned above was connected up to a vacuum apparatus arranged as suggested in figure 6. After being thoroughly evacuated (but not baked out nor having its electrodes degassed) the tube was filled with hydrogen obtained either from a palladium tube or by electrolysis of dilute sulphuric acid. In the latter case the hydrogen was passed over phosphorus pentoxide and through a liquid-air trap before being admitted to the experimental tube. On being tested in the circuit of figure 3 the tube showed very rapid flickers and jumps, much more rapid than any seen before. The outline of the negative glow was in many cases, and particularly at higher pressures, very suggestive of that of a liquid on a greasy surface. If the current through the tube was increased the appearance of the end of the glow often suggested drop-formation, and on some occasions a drop of glow gradually removed itself from the main glow. Figure 7 illustrates this effect, which was also seen in some later experiments with neon. The periodic movements of the negative glow in hydrogen were not as regular nor as persistent as those previously seen, and were usually accompanied by audio-frequency oscillations.

§ 4. EXPERIMENTAL TUBE WITH ADJUSTABLE ELECTRODE

In order to see whether the jumping effect was due to the small size of the anode an experimental tube was used, figure 8, in which the effective length of the wire anode was adjustable. The electrodes were of pure nickel, and at first the wire anode was of the same length as the plate cathode. The tube was joined to the vacuum apparatus, figure 6, which had been modified by the addition of a glass container filled with 98 per cent. of neon and 2 per cent. of helium.

After being evacuated thoroughly, the tube was filled with neon at a pressure of 10 mm. of mercury, and then connected up with the circuit of figure 1. Practically all the phenomena previously seen in the neon tubes were found to occur with this tube. On re-evacuation and refilling with neon, however, the jumping-effect could not be reproduced.

In order to see whether the jumps were more readily obtainable with a small anode, the sealing-wax joint at *S*, figure 8, was gently heated with a small flame, and the anode was pulled down so that only a small piece projected above the glass. Rather irregular jumps were obtained.

In a further series of experiments it was found possible to rid the tube of the cause of the jumping glow, by passing a discharge through it from a small induction

coil and simultaneously heating the glass tube while evacuating. On filling the tube with neon and testing after such treatment it was usually found impossible to persuade the glow to jump, even by the most careful adjustment of current and voltage.

After the wax joint had been softened and the anode wire pushed up, so that again the anode was of the same length as the cathode, it was found that very vigorous jumps could be obtained, either electrode being used as cathode. It was thus clear that the effects were not due to the smallness of the anode in the original tubes. This was confirmed by the fact that one lamp of the commercial I type showed a jumping effect, the wire being used as cathode.

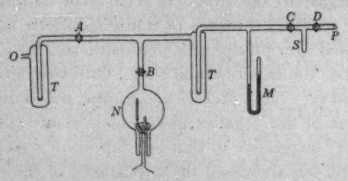


Fig. 6.

Fig. 7.

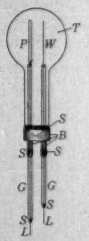


Fig. 8.

Fig. 6. *N*, experimental tube; *M*, mercury manometer; *S*, side tube (or palladium tube); *T*, *T*, liquid-air traps; *A*, *B*, *C*, *D*, taps; *P*, tube leading to gas reservoir; *O*, tube leading to McLeod gauge and pump system.

Fig. 7. (Shaded part indicates negative glow.)

Fig. 8. *T*, glass tube; *P*, plate; *B*, brass cap; *L*, *L*, leads; *W*, wire; *G*, *G*, glass guide-tubes; *S*, *S*, sealing-wax joints.

§ 5. INTRODUCTION OF TRACES OF OTHER GASES OR VAPOURS INTO THE DISCHARGE TUBE

It was noticeable in the experiments just described that jumps could always be obtained after the adjustment in length of the anode wire. This involved heating the sealing-wax joint and it was possible that a little air might have entered the experimental tube.

In order to find out whether traces of oxygen were responsible for the effect, this gas was deliberately introduced by heating potassium chlorate in a side tube. In the first experiment of this kind the potassium chlorate had been slightly contaminated with tap grease. It was found that the jumping effect which had been eliminated by the method previously described was in all cases restored if the potassium chlorate was heated, not too violently. After the heating of the potassium chlorate a higher value of applied voltage was necessary to produce the glow; also flashing occurred more readily.

The next step was to use pure potassium chlorate, uncontaminated by grease. The experimental tube, electrodes and side tube were removed and thoroughly

cleaned, pure potassium chlorate was introduced into the side tube and the degassed electrodes were fitted again into the experimental tube. (Apiezon tap grease was now used for the taps.) It was found that the jumping effect, after elimination in the usual way, could not be restored by heating the potassium chlorate.

Air was then let in through tap *C*, figure 6, the pressure was reduced to about 10^{-3} mm. and neon was introduced. No jumps were obtained on testing. After refilling with pure neon, it was found that on reduction of the effective length of the anode (by heating the sealing-wax joint, and adjusting) the jumps were restored. In pushing the anode up again, some sealing-wax was introduced into the small glass guide tube for the anode above the joint. It was found in a series of experiments that jumps could always be restored by gently heating this sealing-wax. Either electrode used as cathode showed the effect. Thus it appeared that the jumping effect could be restored to the tube by introducing volatile products obtained by heating tap grease or sealing-wax. A comparison of the chemical composition of tap grease and sealing-wax suggested that the effect was possibly produced by a hydrocarbon of the terpene series.

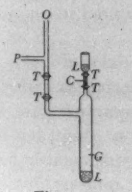


Fig. 9.

Fig. 10.

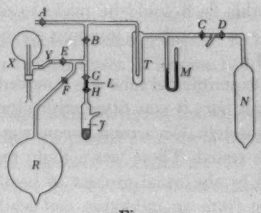


Fig. 11.

Fig. 9. *G*, glass tube; *C*, capillary tube; *O*, tube leading to vacuum apparatus; *T*, *T*, taps; *L*, limonene; *P*, tube leading to experimental tube.

Fig. 10. *P*, plate cathode viewed edgewise; *W*, wire anode; *G*, glow (jumping to positions shown dotted).

Fig. 11. *A*–*H*, taps; *T*, liquid-air trap; *X*, experimental tube; *N*, neon reservoir; *J*, hexane-container; *Y*, connecting-tube; *M*, mercury manometer; *R*, reservoir; *L*, capillary tube.

Accordingly the apparatus was modified so that a volatile terpene, limonene, could be introduced into the experimental tube by means of a side tube, figure 9.

No very marked effect was produced on the introduction of a small quantity of vapour. The glow was smaller and whiter, but no jumps were obtained. If, however, the apparatus was evacuated, limonene vapour introduced, and an induction coil discharge passed while re-evacuating, vigorous jumps were obtained on filling with neon and testing as before. The limonene vapour in the tube did not alone produce the jumping effect; the latter appeared to be due to a modification of the surface of the cathode or of the gas content of the tube as a result of sending a discharge through limonene vapour.

A very significant type of jumping glow was produced in the tube thus prepared: see figure 10. The small glow in the centre of the cathode jumped up and

down at a rate which could be varied from about 2 oscillations per second to one corresponding to a note above audible frequency in the telephones in the circuit. At the slow rate, it was easily seen that a small flash occurred above or below the glow almost simultaneously with its movement up or down.

On removal of the nickel electrodes, the plate cathode was seen to be covered with a brownish deposit on the side nearest the anode. The other side appeared greenish. The electrode when looked at under a strong lens was seen to be pitted all over its surface. The nickel electrodes were now replaced by clean degassed aluminium electrodes of the same shape and size as the original nickel electrodes. They gave similar effects to those noted in the case of nickel, but the jumping effect was much more difficult to remove.

On one occasion, with the minimum possible applied voltage and a small current through the tube, a faint glow was observable in the neighbourhood of the wire anode. A bright flash, which seemed to move downwards over the surface of the plate cathode, appeared at regular intervals; with small current about every 7 seconds. (There was no external capacity in parallel with the tube.) On increasing the current this flash could be made to occur at audible frequency. With sufficient limonene in the tube, a jumping effect similar to that illustrated in figure 10 was obtained.

The experimental tube was removed from the apparatus and thoroughly cleaned and baked out; it was fitted with clean degassed nickel electrodes and replaced. After thorough evacuation, neon was introduced at 12 mm. pressure and the tube was tested. There was a slight tendency for the glow to jump, but this was removed by the usual process of heating the glass and simultaneously passing a discharge from an induction coil while evacuating. After refilling, toluene was introduced as the limonene had been previously. Tap *B*, figure 6, was left open to the liquid-air trap, to remove excess toluene. A high voltage was required to start the discharge and a purplish glow was seen which moved slowly up and down the cathode. This occurred whether the plate or the wire was used as cathode.

The movement was far less jerky than any seen before. When the tube was thoroughly evacuated, and then was refilled with neon at a pressure of 5–6 mm., the glow showed no tendency to jump. An induction coil discharge had not been passed through the tube, nor had the glass walls been heated. Toluene was again introduced, the total pressure was reduced to about 0·04 mm., and more neon was let in. Very vigorous jumps were observed at almost all values of the current for which the discharge did not cover the cathode. The experiments were repeated, greater precautions having been taken to eliminate air. Very vigorous jumps and flashing effects were seen, and an oily deposit was visible on the cathode.

A new tube fitted with degassed nickel electrodes was now used. Hexane was introduced in the same way as limonene and toluene had been previously. The jumping glow was obtained at once. Introduction of more hexane vapour made the glow contract very much and the cathode became blackened.

The method so far used of introducing hydrocarbon vapour into the discharge tube had not been entirely satisfactory, since a little air and some tap grease might

enter with the liquid into the reservoir. Further, it seemed desirable to have a more definite idea of the amount of hydrocarbon vapour introduced into the experimental tube. The apparatus was accordingly modified as shown in figure 11, and the whole was thoroughly evacuated with liquid air on the hexane container, the electrodes having been removed and cleaned and replaced so that wire and plate were of the same length. Considerable difficulty was found in getting rid of jumps, owing to previous absorption of the hexane by the glass of the experimental tube. However, this was accomplished finally by passing an induction-coil discharge while evacuating the tube without heating the glass.

Characteristics of the tube, the plate and wire being used in turn as cathode, were then obtained with the apparatus of figure 1. This was done for five different pressures of neon. The pressure of the neon was adjusted to 14·5 mm. in the experimental and connecting tubes and reservoir, and the characteristic was determined before and after the introduction of a dose of hexane. The hexane-container was immersed in a freezing mixture consisting of solid carbon dioxide and ether, the tap H was opened, G being closed; then H was closed and G opened, F being opened and B and E shut; and finally E was opened. Characteristics, which were in all cases determined by starting with the highest value of current recorded and then reducing it, were also obtained after further doses of hexane had been thus introduced.

From the dimensions of the apparatus, and a knowledge of the vapour pressure of hexane at $-78°$ C., it was calculated that the partial pressure of hexane in the experimental tube after the first dose could not have exceeded 2×10^{-5} mm. and was probably considerably less than this, since the taps were of small aperture and were not left open for more than a few seconds. Moreover, some of the introduced hexane vapour would be absorbed by the glass.

§ 6. EFFECT OF THE INTRODUCTION OF HEXANE ON THE CHARACTERISTIC CURVE AND GENERAL BEHAVIOUR OF THE DISCHARGE TUBE

Figures 12 and 13 show the current/voltage characteristics, of the tube, for wire and plate cathodes respectively, before the introduction of the first dose of hexane. Inset are curves connecting the minimum voltage reached with the pressure of gas.

Figures 14 and 15 show the effect of successive doses of hexane on the characteristic, for wire and plate cathodes respectively.

Figure 16 shows the effect of pressure on the value of the current for which the glow just fails to cover the cathode. Inset is the effect of hexane on this critical current in the case of the wire cathode. Figure 17 shows graphically the currents for which flashing started and stopped, for the series of pressures for which characteristics of the tube were obtained. There was no external capacity in parallel with the electrodes. The effect of introduced hexane on the hysteresis is indicated also. It is clear that hysteresis is increased (markedly in the case of the plate cathode) by the introduction of hexane.

From the results shown in the figures, it appears that a small quantity of introduced hexane has the effect of (*a*) lowering the voltage required to maintain a given current through the tube and altering the shape of the characteristic; (*b*) in-

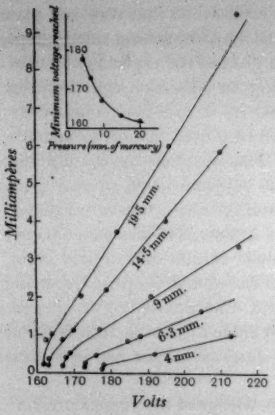


Fig. 12. Effect of pressure on characteristic. (Wire cathode.)

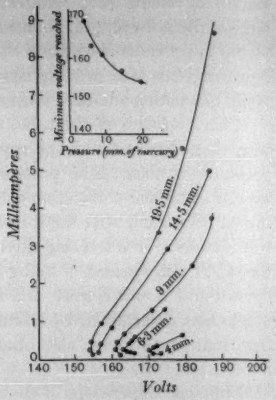


Fig. 13. Effect of pressure on characteristic. (Plate cathode.)

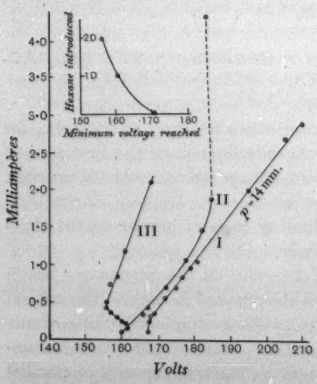


Fig. 14. Effect of introduced hexane on characteristic. (Wire cathode.) I, before introduction of hexane; II, 1st dose; III, 2nd dose.

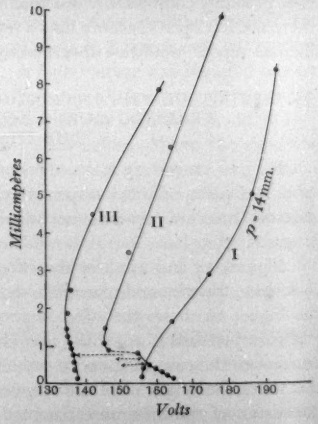


Fig. 15. Effect of introduced hexane on plate characteristic. I, no hexane; II, two doses; III, further dose.

creasing the value of the current at which the glow just fails to cover the cathode; (*c*) causing jumping of the negative glow when the discharge does not cover the cathode; (*d*) increasing the hysteresis for flashing of the tube, there being no external capacity in parallel with it.

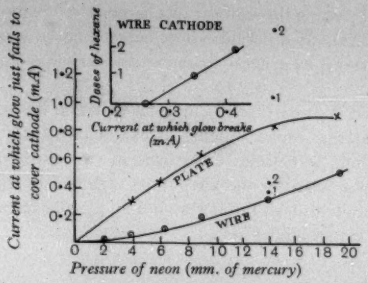


Fig. 16. 1, one dose of hexane; 2, two doses of hexane.

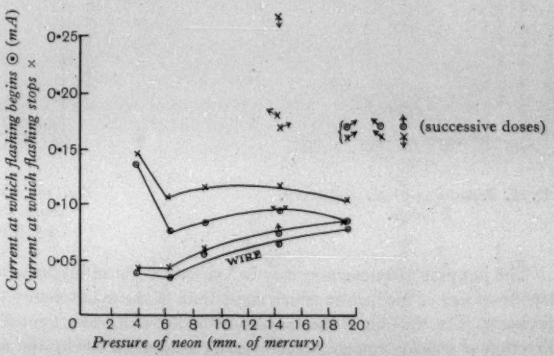


Fig. 17. Effect of successive doses of hexane on hysteresis. (Plate cathode.)

§ 7. DISCUSSION OF RESULTS AND OF POSSIBLE CAUSES OF THE JUMPING GLOW PHENOMENON

The results of the experiments described in the preceding sections suggest that the jumping-glow phenomenon is not due to a particular shape, disposition or chemical composition of the electrodes, nor is it, apparently, confined to a discharge through a gas of a particular molecular structure. It occurs, however, when minute quantities of certain hydrocarbon vapours are present in the discharge tube.

Some current/voltage curves for tubes which showed the jumping-effect are given in figures 18 and 19, in order to illustrate the characteristic discontinuities

of such curves. When a jump took place the voltage across the tube sometimes changed appreciably, on occasions by as much as 10 volts, but reverted to its original value on the return jump.

In an attempt to gain more insight into the mechanism of the phenomenon, a number of observations were made on the relation between the periodic time of jump and the current passing through the discharge tube. Some of these are shown in figures 20 (*a*) and (*b*). If a periodic-time/current curve was taken before and after a reversal of current, its general character was on some occasions found to be altered considerably.

The curve marked *X*, figure 20 (*a*), was taken while flashing of the ordinary type occurred, no external capacity being in parallel with the electrodes. Jumps obtained during flashing sometimes took place in two or more stages, so that a little repeated tune was heard in the telephones attached to the circuit. Increase of current raised the pitch of all the notes, and so usually left the musical phrase unaltered but raised it to a higher key.

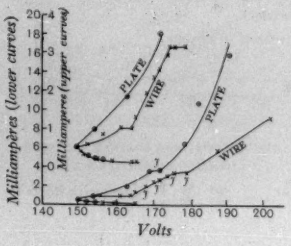


Fig. 18. Pressure 34·5 mm. of mercury. *J*, jumps.

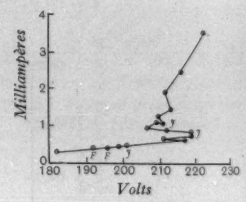


Fig. 19. Plate cathode, small anode. Pressure 10 mm. mercury. *J*, *J*, jumps; *F*, *F*, flickers.

The jumping phenomenon may be caused by the influence of the hydrocarbon vapour on any of the factors which contribute to the maintenance of a steady glow-discharge. The shifting of the characteristic curve in neon, figures 1 and 15, in the direction of smaller voltage may be partly accounted for by the additional energy available as a result of collision of hydrocarbon molecules with metastable neon atoms; the formation of slow-moving negative ions may account for the very marked diminution in the size of the glow on the introduction of traces of hydrocarbon vapour. The electrons which must be supplied in a steady discharge to make up for losses due to recombination and collection of charges by the glass walls are, it is usually supposed, obtained from the cathode, although ionization by positive ions has also been suggested by Townsend as a possible source of them.

The whole discharge takes place in such a way that the necessary electrons are produced with the minimum expenditure of energy[6]. They may be ejected from the cathode thermionically, as a result of local heating due to the impact of positive ions on the cathode surface; by photoelectric emission, due to the light given out

by positive ions in recombining with electrons, or in processes of ionization occurring in the discharge; or by autoelectronic emission due to the intense local electric field of an approaching positive ion[7]. If there is any possibility of chemical action at the cathode surface, the energy liberated by this action may also free electrons from the cathode[8].

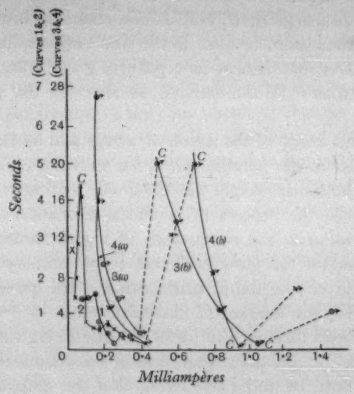
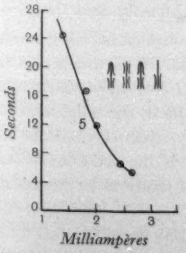
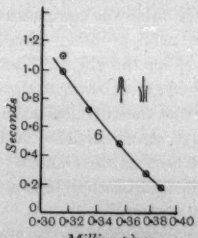


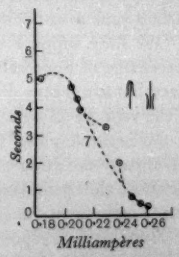
Fig. 20 (*a*). Curves 1 and 2, wire cathode. Curves 3 and 4, plate cathode. *C* indicates change in type of jump.



$p = 34{\cdot}5$ mm. Plate anode. Wire cathode. Diode in series.



$p = 7{\cdot}5$ mm. Small anode. Plate cathode. Resistance in series.



$p = 4$ mm. Small anode. Plate cathode. Diode in series.

Fig. 20 (*b*).

The ease with which electrons are liberated from the cathode depends very largely on its surface condition; this is well known in the cases of photoelectric and thermionic emission, and it has recently been demonstrated in the case of autoelectronic emission[7]. Langmuir[10] has shown that a surface in a gas (or

vapour) tends to become covered with a monomolecular layer of the gas which is then said to be adsorbed on the surface*.

In any gaseous discharge it is extremely likely that there will be a film of gas formed on the cathode surface, since positive ions are continually arriving and being neutralized there. The extent to which the film will approach to a complete monomolecular layer will depend on the rate of evaporation, which may be altered profoundly by the discharge itself, and on the chemical interaction between the metal and the adsorbed atom or ion. If the film consists in a layer of adsorbed positive ions, this layer may behave as a positive grid[12] and assist materially in the ejection of electrons from the cathode, so lowering the sparking potential in the gas.

S. C. Lind[13], in a study of the action of α rays and of electrical discharges in gaseous hydrocarbons of the paraffin series, has shown that, in both cases, hydrocarbons of higher molecular weight are obtained, hydrogen and methane being liberated. He ascribed the process to an initial formation of ion clusters, with subsequent condensation to the compounds of higher molecular weight. E. G. Linder[14] found that for the glow discharge in paraffin hydrocarbons a similar chemical action occurred; he found further that if the products of the reaction were rapidly removed, the amount of chemical action in a given time was proportional to the current through the tube. In discussing his results, with those of other investigators, he concluded that most of the chemical action took place in the gas phase, but both he and Lind found that the cathode gradually became covered with a wax-like material insoluble in all organic reagents. This finally became carbonized if the discharge was continued. Similar effects have been observed in the course of the present investigation when the pressure of hydrocarbon vapour in the discharge tube was comparatively high.

We may suppose that in neon or other discharge tubes containing a small percentage of hydrocarbon vapour the chemical actions described above also take place, but to a greatly diminished extent owing to the smallness of the partial pressure of the vapour. If we assume that the chemical action occurs in the neighbourhood of the negative glow, then hydrogen, methane and heavy hydrocarbon ions may accompany the other positive ions on their journey towards the cathode; of these the more slowly moving heavy hydrocarbon ions are most likely to be neutralized before arriving at the cathode surface[7].

Hydrocarbon vapour will condense on and be absorbed by the glass walls of the discharge tube, which also collect ions and electrons diffusing from the main discharge.

In attempting to form a theory of the jumping phenomenon the following possible processes suggest themselves: (*a*) The average work function of the composite surface directly supplying the negative glow might be increased by the action of the discharge, owing to deposition of hydrocarbon molecules; when the glow jumped to a part of the cathode where the effective work function was less, the original surface would lose these molecules by evaporation. The periodic time

* An account of recent work on the properties of surface layers has been given by Adam[11].

would then be determined by the rate of evaporation and the rate of deposition of hydrocarbon molecules. The work functions of the two patches of surface would change at rates that would be independent of the current strength if we may assume that the area of the cathode covered by the glow is proportional to the current strength, and if temperature variations can be ignored. Hence the periodic time of jump would be independent of current strength, which is certainly not the case in the experiments here described (see figure 20).

(*b*) The effect might be due to collection of ions by the glass walls. The regularity of the jumping glow, and the very small effect (if any) on it of bringing earth-connected conductors in contact with the walls, discounts this hypothesis.

(*c*) The effect might be due to the slow chemical changes going on in the discharge, a sudden change occurring when the reaction had reached a given stage. If we assume that the amount of chemical action is proportional to the quantity of electricity transferred between the electrodes, then the periodic time of jump would be inversely proportional to the current. The results obtained are not in accord with this relationship.

(*d*) The part of the cathode not covered with glow might become covered with a more or less insulating layer of hydrocarbon molecules. Positive ions diffusing from the main glow would form a layer on this, and as they collected the potential difference across the layer would increase, until finally the breakdown voltage of the layer was reached. A flash would then occur, and the cleaned cathode surface might then have a smaller work function than the original part covered by glow, which would accordingly jump to the new position. The process would be reversible so that a periodic jump might result.

Suppose that a fraction x, proportional to the current strength i, of the cathode is covered with glow; $(1 - x)$ will be covered by the insulating layer. Further, suppose that the rate of collection of positive ions by the insulating layer is proportional to the current strength, and that the breakdown voltage V of the insulating layer is a constant quantity.

Then $VC = \sigma$ where σ is the surface density of positive charge on the insulating layer for which breakdown occurs, and C is the capacity per unit area.

Also $\sigma = \alpha it/(1 - \beta i)$ where α, β are constants, $x = \beta i$, and t is the time-interval between two successive jumps. Therefore curves connecting it and i should approximate to straight lines.

This is found to be the case for simple types of jump, when the two areas successively covered by glow at the beginning of the jump do not appreciably overlap. It is also found to be true for the more complicated types, provided that the actual type of jump does not change. If the latter happens, there is sometimes a break in the curve connecting it with i and another straight line results; see figure 21. This explanation of the jumping-glow phenomenon is parallel to that given by Copeland[15] to account for his results on secondary-electron-emission from metal surfaces contaminated by oil films.

A somewhat similar explanation may account for the rapid movements of small glows in some of the tubes originally tested, and the effect seen in the experi-

mental tube with aluminium electrodes when the cathode was covered with an oily layer (see § 5).

We may here assume that a flash occurs in the immediate neighbourhood of the main glow which then moves to the new position, whereupon a flash occurs again, so that the movement is continued until the end of the electrode is reached, when the glow disappears and the tube lights up with the glow in the original position. The process is then repeated. In this case as in others local heating due to the main glow may play a part in the process. The mechanism of movement is reminiscent of the kind of action which is supposed to occur when a nerve impulse travels down a nerve as a result of electrical or other stimulation.

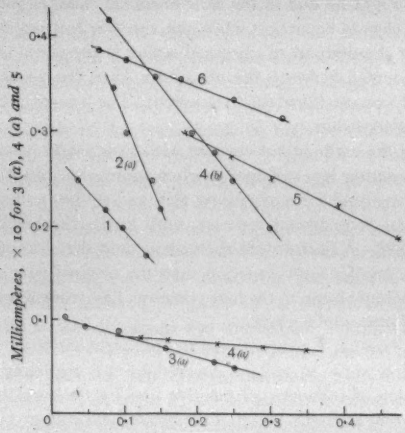


Fig. 21. Curves obtained from those of figures 20 (*a*) and (*b*).

The internal-flashing theory may also account for the flickers sometimes seen above the glow before a jump occurred, and it is upheld by the results of experiments with the short-period galvanometer and with the cathode-ray oscillograph, described in § 1. The theory also accounts for the very large range in frequency of jump observed under different conditions.

It is possible that the flash is due to surface chemical action occurring on that part of the cathode not covered by glow, when the reaction products of the discharge have reached a certain concentration in the neighbourhood of this surface. Such an explanation of the jumping glow would to some extent link up the phenomena with those of periodic chemical action[16].

When jumps occur during flashing, or while other audio-frequency oscillations are occurring in the tube, the same type of curve connecting it with i is not obtained; see figure 20 (*a*). In the former case, increasing the current increases the period of the jumping glow.

§ 8. ACKNOWLEDGMENTS

The experiments described have been carried out in the Physics Department of the London (R.F.H.) School of Medicine for Women. I should like to express my thanks to Dr W. H. Eccles, F.R.S., for lending me a Langmuir mercury condensation pump, and for the interest he has taken in the work; to Miss M. D. Waller for lending me a Hyvac oil pump, and a Gambrell short-period galvanometer; to the Council of the London (R.F.H.) School of Medicine for Women for grants from the Waller Memorial Research Fund for the purchase of apparatus, and to Mr W. Burrows of King's College for his help in making the experimental discharge tubes and setting up the vacuum apparatus.

REFERENCES

(1) Leyshon, W. A. "Characteristics of discharge tubes under flashing conditions." *Proc. Phys. Soc.* **42**, 157 (1930).
(2) Leyshon, W. A. "Demonstration of a new periodic effect in a neon discharge tube." *Proc. Phys. Soc.* **42**, 147 (1930).
(3) Finch, G. I., Sutton, R. W. and Tooke, A. E. "A time base for the cathode ray oscillography of irregularly occurring phenomena." *Proc. Phys. Soc.* **43**, 502 (1931).
(4) Taylor, J. Letter. *Nature*, Sept. 13th (1924).
(5) Taylor, J. "Sparking potentials of glow discharge tubes." *Phil. Mag.* **48**, 3, 368 (1927).
(6) Thomson, J. J. "On striations, cathode dark space, and negative glow in the electric discharge." *Phil. Mag.* **48**, 8 (1929).
(7) Oliphant, M. L. E. and Moon, P. B. "Liberation of electrons from metal surfaces by positive ions." *Roy. Soc. Proc.* A, **127**, 373 (May 7, 1930).
(8) Richardson, O. W. *Emission of Electricity from Hot Bodies*. Chap. IX.
(9) Fowler, R. H., Gossling, B. S. and Stern, T. E. "Further studies in the emission of electrons from cold metals." *Roy. Soc. Proc.* A, **124** (1929).
(10) Langmuir, I. "Constitution and fundamental properties of solids and liquids." *Amer. Chem. Soc.* **38**, 2221 (1916).
Compton, K. T. and Langmuir, I. "Electrical discharges in gases." *Reviews of Modern Physics*, **2** (1930).
(11) Adam, N. K. *Physics and Chemistry of Surfaces*. (Oxford: The Clarendon Press.)
(12) Becker, J. A. "Adsorbed atoms and adions." *Amer. Electrochem. Soc. Trans.* **55**, 153 (1929).
(13) Lind, S. C. "The theory of chemical action in electrical discharge" (and other papers). *Science* (June 8, 1928).
(14) Linder, E. G. "Organic reactions in gaseous electrical discharges: normal paraffin hydrocarbons." *Phys. Rev.* **35**, 1375 (1930).
(15) Copeland, P. L. "Secondary electrons from contaminated surfaces." *Phys. Rev.* **35**, 982 (1930).
(16) Hedges, E. S. and Myers, J. E. *The Problem of Physico-chemical Periodicity*. Edward Arnold and Co. (1926).

DISCUSSION

Dr L. SIMONS remarked that Prof. O. W. Richardson, when baking out thermionic tubes by means of an alternating magnetic field, had found that some points on the walls of the tubes glowed brightly. When the glow had been removed by a strong heating it reappeared at other points. Possibly the effects described in the paper are due to a poisoning skin similar to that noticed by Prof. Richardson.

Mr P. B. MOON. I should like to ask Dr Leyshon whether the nature of the enclosing walls influenced the phenomenon. It seems possible that variation of the charge-distribution on the walls may affect the position of the discharge, and that hydrocarbon vapour may cause changes in the electrical properties of the glass surface.

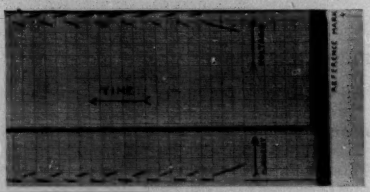


Fig. *A*. Current and voltage as functions of time.

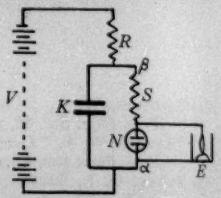


Fig. *B*. Circuit for measuring current and voltage.

Dr L. F. RICHARDSON. The photograph, figure *A*, shows the course in time of the voltage and current when a neon lamp, instead of giving the usual alternation between flash and extinction, showed an alternation between two shapes of glow without any extinction.

The circuit, figure *B*, was that of Anson and Pearson with the additional resistance *S* and the Wulf electrometer *E*. The current was measured by transferring the electrometer contact from the point α to the point β. The time marks were 0·2 sec. apart. The voltage scale is $2{\cdot}01_6$ V per division. The current scale is $5{\cdot}4_2\,\mu$A per division.

The lamp was an Osglim with plate and spiral electrodes, the makers' coil having been removed. The glow was a small tuft fluttering to and fro on the spiral with about the period shown in figure *A*; so that we may suppose that the two slowly-changing parts of the record relate to the two positions of the tuft, and the sudden jumps of voltage and current relate to the sudden changes of position. It is seen that both in the jumps and in the slower changes an increase of current synchronizes with a decrease of voltage and vice versa. In one position of the glow the current rose slowly from 1 to 4 μA; in the other it fell slowly from 22 to 14 μA. The portion of the photograph on the right shows a reference mark at $156{\cdot}_3$ V from which we deduce that the extremes of voltage were $160{\cdot}_5$ and $151{\cdot}_5$ V in the periodic part of the record, which follows the introductory 0·6 sec.

The observed time-curves closely resemble a figure in B. van der Pol's paper on "Relaxation-oscillations"* showing a computed integral of his differential equation

$$\frac{d^2v}{dt^2} - \epsilon\,(1 - v^2)\,\frac{dv}{dt} + v = 0, \text{ when } \epsilon = 10,$$

* *Phil. Mag.* 2, 986 (1926).

t being time, and v in our case either voltage or current, both reckoned as deviations from their means.

Further particulars. $S = 0{\cdot}372$ megohm, mostly Ferranti wire-wound anode feed resistances, $R = 0{\cdot}93$ megohm (a Dumetome grid-leak), battery voltage $185{\cdot}_3$ V, from exide accumulators; $K = 3{\cdot}42\,\mu$F in good paper condensers. The electrometer had a period of 0·02 sec. and a damping ratio of $7{\cdot}_5 : 1$, so that detail briefer than 0·02 sec. should be ignored. The falling-plate camera was made by the Cambridge Instrument Co.

Author's reply. In reply to Dr Simons: it seems very likely that the effects he mentions as having been observed by Prof. O. W. Richardson are allied to those I have described.

Undoubtedly, as Mr Moon suggests, it is possible that variations in the electrical state of the walls of the tube due to the presence of adsorbed hydrocarbon may occur as the discharge proceeds, and that in this way the walls might produce a variable third-body effect on the main discharge between the metal electrodes. One would expect this effect to be most marked when the main discharge is a heavy one, and is comparatively near to the glass walls. I do not think, however, that the phenomena I have described are due to this cause, for the following reasons:

(1) In the original tubes, very little effect on the periodic time of jump is produced when earth-connected conductors are brought into contact with the outer walls of the tube; I tried this experiment some time ago at the suggestion of Mr J. E. Calthrop. Further, the glow cannot be induced to jump, unless it is nearly ready to do so, by bringing highly charged conductors near to the tube. (2) I have recently observed jumps, of the same type as those already described, in a large experimental tube, with greatly increased distance between the electrodes and the walls. (3) A visible flash at a part of the cathode surface often precedes the jump of the main glow to this part. (4) It is surprising to find, in spite of all the assumptions made in the simple theory put forward in the paper, that the prediction it gives of a linear relationship between it and i is found to be verified in most of the cases suitable for study. It is difficult to see how these results could be accounted for if varying changes on the glass walls are responsible for the phenomenon.

Dr L. F. Richardson's beautiful photograph, with its very close correspondence to a theoretical relaxation-oscillation curve given by Dr van der Pol, shows much more clearly than my original observations made in 1929 that the phenomenon is, as I suggested then, a form of relaxation oscillation.

Addendum by Dr L. F. Richardson. In view of Dr Leyshon's interesting researches on the effect of hydrocarbons, I have examined the spectrum of my Osglim lamp which showed the jumping glow. The spectrum shows no trace of the Swan bands of hydrocarbons. Neon, helium and argon are evident. The following faint lines suggest the presence of iodine: 5119, 5115, 4864, 4410, 3483 Å. The Balmer lines of hydrogen are absent, although they show plainly in four other Osglim lamps.

A CATHODE-RAY OSCILLOGRAPHIC METHOD OF MEASURING INDUCTANCE

By G. I. FINCH and R. W. SUTTON
Imperial College of Science and Technology

Received September 29, 1931. *Read January* 16, 1932

ABSTRACT. The voltage fluctuations across the condenser in a damped oscillatory circuit, comprising inductance, capacity and resistance, are recorded by means of a cathode-ray oscillograph. The following relationship is derived:

$$L = CV_1^2\{1 + \lambda + (\tfrac{1}{2} + 2/\pi^2)\lambda^2 + (\tfrac{1}{6} + 2/\pi^2)\lambda^3\}/i_0^2,$$

where L is the inductance, C the capacity, V_1 the first peak voltage across C in a train of damped oscillations, i_0 the primary current through L at break of a charging circuit and λ the logarithmic decrement of the oscillatory circuit. The measurement of V and λ from the oscillogram is discussed.

§ 1. INTRODUCTION

In connexion with a study of the coil ignition of gases, upon which we have recently been engaged, the need arose for some convenient means of measuring the values of the inductances of certain oscillatory circuits under actual working conditions. The frequencies of the circuits were unknown, nor would it have been convenient at the time to determine them.

In what follows an account is given of a cathode-ray oscillographic method by means of which the inductance of a coil may be measured, under working conditions, in terms of the value of the current flowing at the moment of break in a charging circuit and of the maximum voltage across a condenser of known value in an oscillatory circuit, without a knowledge of the natural frequency of the latter. Alternatively, if this frequency be known, the circuit inductance and capacity can both be determined.

§ 2. THEORY

LCR At any time after break, the general equation for the circuit *LCR*, figure 1, is

$$L\,di/dt + Ri + V = 0,$$

V where V is the potential across C. Now

i $$i = C\,dV/dt;$$

$$\therefore\ d^2V/dt^2 + (R/L)\,(dV/dt) + V/LC = 0.$$

When $R^2/4L^2 < 1/LC$, the solution is

ωt $$V = e^{-kt}\,(A\cos\omega t + B\sin\omega t),$$

k where $k = R/2L$, and $\omega = (1/LC - R^2/4L^2)^{\frac{1}{2}}$.

On differentiating and substituting in $i = C\,dV/dt$, we have

$$i = Ce^{-kt}\{(B\omega - Ak)\cos\omega t - (A\omega + Bk)\sin\omega t\}.$$

The initial conditions are that, at break, $t = 0$, $i = i_0$ and $V = -Ri_0$, each being equal to $-E$, where E is the e.m.f. of the battery. i_0 E

$$\therefore\ i_0 = C(B\omega - Ak), \qquad B, A$$

$$A = -Ri_0,$$

and

$$B = i_0(1 - R^2C/2L)/\omega C.$$

The expression for i can be written

$$i = Ce^{-kt}\{(B\omega - Ak)^2 + (A\omega + Bk)^2\}^{\frac{1}{2}}\cos(\omega t + \theta)$$

$$= Ce^{-kt}\{(A^2 + B^2)(\omega^2 + k^2)\}^{\frac{1}{2}}\cos(\omega t + \theta),$$

where $\tan\theta = (A\omega + Bk)/(B\omega - Ak) = -R/2\omega L.$ θ

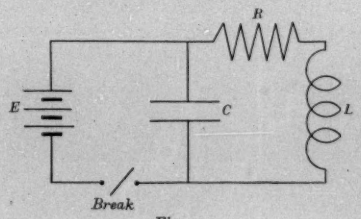


Fig. 1.

Now V has its maximum values when $i = 0$, i.e. when $\cos(\omega t + \theta) = 0$, or $\omega t + \theta = \frac{1}{2}\pi, \frac{3}{2}\pi$, etc., or $\omega t = \frac{1}{2}\pi - \theta, \frac{3}{2}\pi - \theta$, etc. Therefore, the maximum values of V are given by

$$V = e^{-kt}(A\sin\theta + B\cos\theta),$$

where

$$t = (\tfrac{1}{2}\pi - \theta)/\omega,\ (\tfrac{3}{2}\pi - \theta)/\omega, \text{ etc.},$$

$$= e^{-kt}(A^2 + B^2)\,\omega/\{(A^2 + B^2)(\omega^2 + k^2)\}^{\frac{1}{2}}.$$

Now

$$\omega^2 + k^2 = 1/LC,$$

and

$$A^2 + B^2 = i_0^2\{R^2C^2\omega^2 + (1 - R^2C/2L)^2\}/\omega^2C^2$$

$$= i_0^2/\omega^2C^2.$$

$$\therefore\ V = i_0(L/C)^{\frac{1}{2}}e^{-kt}.$$

If V_1, V_2 be the first and second peak values of V, then V_1, V_2

$$V_1 = i_0(L/C)^{\frac{1}{2}}e^{-k(\frac{1}{2}\pi - \theta)/\omega},$$

and

$$V_2 = i_0(L/C)^{\frac{1}{2}}e^{-k(\frac{3}{2}\pi - \theta)/\omega}.$$

$$\therefore\ V_1/V_2 = e^{k\pi/\omega},$$

and λ, the logarithmic decrement, is equal to $k\pi/\omega$.

$$\therefore\ V_1 = i_0(L/C)^{\frac{1}{2}}e^{-\lambda(\frac{1}{2}\pi - \theta)/\pi}.$$

Now, $\tan\theta = -R/2\omega L = -\lambda/\pi$; and, as θ is small, we can write

$$\tan\theta = \theta = -\lambda/\pi, \text{ neglecting } \lambda^3.$$

$$\therefore V_1 = i_0 (L/C)^{\frac{1}{2}} e^{-\frac{1}{2}\lambda - \lambda^2/\pi^2}, \text{ neglecting } \lambda^4$$

$$L = CV_1^2 e^{\lambda + 2\lambda^2/\pi^2} \div i_0^2$$

$$= CV_1^2 \{1 + \lambda + (\tfrac{1}{2} + 2/\pi^2)\lambda^2 + (\tfrac{1}{6} + 2/\pi^2)\lambda^3\}/i_0^2,$$

λ^4 being neglected. Thus the value of the inductance can be obtained from

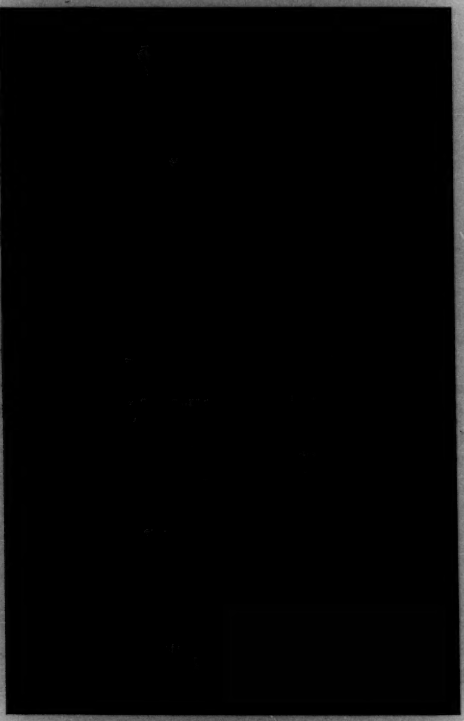


Fig. 2.

measurements of the peak voltage across the condenser, C, of known capacity, and the logarithmic decrement of the oscillations as recorded on a cathode-ray oscillogram such as is shown in figure 2.

§ 3. EXPERIMENTAL METHOD

A cathode-ray oscillograph, operating in conjunction with the type of time base previously described*, does not in the case under consideration record a base line. Before the necessary measurements can be carried out, this line must be located. This may be done as follows:

In figure 2 let a_1, a_2, a_3, ... be the amplitudes of successive oscillations from the base line, the position of which is as yet unknown. The quantities which can be actually measured to a sufficiently high degree of approximation are A_1, A_2, etc., where a_1, a_2, a_3

$$A_1 = a_1 + a_2, \qquad A_1$$

$$A_2 = a_2 + a_3, \qquad A_2$$

.....

Now $$a_2/a_1 = a_3/a_2 = \ldots = l, \qquad l$$

$$\therefore A_1 = a_1 + a_1 l$$

$$= a_1 (1 + l),$$

and $$A_2 = a_1 l (1 + l).$$

$$\therefore A_2/A_1 = l;$$

from which the logarithmic decrement can be determined.

The value of l having been thus determined, the values of a_1, a_2, a_3, etc. follow, because

$$a_1 = A_1/(1 + l).$$

Thus the base line may be located. In practice, for the purpose of measuring up an oscillogram, a suitable voltage deflection scale is employed, such as is shown in figure 3 of the previous paper*.

DISCUSSION

Dr D. Owen. The theory worked out in the paper is based on the assumption of constancy of the inductance and resistance of the coil under test, and it also neglects the self-capacity of the coil. As regards the resistance term, the effective resistance to varying currents of an iron-cored coil may be very high compared with the ohmic resistance of the winding; and this extra resistance, arising from hysteresis and eddy currents, must be dependent on the current flowing as well as on the frequency. Unfortunately no actual example of results is given in the paper, and in the absence of such figures further commentary is barred.

Prof. Finch's success in the application of the cathode-ray oscillograph has been signally demonstrated in an earlier paper read before the Society, and doubtless the difficulties here referred to will be surmounted by the authors.

* Finch, Sutton and Tooke, *Proc. Phys. Soc.* **43**, 502 (1931).

AUTHORS' reply. It is clear from the general circuit equation set forth at the beginning of § 2 that the theory is only applicable to cases where the self-capacity of the coil is negligible in comparison with C, and R is independent of the current traversing the coil. Thus the presence of iron in the magnetic circuit can give rise to very considerable inaccuracy, for the reasons pointed out by Dr Owen. It is our intention, however, in a further communication to the Society, to show how the method enables the various damping-losses in an oscillating circuit to be determined separately by simple oscillographic means based upon observed deviations from the theory as outlined above.

THE MEASUREMENT OF ELECTRICAL RESISTANCE IN TERMS OF A MUTUAL INDUCTANCE AND A PERIOD

By H. R. NETTLETON, D.Sc., Lecturer in Physics, Birkbeck College
AND F. H. LLEWELLYN, B.Sc., Birkbeck College

Received October 14, 1931. *Read January* 15, 1932

ABSTRACT. It is shown that if the ratio α/a of the radii of the concentric circles forming a simple inductometer is $0{\cdot}50607_8$, the mutual inductance is so accurately proportional to the angle through which the turning coil is displaced that the rising deviation from a linear law is less than 5 parts in a million at 7° of deflection. If the outer circle is replaced by two twin circles which are separated symmetrically with regard to the turning coil, the ratio α/a must be increased to bring about a similar approach to linearity.

If a heavy dynamometer is constructed on this principle, then, over the range indicated, the deflecting couple is strictly proportional to the current-products. Moreover if such a dynamometer be damped electrically, the damping-couple is accurately proportional to the velocity and, over a range of oscillation of 14° we have a remarkably close approach to true damped simple harmonic motion.

It is shown that the resistance of the oscillator circuit is very accurately given by the expression

$$R = \frac{CM}{c} \Big/ \left[\frac{\lambda T}{\pi^2 + \lambda^2} - \frac{\lambda_0 T_0}{\pi^2 + \lambda_0^2} \right],$$

where λ is the logarithmic decrement and T the period on closed circuit of the oscillator when a current C traverses the field coils, while λ_0 and T_0 correspond to open circuit and M is the mutual inductance between the oscillating coil and the field coils when the former is in the position of steady deflection due to the passage through it of a steady current c, in conjunction with the current C through the field coils. This expression eliminates quantities difficult to measure in Weber's method. Experimental tests are described. Table of Legendre and other functions, which facilitate the calculation of mutual inductances of the nature here considered, are appended.

§ 1. INTRODUCTION

A METHOD of measuring electrical resistance absolutely was suggested by W. Weber*. In this method a magnet of moment m is suspended within a coil and the period T_0 and logarithmic decrement λ_0 are observed when the circuit is open. On closure of the circuit induced currents assist the damping, and from a measurement of the new logarithmic decrement λ, together with other constants, the resistance of the coil can be calculated.

m

T_0, λ_0

λ

This historic method of damping has been carried out with various modifications by H. F. Weber, Dorn†; Wild‡; and F. Kohlrausch§.

* *Pogg. Ann.* **82**, 337 (1851). † *Wied. Ann.* **17**, 773 (1882); **36**, 22 and 398 (1889).
‡ *Mém. de l'Acad. des Sc. St Pétersbourg*, **32**, 2 (1884).
§ *Phil. Mag.* **47**, 302 and 342 1874); *Abh. d. bayr. Akad.* **16** (1888).

Unfortunately for the method, the basic formula for the resistance, namely

R $$R = m^2 G^2 T_0 / 4I\,(\lambda - \lambda_0),$$

G involves the square of a magnetic moment, the square of a galvanometer constant *G*
I and a moment of inertia *I* and in addition there are corrections for the arc of oscillation, for the length of the magnet and for the self-inductance of the coil. The attempts of other investigators to avoid these difficult quantities has resulted in the substitution of other quantities almost as difficult to measure accurately. Thus, in the opinion of Lord Rayleigh, Rowland and Glazebrook, the formula is sufficient to show that the method cannot compete with the other methods used for absolute measurements of resistance.

In 1930 the present writers* showed that, by the aid of a rotating coil magnetometer, the quantities $m^2 G^2 T_0 / 4I$ could be replaced by $\pi^2 MC / cT_0$, where M is a mutual inductance and C/c a current ratio.

In the present communication the magnet is replaced by a large, heavy coil of selected dimensions, the rotating coil magnetometer is no longer necessary, the swings are strictly isochronous, no length-correction is needed, the effect of self-
T inductance can be eliminated by observation of *T*, the period on closed circuit, and the basic expression for the resistance of the oscillating coil is

$$R = \pi^2 MC / c\, T_0\,(\lambda - \lambda_0).$$

Indeed the sole surviving measurements of Weber's historic method are those of period and logarithmic decrement, which, with a heavy system executing slow and true damped-simple-harmonic motion, can be measured with great accuracy. In order to attain these ends it is here shown that a suitable coil oscillating about a vertical axis may be so placed between two larger parallel field-coils that, over a range of some 7° on each side of the position of rest, the mutual inductance is so strictly proportional to the angle of displacement that the maximum deviation from a straight-line law is only a few parts in a million. Accordingly, when such a coil system is damped electrically, the damping-couple is accurately proportional to the angular velocity; and when it is used as a dynamometer, the deflecting couple is strictly proportional to the current-products.

As regards sensibility, it is practically immaterial whether the channel of the oscillating-coil-former is filled with copper wire of high or of low resistance. In either case it is equally convenient to standardize a low resistance.

§ 2. PRINCIPLE OF THE METHOD

A dynamometer is composed of two large field coils *A*, *B*, figure 3, of identical dimensions, together with a smaller heavy coil *C* supported by a thick conducting torsion wire and fitted below with damping devices and a thin outlet spiral. The oscillating coil *C* is wound upon a former in a channel of square section, and is of
α such dimensions that the ratio of the mean radius α of the turns composing it to
a the mean radius *a* of the turns of the field coils lies within the limits expressed by the relationship $0{\cdot}56 > \alpha/a > 0{\cdot}51$.

* *Proc. Phys. Soc.* **42**, 501 (1930).

The position of rest of C is arranged to be that of zero mutual inductance with A and B when the latter are in series and conjunction. The mutual inductance must also be zero when C is swinging and A and B in opposition. The distance of separation of A and B is then adjusted so that for deflections over the range of the scale the mutual inductance is proportional, within the accuracy of measurement, to the angle of displacement of C.

Observations of T_0, the period on open circuit, and λ_0, the corresponding logarithmic decrement, are then made with the aid of a telescope and circular scale. Next, with the oscillating coil short-circuited, a current C, between 1 and 2 amperes, is passed through the field coils, and the heavy logarithmic decrement λ of the oscillating coil is measured. The current C in the field coils being maintained, a small current c (whose value relative to C must be accurately known) is passed through the oscillating coil now critically damped, and a suitable scale-deflection is obtained. The oscillating coil is then clamped at this deflection, the currents are broken, and the mutual inductance M between the field coils in conjunction and the oscillating coil is measured. By reversal of the current through the latter, a deflection on the other side of the zero position, and hence the mean mutual inductance corresponding to the current product Cc, is obtained. The total resistance of the closed oscillating coil circuit can then be found to a high order of accuracy from the simple expression

T_0, λ_0 λ C c M

$$R = \frac{\pi^2 MC}{cT_0(\lambda - \lambda^3/2\pi^2 - \lambda_0)}.$$

§ 3. THEORY OF THE METHOD

Design of dynamometer coils so that over a small range the mutual inductance is very accurately proportional to the angle of rotation. It was pointed out by Lord Rayleigh* as far back as 1886 that if the ratio of the mean diameters of the coils of a simple inductometer is about 0·55, the mutual inductance is very nearly proportional to the angle through which the turning coil is displaced from the zero position up to some 70° of range. Lord Rayleigh illustrated this by taking four terms of a slowly converging series for the case where the coils are concentric. The problem before us is to find the best value of the ratio α/a of the coil radii so that the law $M = k\theta$, where k is a constant, shall hold with the highest precision over some few degrees of arc; and to investigate, further, how this ratio is affected when the rotating coil has its centre so situated as to lie at a distance x from the centres of two twin coils symmetrically placed about it.

θ, k x

The mutual inductance between a circle A and a smaller circle C in the position shown in figure 1 is given† by

$$M_{AC} = 4\pi^2 s \sin^2\psi \sin^2\phi \sum_{n=1}^{n=\infty} \frac{1}{n(n+1)} P_n'(\cos\psi) P_n'(\cos\phi) P_n(\sin\theta) \left(\frac{s}{r}\right)^n \quad \ldots\ldots(1),$$

where P_n is the Legendre function of the first kind, of order n, P_n' its differential

* *Phil. Mag.* **22**, 469 (1886).

† Gray, *Absolute Measurements in Electricity and Magnetism*, pp. 201, 227.

$n, s, r, \psi, \phi, \theta$ coefficient, and n a positive integer, and where the symbols s, r, ψ, ϕ, and θ are sufficiently defined by figure 1.

The presence of a second circle B identical with A and symmetrically placed with regard to C simplifies the expression by removing even terms. We may then write for the mutual inductance between C and the twin coils A and B in conjunction

$$M = Gq \sum_{n=1}^{n=\infty} \frac{2}{n(n+1)} \left(\frac{s}{r}\right)^{n-1} P_n'(\cos\psi)\, P_n'(\cos\phi)\, P_n(\sin\theta) \quad \ldots\ldots(2),$$

q where $q = \pi\alpha^2$ and is the area of the small circle C, while $G = t.2\pi a^2/r^3$ and is the
t galvanometer constant of A and B in conjunction at the origin O; t is the total number of turns on A and B together, if the turns on each coil may be regarded as
n coincident, and n is now an odd positive integer.

As the complications attendant on the problem of multiple layers are usually best dealt with by Maxwell's method of subsequent correction rather than by direct integration of equation (2), we shall examine in the first place two cases of importance with simple circles.

Case of two concentric circles. When A and B are coincident and concentric with C we have, figure 2, $r = a$, $s = \alpha$, $\phi = \psi = 90°$ and hence since

$$P_n'(\cos 90) = (-1)^{(n-1)/2}(1.3.5.7 \ldots n)/\{2.4.6 \ldots (n-1)\} \quad \ldots\ldots(3),$$

$$M = Gq \sum_{n=1}^{n=\infty} \frac{2}{n(n+1)} \frac{1^2.3^2.5^2.7^2 \ldots n^2}{2^2.4^2.6^2 \ldots (n-1)^2} P_n(\sin\theta)\, y^{(n-1)/2} \quad \ldots\ldots(4),$$

y where n is odd and $y = \alpha^2/a^2$.

By expanding $P_n(\sin\theta)$ in powers of $\sin\theta$ and these again in powers of θ we obtain the following useful expression

$$\begin{aligned}
\frac{M}{Gq} = {} & \theta \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} \\
& - \frac{\theta^3}{6} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} [1 + (n+2)(n-1)] \\
& + \frac{\theta^5}{120} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} [1 + 10(n+2)(n-1) \\
& \qquad + (n-3)(n-1)(n+4)(n+2)] \\
& - \frac{\theta^7}{5040} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} \\
& \qquad \times [1 + 91(n+2)(n-1) + 35(n-3)(n-1)(n+4)(n+2) \\
& \qquad + (n-5)(n-3)(n-1)(n+6)(n+4)(n+2)] \\
& + \ldots\ldots\ldots \qquad \ldots\ldots(5),
\end{aligned}$$

where n is odd and by the usual convention the first term ($n = 1$) under the summation sign is always unity.

When $y = 0$, that is when, in the limit, the rotating coil is of infinitely small area, the mutual inductance clearly follows a sine law deviating from a straight line as $\sin\theta$ deviates from θ. When $y = 0{\cdot}256115$ or $\alpha/a = 0{\cdot}506078$ the term in θ^3

vanishes, the coefficient being less than 1×10^{-7}, and on substituting this value in the other terms we obtain

$$M/Gq = 0{\cdot}87963773_0\,\theta - 0{\cdot}01687_9\,\theta^5 - 0{\cdot}00483\,\theta^7 \qquad \text{.....(6).}$$

This result, arrived at by summation of terms as far as $n = 31$, reveals a rising deviation from the straight-line law amounting to 4·2 parts in a million at 7° of deflection, 18 parts in a million at 10°, and 27 parts in a million at 11° of deflection.

The remarkable nature of this result is confirmed by direct calculation from the fundamental equation (4), and in table 1 we show the values of M/Gq thus directly calculated for the angles 0° to 7° inclusive and when $\theta = 11°$, $y = 0{\cdot}2561$

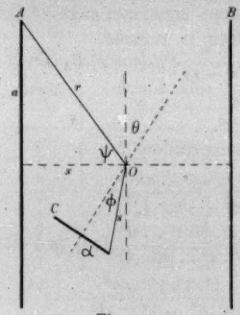


Fig. 1.

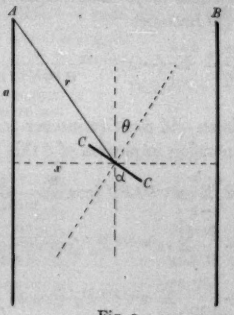


Fig. 2.

and $y = 0{\cdot}2567$. This calculation necessitates a knowledge of $P_n(\sin\theta)$ for the angles involved up to $n = 21$. When $y = 0{\cdot}2567$, and is therefore slightly larger than the limiting solution, the deviation is distributed, the maximum deviation over the range 0° to 7° never exceeding 0·75 part in a million.

Table 1. Variation of mutual inductance with angle of deflection for special cases of concentric circles

Angle (°)	Actual value of M/Gq $a^2/a^2 = 0{\cdot}2561$	Value of M/Gq if linear law be assumed	Difference $\times 10^8$	Actual value of M/Gq $a^2/a^2 = 0{\cdot}2567$	Difference from linear-law value $\times 10^8$
0	0·00000000	0·00000000	0·0	0·00000000	0·0
1	$0{\cdot}01535267_8$	$0{\cdot}01535267_8$	0·0	$0{\cdot}01534855_8$	0·0
2	$0{\cdot}03070535_6$	$0{\cdot}03070535_6$	0·0	$0{\cdot}03069712_2$	+ 0·6
3	$0{\cdot}04605802_7$	$0{\cdot}04605803_4$	− 0·7	$0{\cdot}04604569_5$	+ 2·1
4	$0{\cdot}06141068_1$	$0{\cdot}06141071_2$	− 3·1	$0{\cdot}06139427_5$	+ 4·3
5	$0{\cdot}07676330_1$	$0{\cdot}07676339_0$	− 8·9	$0{\cdot}07674284_4$	+ 5·6
6	$0{\cdot}09211584_8$	$0{\cdot}09211606_8$	− 22·0	$0{\cdot}09209138_4$	+ 3·6
7	$0{\cdot}10746827_8$	$0{\cdot}10746874_8$	− 47·3	$0{\cdot}10743984_8$	− 6·1
11	$0{\cdot}16887496_2$	$0{\cdot}16887945_8$	− 449·5	$0{\cdot}16883127_9$	− 285·8

It might well be questioned whether a coil could in practice be constructed near enough to the theoretical specification to yield anything like the best possible

linearity between M and θ. It should be pointed out at once therefore that this difficulty is largely overcome by the fact that if the oscillating coil is somewhat larger than the value demanded by the concentric position, twin field-coils may be placed on either side of it and separated outwards till linearity is attained; for it will be seen below that as the coils are separated a larger and larger value of y is required—a fact, moreover, which is likely to be of great importance when correction is made for the action of multiple layers.

Case of circle with centre symmetrically on the axis of two larger equal parallel coils. This case, shown in figure 2, is derived from the more general case of figure 1 by putting ϕ equal to 90° and s equal to α. This case is of practical importance and equation (2) becomes

$$\frac{M}{Gq} = \sum_{n=1}^{n=\infty} (-z)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1.3.5 \dots n}{2.4 \dots (n-1)} P_n'(\cos\psi) P_n(\sin\theta) \qquad \dots\dots(7),$$

n, z where n is an odd positive integer and $z = \alpha^2/r^2$.

By expansion in powers of θ this gives the expression

$$\begin{aligned}\frac{M}{Gq} = {} & \theta \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2.3^2.5^2 \dots n^2}{2^2.4^2 \dots (n-1)^2} \\ & - \frac{\theta^3}{6} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2.3^2.5^2 \dots n^2}{2^2.4^2 \dots (n-1)^2} [1 + (n+2)(n-1)] \\ & + \frac{\theta^5}{120} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2.3^2.5^2 \dots n^2}{2^2.4^2 \dots (n-1)^2} \\ & \qquad \times [1 + 10(n+2)(n-1) + (n+4)(n+2)(n-3)(n-1)] \\ & - \frac{\theta^7}{5040} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2.3^2.5^2 \dots n^2}{2^2.4^2 \dots (n-1)^2} \\ & \qquad \times [1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-3)(n-1) \\ & \qquad\qquad + (n+6)(n+4)(n+2)(n-5)(n-3)(n-1)] \\ & + \dots\dots\dots \qquad \dots\dots(8).\end{aligned}$$

A preliminary investigation of this expression up to the fourth term clearly reveals that as x/a or $\cot\psi$ increases continuously the term in θ^3 vanishes for larger and larger values of z, a second and higher root also appearing; y or α^2/a^2 attains a value of about 0·4 at $x^2/a^2 = 0{\cdot}05$, and the terms in θ^5 and θ^7 are still very small. Above this the roots approach and merge, the term in θ^3 becoming a minimum and almost vanishing for very slowly diminishing values of z.

As our actual experiments were performed with ψ equal to 79° approximately, we have worked out this case completely, using accurate values of $P_n'(\cos 79)$ up to $n = 31$.

When $\psi = 79°$, $x^2/a^2 = 0{\cdot}037784$, $x/a = 0{\cdot}19438$, the term in θ^3 vanishes if $z = 0{\cdot}2874$*, i.e. when $y = \alpha^2/a^2 = 0{\cdot}29826$ or $\alpha/a = 0{\cdot}5461$. On substituting this value in the other terms of equation (8), we obtain

$$M/Gq = 0{\cdot}88498462\,\theta - 0{\cdot}02030\,\theta^5 - 0{\cdot}0063\,\theta^7 \qquad \dots\dots(9).$$

* The root is possibly nearer 0·2873. When $z = 0{\cdot}2874$ the residual term in θ^3 is $+0{\cdot}0000245\theta^3$.

This result reveals a deviation of 5·1 parts in a million from the straight line law at $\theta = 7°$, of 21 parts in a million at 10° and of 31 parts in a million at 11°. It is confirmed by direct calculation from the fundamental equation (7). In table 2 we give the values of M/Gq thus directly calculated for the angles 0° to 7° inclusive as well as for 11°; this direct calculation needs an accurate knowledge of $P_n(\sin\theta)$ for the angles involved up to $P_{21}(\sin\theta)$. It should be noted that in evaluating the first one or two terms, direct multiplication is necessary, seven-figure logarithms not being sufficiently accurate.

Table 2. Variation of mutual inductance with angle of deflection for circle between two larger equal circles. (Ratio of radii $\alpha/a = 0{\cdot}5461$; measure of displacement $x/a = 0{\cdot}19438$)

Angle (°)	Actual value of M/Gq	Value of M/Gq on assumption of a linear law	Difference $\times 10^8$
0	0·00000000	0·00000000	0·0
1	0·01544589$_5$	0·01544589$_5$	0·0
2	0·03089179$_0$	0·03089179$_0$	0·0
3	0·04633768$_0$	0·04633768$_5$	− 0·5
4	0·06178355$_5$	0·06178358$_0$	− 2·5
5	0·07722939$_0$	0·07722947$_5$	− 8·5
6	0·09267514$_3$	0·09267537$_0$	− 22·7
7	0·10812075$_7$	0·10812126$_5$	− 50·8
11	0·16989967$_0$	0·16990484$_5$	− 517·5

Effect of coils possessing multiple layers. Though we are unable to offer a satisfactory mathematical treatment of this problem, we have derived useful information by making use of a principle due to Maxwell. If a coil of rectangular cross-section has axial breadth $2b$ and radial depth $2d$, and P_0 is any term in the expression of action of any kind at a point, due to the central circular filament of radius α, then, to a first order, the average term $\bar{P}$ for the action of the entire coil is given by

b, d

$\bar{P}$

$$\bar{P} = P_0 + \frac{(b^2 + d^2)}{6}\frac{\partial^2 P_0}{\partial a^2} \qquad \text{.....(10).}$$

It is much easier to apply this equation to the oscillating coil than to the field-coils and as a result we aim at so constructing the coil former that $1 > d/b > 0{\cdot}9$, under which circumstances, if d and b are small compared with the radius α, the disturbing effect is very slight. If the oscillating coil is unduly lengthened so as to approach the nature of a solenoid, its radius α must be increased to maintain proportionality between mutual inductance and deflection. As the primary effect of multiple layers is to alter the effective radii of coils, it is highly probable that the effect may in the main be compensated by adjustment of the distance between the field-coils. At the moment we are submitting problems of this nature to direct experimental test.

Determination of the resistance by observation of the damping of the oscillating coil of a dynamometer fulfilling the law $M = k\theta$. Consider a coil suspended symmetrically between twin field coils so adjusted that over 7° on either side of the

position of rest (which is that of zero mutual inductance) the mutual ind*M*uctance
θ is very accurately proportional to the angle of deflection θ. When the field-coils
C, c are carrying a current C and the other coil a current c, the mutual energy of the system is $Cck\theta$, and the couple tending to increase θ is Cck, being independent of the angle. The action of the earth's magnetic field not only is extremely minute but is completely neutralized in our experiments by large square coils arranged in the Helmholtz manner, and hence it need not be considered further.

Now if while a current C traverses the field-coils the oscillating-coil circuit is
c_1 closed through a resistance, a current c_1 which varies with time will be induced in
F the moving coil and give rise to electrical damping. As the flux F in the position θ is given by

$$F = Ck\theta + Lc_1 \qquad \text{.....(11)},$$

L where L is the self-inductance of the moving coil, we have for the induced current

$$c_1 = (-Ck\dot{\theta} - L\dot{c}_1)/R \qquad \text{.....(12)},$$

R where R is the total resistance of the closed oscillator circuit.

Since the effect of self-inductance is small, we may take $\dot{c}_1$ in the $L\dot{c}_1$ term as the value of $\dot{c}_1$ when inductance is neglected, namely:

$$-Ck\dot{\theta}/R,$$

and hence

$$c_1 = -Ck\dot{\theta}/R + LCk\ddot{\theta}/R^2 \qquad \text{.....(13)}.$$

Thus the deflecting couple, tending to increase θ, which is produced by the current C in the field coils and the induced current c_1 in the oscillating coil, is given algebraically by

$$Cc_1k = -C^2k^2\dot{\theta}/R + LC^2k^2\ddot{\theta}/R^2 \qquad \text{.....(14)},$$

so that on equating the resultant torque, tending to increase θ, to the product of moment of inertia and angular acceleration, we have

$$(I - LC^2k^2/R^2)\,\ddot{\theta} + (p + C^2k^2/R)\,\dot{\theta} + u\theta = 0 \qquad \text{.....(15)},$$

p, u where p is the coefficient of air damping and u the torsional restoring couple per radian, so that, without approximation,

$$C^2k^2/R = 4I\,(\lambda/T - \lambda_0/T_0) - (4\lambda/T)\,LC^2k^2/R^2 \qquad \text{.....(16)},$$

λ_0, T_0 where λ_0, T_0 are the logarithmic decrement and period respectively on open circuit
λ, T and λ, T the corresponding quantities when the coil is closed through the total resistance R. Since the effect of self-inductance is very small, this quadratic in R is conveniently written without approximation in the form

$$R = \frac{C^2k^2}{4I}\,\frac{1}{\lambda/T - \lambda_0/T_0}\left[1 + \frac{4\lambda}{T}\,\frac{L}{R}\right] \qquad \text{.....(17)}.$$

Now since in oscillation on open circuit

$$u/4I = (\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(18)},$$

and since on deflection by a steady current c through the oscillating coil while the current C traverses the field coils

$$Cck = u\theta \qquad \text{.....(19)},$$

where

$$k\theta = M, \qquad \text{.....(20)},$$

M being the measured mutual inductance when the oscillating coil is damped in the position which it takes up when a steady deflection is produced by the combined currents C and c, we have, on multiplying equations (18), (19) and (20) together, M

$$C^2k^2/4I = M(C/c)(\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(21),}$$

and on making this substitution for $C^2k^2/4I$ in equation (17)

$$R = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_0^2(\lambda/T - \lambda_0/T_0)}\left[1 + \frac{4\lambda}{T}\frac{L}{R}\right] \qquad \text{.....(22),}$$

or very approximately

$$R = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_0^2(\lambda/T - \lambda_0/T_0)} + \frac{4\lambda L}{T} \qquad \text{.....(23).}$$

As, within the accuracy of our experimental tests, $4\lambda L/RT$ is negligible compared with unity, the effect of self-inductance may be neglected in our case and accordingly

$$(\pi^2 + \lambda^2)/T^2 = (\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(24);}$$

hence T can be calculated from T_0, and indeed with ample accuracy

$$T = T_0(1 + \beta) \qquad \text{.....(25),}$$

where

$$\beta = (\lambda^2 - \lambda_0^2)/2\pi^2 \qquad \text{.....(26),} \qquad \beta$$

and thus

$$R = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_0[\lambda(1 - \beta) - \lambda_0]} \qquad \text{.....(27);}$$

or in our experiments, on account of the smallness of L and λ_0,

$$R = M(C/c)\pi^2/T_0(\lambda - \lambda^3/2\pi^2 - \lambda_0) \qquad \text{.....(28),}$$

with an accuracy never less than one part in ten thousand.

Elimination of the primary effect of self-inductance. In the solution of the fundamental differential equation (15) we have, before introducing the special elimination of $C^2k^2/4I$, followed the usual lines associated with Weber's method of damping, by treating the effect of self-inductance as a very small addition term—a procedure most useful in showing the smallness and estimating the order of the effect.

It may, however, ultimately be desirable to use larger coils in which the ratio L/R of the oscillator circuit is increased, and we here show that the effect of self-inductance may be completely eliminated from equation (15), which recognizes the primary effect of such inductive action in diminishing the effective moment of inertia of the system. From equation (15) we have, under closed-circuit conditions,

$$u/(I - LC^2k^2/R^2) = 4(\pi^2 + \lambda^2)/T^2 \qquad \text{.....(29),}$$

and combining this with equation (18) for open circuit we find

$$LC^2k^2/R^2I = 1 - (\pi^2 + \lambda_0^2)T^2/(\pi^2 + \lambda^2)T_0^2 \qquad \text{.....(30),}$$

and so on, making this substitution in equation (16),

$$\frac{C^2k^2}{R} = 4I\left[\frac{\lambda}{T}\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda^2}\frac{T^2}{T_0^2} - \frac{\lambda_0}{T_0}\right] \qquad \text{.....(31);}$$

whence, on making the special substitution of equation (21), we obtain

$$R = MC/c\{\lambda T/(\pi^2 + \lambda^2) - \lambda_0 T_0/(\pi^2 + \lambda_0^2)\} \qquad \text{.....(32),}$$

an accurate expression eliminating the primary inertia effect of self-inductance and derived without approximation from equations (15) and (21).

This equation could be used directly, to the fullest advantage of the method, if the period T (as the number of swings is limited) were taken photographically by a running film on which an accurate time trace was simultaneously recorded*.

If T cannot be observed experimentally with sufficient accuracy and the effect of self-inductance is not negligible but small (i.e. $4\lambda L/RT_0$ is small compared with unity), L/R must be estimated by measuring L and using the value of R obtained by neglecting inductance. From equations (30) and (16) or (31) we then have, as a good approximation,

$$(\pi^2 + \lambda_0^2)\, T^2/(\pi^2 + \lambda^2)\, T_0^2 = 1 - 4L\,(\lambda - \lambda_0)/RT_0 \qquad \text{.....(33),}$$

or

$$T = T_0\,(1 + \beta - \gamma) \qquad \text{.....(34),}$$

where $\beta = (\lambda^2 - \lambda_0^2)/2\pi^2$ as before (26) and

$$\gamma = 2L\,(\lambda - \lambda_0)/RT_0 \qquad \text{.....(35),}$$

whence from equation (32)

$$R = M\,(C/c)\,(\pi^2 + \lambda_0^2)/T_0\,[\lambda\,(1 - \beta - \gamma) - \lambda_0] \qquad \text{.....(36),}$$

or (very closely, as λ_0 is very small)

$$R = M\frac{C}{c}\frac{\pi^2}{T_0\,(\lambda - \lambda^3/2\pi^2 - \lambda_0)} + \frac{2L}{T_0}\,(\lambda - \lambda_0) \qquad \text{.....(37).}$$

These last equations emphasize the care that must be taken in estimating T if the inductance term is too large to be neglected.

It is unsound, for example, in using equation (23) to estimate T as in equations (24) and (25), neglecting inductance, and then to add on to the value of R the quantity $4\lambda L/T$ as a correction for inductance.

Separation of the dynamical and statical experiments. The logarithmic decrements λ and λ_0 are taken under periods defined by T and T_0. We have assumed that when the deflection experiments are performed the control is unchanged and that the natural period on removal of the damping vanes is still T_0. If, owing to change of temperature, the natural period taken before and after the deflection experiments differs from T_0 and has a value T_1, equation (21) should be replaced by

$$\frac{C^2k^2}{4I} = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_1^2} = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_0^2}\frac{T_0^2}{T_1^2} \qquad \text{.....(38),}$$

and accordingly any expression for R must be multiplied by the factor T_0^2/T_1^2. Thus, since inductance is negligible in our experiments, our working formula, as derived from equation (28), is

$$R = M\frac{C}{c}\frac{\pi^2}{(\lambda - \lambda^3/2\pi^2 - \lambda_0)}\frac{T_0}{T_1^2}, \qquad \text{.....(39),}$$

though in practice the difference between T_0 and T_1 is normally only one or two parts in ten thousand.

* Cf. *Dictionary of Applied Physics*, **2**, 235.

The sensibility in relation to the dimensions and windings of the apparatus. The fundamental feature of the important problem of sensibility lies in the necessity, on changing from mechanical to electrical damping, of securing a marked and accurately measurable increase of logarithmic decrement. Whatever the moment of inertia of the oscillating coil may be, it is desirable to maintain a period of about 16 seconds by suitable choice of the length and diameter of the suspending wire—at all events if logarithmic decrements are measured by direct eye-observation. Regarding then T_0 as fixed, we examine the sensibility by means of the expression

$$(\lambda - \lambda_0)/T_0 = C^2k^2/4IR = C^2\,(0{\cdot}88Gq)^2/4IR \qquad \text{.....(40).}$$

Now if the channel of the oscillating coil is of volume V and mean radius α and if σ is the resistivity and l the length of wire filling it, then whatever the gauge of wire chosen, since the resistance of the suspension may be regarded as negligible, we have

V, α
σ, l

$$R = \sigma l^2/V \qquad \text{.....(41)}$$

and

$$q = \alpha l/2 \qquad \text{.....(42),}$$

and hence

$$(\lambda - \lambda_0)/T_0 = 0{\cdot}048\; C^2G^2\alpha^2V/I\sigma \qquad \text{.....(43),}$$

which reveals that the sensibility is independent of the diameter and length of the wire.

If the former of the oscillating coil is of negligible moment of inertia in comparison with the wire it contains, α^2V is proportional to I. Thus the sensibility is proportional to CG^2 and inversely proportional to the resistivity of the material with which the oscillating coil is wound. Now G is dependent on the dimensions of the field coils, which have double the radius of the oscillating coil and if the field coils are made larger in all proportions and wound with a fixed gauge of wire, G will increase with a, the mean field coil radius.

a

Thus the larger the entire apparatus the greater is the sensibility, the surplus of which may well be utilized in reducing C, the current through the field coils, or the channels of the field coils or possibly in substituting manganin for copper in the oscillating coil. Other advantages of working with a large apparatus are that λ_0 becomes very small and constant, that a thick fatigueless suspension may be used, that the stability is excellent and that ample clearance is left between the field-coil formers and the sphere surrounding the oscillating coil.

§ 4. THE APPARATUS AND METHOD OF EXPERIMENTING

Under difficult circumstances, due to insufficient space and isolation and to the vibration of the building, we have tested experimentally the general theory of the method with the apparatus shown in figure 3.

The field coils A, B were constructed of dexonite specially cast by the Dexine Co. Ltd., the channels being of radial depth and axial breadth 3·6 cm. and the mean diameter of the winding 31·0 cm. Each channel was filled with 504 turns o[f] double-silk-covered copper wire (s.w.g. 16), and the galvanometer-constant G with the coils touching and joined in series and conjunction was 368 cm.$^{-1}$ at the origin

of symmetry. The coils could be separated and firmly locked and were capable of rotation on the carrier *D* about a vertical axis. They were set with the plane of the windings in the magnetic meridian.

Several coils were used as the oscillating coil *C*. Each was of mean channel-diameter about 17 cm. but of varying dimensions which are given in the next section,

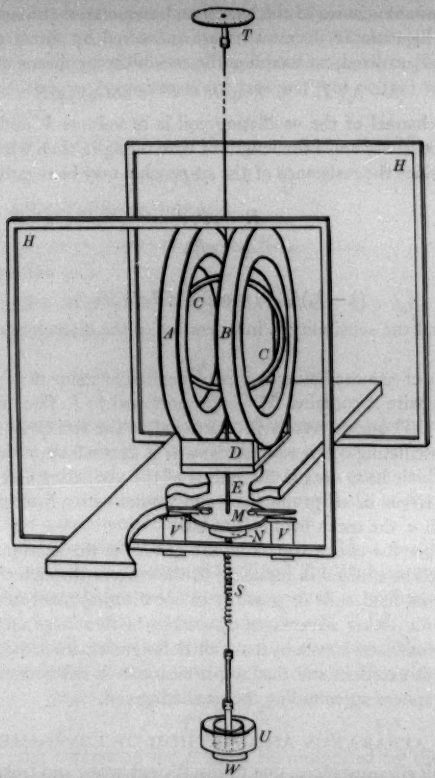


Fig. 3.

and was wound on a wooden former, the ends of the windings being brought to brass cross-pieces 180° apart. It was encased in a papier-mâché sphere (a globe of the world was used for this purpose) which not only diminished and smoothed the air damping but also enabled the moving system to be neatly clamped, without jolt or displacement, by means of wooden jaws capable of adjustment and controlled by a link work slider.

Current is led into the oscillating coil via the torsion head T and suspension wire, and passes out via a vertical brass rod E, finishing in a spiral S attached to a firm base W. The connecting leads to T and W, along the ceiling and floor respectively, are parallel to the axis of the field coils.

Attached to the rod E is a horizontal disc M to which, when necessary, can easily be clipped two vanes of aluminium, VV, dipping into troughs of liquid paraffin and giving critical damping to the rotating system. This liquid was most satisfactory and vastly superior to glycerin, which is not only hygroscopic but, owing to the heaping up of the liquid by the vanes in rotation, produces a kind of fatigue. Under the disc M is a concentric copper cylinder N which dips into a small annular glass vessel U (shown in figure 3 as resting on the base W) surrounding the lower suspension and filled with xylene. The effect of this is to damp out lateral movements of the rotating system due to the vibration of the room, and to ensure smoothness of rotation without producing any appreciable damping of the torsional oscillations. A device of this nature was essential for the accurate measurement of logarithmic decrements.

The weight of the oscillating coil and its accessories was often as large as 2000 gm., and the suspension was a metre of German-silver wire, s.w.g. 22. This was quite fatigueless. Earlier suspensions of silicon-bronze and cadmium copper showed slight traces of fatigue, and our attempts to obtain elinvar were unsuccessful. Thick wires show far less fatigue than thin ones, but no other wire below gauge 18, examined in a careful search, behaved with the striking perfection of this German silver. The outlet spiral in the earlier experiments was of silicon bronze, s.w.g. 30, and in later experiments of copper wire, s.w.g. 36.

A plane mirror was attached to the rod socket where it emerged from the globe at the bottom of the coil. This mirror, in conjunction with a telescope and well illuminated scale, allowed deflections, period and logarithmic decrements to be measured. The scale was circular, its radius of curvature was 198·5 cm. and its length 1 m., and the zero was in the middle. Readings of steady deflections could be measured to 0·01 cm.

The earth's horizontal field at right angles to the axis of the field coils was neutralized by two large rectangular coils, figure 3, of sides 49 cm. by 55 cm., HH, placed in the Helmholtz position and worked from a single accumulator. The neutralization of the earth's field by the flux of these earth coils was finally checked with all *in situ* by timing the swings of the oscillating coil when it carried (*a*) no current, (*b*) a current of 1·5 A, (*c*) a similar current in the contrary direction; equality of all these periods being secured by adjustment of the earth-coil current. For this purpose the lower spiral was removed and replaced by a pin dipping into mercury.

The entire dynamometer was enclosed in a case and the suspensions and damping appliances were shrouded so that the entire system was free from draughts. The leads from the oscillating coil were brought through a resistance box to opposite quadrants of a good key situated close to the telescope, where either a short circuit could be effected or connection could be made through potential leads to

a potentiometer in either direction, or through current leads to an accumulator circuit or alternatively a mutual inductometer. The leads from the field coils were led well away to keys which enabled them to be joined in conjunction or opposition, and thence in either direction to the mutual inductometer or to an accumulator circuit.

Before an experiment the torsion head was so adjusted that the oscillating coil in its position of rest, with no torsion in either top or bottom suspension, had zero mutual inductance with the field coils. The mirror was then adjusted so that the reading in the telescope was very near the middle of the scale. Furthermore the symmetry was so adjusted that when the field coils were in opposition the mutual inductance was very closely zero over the entire scale-range of oscillation. Readings were then taken, at every 10 cm. of the scale, of the mutual inductance between the oscillating coil and the combined field coils. Inductances and deflections of nearly equal magnitudes on opposite sides of the zero were subtracted algebraically to eliminate any slight displacement of the scale and any irreversible inductance. While the linearity between mutual inductance and displacement were thus being tested the deflections were always produced electrically.

The mutual inductometer was of the latest Campbell pattern of range 0–11,000 μH with $\frac{1}{5}$ factor switch for the scale, and the mutual inductances were measured in the first place by Hartshorn's modification* of Felici's method using a.c. of low frequency from a valve oscillator, the readings being standardized by d.c. and ballistic-galvanometer observation. Later we often worked directly by d.c., which, to our surprise, we found to be sensitive to about 0·2 μH at mean range. By suitable switches we could change immediately from a.c. to d.c. The inductometer, dynamometer and valve oscillator were all well separated.

A typical day's experiment is illustrated by the following order of observation: (*a*) The zero and symmetry are tested and if need be corrected and the μH/scale-division checked by taking readings near the ends of the scale and at about 20 cm. on each side of the zero. (*b*) A period T_0 is taken and λ_0, the air-damping decrement, observed at the same time. (*c*) A current C, usually between 1·25 and 1·55 A., is passed through the field coils and maintained very steady. The current value is read off a standard 0·01 ohm on a Cambridge thermoelectric potentiometer reading from 0 to 90 mV. While one observer watches the current the other deflects the oscillating coil electrically, short-circuits it, and takes readings of the decrement. λ is observed again almost immediately. Directly this reading is completed, the resistance R of the oscillating-coil circuit is compared on the potentiometer with a 10- or 20-ohm resistance derived from the ratio arms of a good Post Office box. More decrements and more resistance comparisons are then taken, and the whole process is then repeated with different values of C. (*d*) A period T_0 and an air-logarithmic-decrement λ_0 are again taken. (*e*) The damping vanes are now most carefully clipped to the carrier disc and the troughs containing liquid paraffin are arranged to give critical damping. With a current C (of the same magnitude as that used for the electrical-damping experiments) traversing the field coils, a

* *J. of Sci. Inst.* **2**, 145 (1925).

current c is passed through the oscillating coil to produce a large steady deflection, which is noted. The current c is then reversed and the deflection on the other side is read. This process is repeated with different values of c, and then again with other values of C in the field coils, just as in the decrement experiments. As the μH. per scale-division is accurately determined, values of M/Cc are easily calculated. The current c, of 1 to 5 mA, is measured on the Cambridge potentiometer by drawing the e.m.f. off the very same 10- or 20-ohm resistance that was used for comparison with R between the decrements in process (c) above. (f) The damping vanes are removed and a period is taken which, averaged with the period in (d) above, gives T_1 of equation (39).

It will readily be seen that by measuring the current c off the very resistance R_s used for checking R, we throw the whole of the onus of the agreement of R, as measured by our method and by comparison with R_s, on the value of the standard 0·01 ohm, which was correct to one part in two thousand.

Experiments were only useful if performed in the quiet of the night or a week-end, for logarithmic decrements were completely spoilt if people were moving on the floor above us.

§ 5. EXPERIMENTAL TESTS

Preliminary experiments to test the possibilities of the method were made with an oscillating coil C_1 of about 85 turns of double-silk-covered copper wire of s.w.g. 22 supported by a silicon bronze suspension of s.w.g. 26. This coil, of mean radius $\alpha = 8{\cdot}6$ cm., yielded a remarkably linear scale-calibration of 17·24 μH./cm., and a value at 14° C. for its resistance R of 2·51 ohms, in close agreement with a Post Office box check. As a result of these initial tests, the equipment was at once improved by introducing a telescope and scale, a potentiometer system for the rapid checking of R, the Hartshorn impurity device with d.c. calibration, and it was decided to prepare coils of greater weight and to investigate the elastic properties of thick suspensions.

A coil C_2 was accordingly prepared of the following dimensions: depth of channel = breadth of channel, 2 cm.; internal radius of channel in coil former, 7·68 cm.; mean radius of winding α, 8·7 cm.; number of turns, 230½ of double-silk-covered copper wire of s.w.g. 18; self-inductance L, 13680 μH. This coil was suspended by fatigueless German silver wire, s.w.g. 22. In tables 3 and 4 we

Table 3. Microhenries and scale-displacement for oscillating coil C_2

Scale-displacement (cm.)	Microhenries	μH./cm. from nth and $(n+3)$rd readings
98·72	4383·4	—
80·50	3573·1	—
60·68	2694·4	—
40·25	1786·2	$44{\cdot}4I_9$
20·70	917·2	$44{\cdot}4I_3$
0·00	− 0·9	$44{\cdot}4I_8$

Mean result $44{\cdot}4I_7$ μH/cm.

illustrate the scale-calibration and the constancy of the ratio M/Cc; and in tables 5 and 6, determinations of the resistance of the coil and its attachments and leads, together with the values obtained by potentiometer comparison.

Table 4. Values of M/Cc for oscillating coil C_2

μH/cm. = 44·41$_7$, T_1 = 15,680 sec.

Field-coil current C (A.)	Oscillating-coil current c (A.)	Deflection due to reversal from $+c$ to $-c$ (cm.)	M/Cc (μH./A.^{2})
2·4500	0·0035760	98·07	248595
2·2500	0·0035580	89·62	248620
2·0500	0·0042855	98·35	248621
1·7500	0·0050269	98·48	248616
1·5500	0·0050261	87·24	248698
1·3600	0·0058854	89·61$_5$	248648

Mean result 248630 μH./A.^{2}.

Table 5. Determination of resistance on April 24, 1931

$M/Cc = 2{\cdot}486_3 \times 10^8$ cm./A.^{2}; $T_1 = 15{\cdot}680$ sec.; $T_0 = 15{\cdot}682$ sec.; λ_0 (common) = 0·00096.

C	λ (common)	$\lambda(1-\beta)-\lambda_0$	$R \times 10^9$ (c.g.s.)	Resistance by comparison (ohms)	Remarks
—	—	—	—	2·8459	
1·7500	0·07416	0·1683$_0$	2·848$_2$	—	Resistance rising
1·7500	0·07390	0·1677$_0$	2·858$_4$	—	C too strong
—	—	—	—	2·8574	
1·5500	0·05819	0·1316$_8$	2·855$_8$	—	Resistance steady
1·5500	0·05820				
1·5500	0·05819				
—	—	—	—	2·8534	
1·3600	0·04502	0·1013$_9$	2·855$_4$	—	
—	—	—	—	2·8531	

Another coil C_3 was prepared on a former of the same dimensions as that of C_2, wound with about 600 turns of double-silk-covered copper wire, s.w.g. 22, the mean radius of winding α being 8·6 cm., the resistance without suspensions 14·9 ohms at 20° C., and the self-inductance L 104700 μH. The suspension was of German silver, s.w.g. 22, and additional resistance was added from a non-inductive box to make the resistance approximately 20 ohms and hence more quickly and accurately comparable with the 20-ohm ratio-arm standard.

In table 7 we show how the microhenries varied with the scale deflection. The result clearly reveals that for this multiple layered coil α/a was rather small: the field coils were as close as possible.

Table 6. Determination of resistance of oscillating coil C_2 on May 3, 1931

$M/Cc = 2{\cdot}487_2 \times 10^8$ cm./A.^{2}; $T_1 = 15{\cdot}702$ sec.; $T_0 = 15{\cdot}704$ sec.;
λ_0 (common) = 0·000945.

C	λ (common)	$\lambda(1-\beta)-\lambda_0$	$R \times 10^9$ (c.g.s.)	Resistance by comparison (ohms)	Remarks
—	—	—	—	2·8616	
1·4500	0·05082	$0{\cdot}1147_7$	$2{\cdot}864_0$	—	Resistance rising
1·3800	$0{\cdot}0460_4$	$0{\cdot}1037_8$	$2{\cdot}869_2$	—	Insufficient comparisons taken
—	—	—	—	2·8671	
1·3500	0·04414	0·09941	$2{\cdot}866_6$	—	
—	—	—	—	2·8656	
1·2500	0·03794	0·08515	$2{\cdot}869_1$	—	Extra resistance added to oscillating-coil circuit
—	—	—	—	5·8580	
1·4500	0·02533				
1·4500	0·02533	0·05614	$5{\cdot}855_7$	—	Resistance very steady
1·4500	0·02533				

Table 7. Microhenries and scale-displacement for coil C_3

Scale-displacement (cm.)	Microhenries	μH./cm. from nth and $(n+5)$th readings
99·68	12300·5	—
90·20	11130·5	—
80·36	9917·0	—
68·80	8489·6	—
63·60	7849·5	—
49·75	6142·5	$123{\cdot}33_3$
37·23	4596·5	$123{\cdot}35_3$
24·20	2988·2	$123{\cdot}37_6$
9·81	1211·3	$123{\cdot}38_9$
0·00	0·0	$123{\cdot}42_6$

We give in tables 8, 9 and 10 the complete data of a long experiment performed on June 21 with the coil C_3 after careful resetting for zero and symmetry.

Table 8. Calibration of scale

Scale reading	Microhenries	Scale deflection	Microhenries
+ 49·74	+ 6158·0	99·52	12271·5
− 49·78	− 6113·5		
+ 20·00	+ 2492·5	39·98	4931·5
− 19·98	− 2439·0		
		59·54	7340·0

Result 123·28 μH./cm. at readings ± 35 cm. of scale.

Table 9. Value of M/Cc

μH./cm. = 123·28 (table 8); $T_1 = 16{\cdot}571$ sec.

Field coil current C (A.)	Oscillating coil current c (A.)	Deflections in cm. due to reversal from $+c$ to $-c$	M/Cc (μH./A.^{2})
1·4500	0·0018567	91·28	2089900
1·4500	0·0014610	$71{\cdot}82_5$	2089900
1·3500	0·0015728	72·00	2090200
1·3500	0·0018910	$86{\cdot}56_5$	2090200
1·2500	0·0016618	70·45	2090500

Mean result: $2{\cdot}090_1 \times 10^9$ cm./A.^{2}

Table 10. Logarithmic decrements and value of resistance

$M/Cc = 2{\cdot}090_1 \times 10^9$ cm./A.^{2}; $T_1 = 16{\cdot}571$ sec.; $T_0 = 16{\cdot}563$ sec.;
λ_0 (common) = $0{\cdot}00088_0$.

C	λ (common)	$\lambda(1-\beta)-\lambda_0$	$R \times 10^9$ (c.g.s.)	Resistance by comparison (ohms)
				19·842
1·4500	0·05818	$0{\cdot}1318_2$	$19{\cdot}84_6$	
1·4500	0·05815	$0{\cdot}1317_4$	$19{\cdot}85_8$	
				19·853
1·4500	0·05811	$0{\cdot}1316_6$	$19{\cdot}87_0$	
1·4500	$0{\cdot}058074$	$0{\cdot}13157$	$19{\cdot}88_3$	
				19·862
1·3500	0·05048	$0{\cdot}1141_2$	$19{\cdot}86_9$	
				19·875
1·3500	0·05048	$0{\cdot}1141_3$	$19{\cdot}86_9$	
				19·888
1·2500	0·04336	0·09776	$19{\cdot}88_7$	
1·2500	$0{\cdot}043325$	0·09768	$19{\cdot}90_3$	
				$19{\cdot}906_5$
1·2500	$0{\cdot}043265$	0·09754	$19{\cdot}93_2$	
				19·924

Owing to the fact that logarithmic decrements play such an important part in the method, we illustrate the accuracy attained and the method of evaluation by giving two examples recorded in table 11 below.

It should be pointed out that a logarithmic decrement worked out thus by differences inevitably yields the same result as if the slope of the corresponding graph were evaluated by zero sum or by the method described by Awbery*.

§ 6. SUGGESTIONS FOR FURTHER DEVELOPMENT

Though we have attempted to show both theoretically and practically the possibilities of the method, much on both sides remains to be done. Owing to our experimental conditions and the expenses involved in the construction of coils and other accessories, we would gladly see the method pushed to its limits in the hands of other observers.

* *Proc. Phys. Soc.* **41**, 384 (1929).

We believe that the measurement of logarithmic decrements, which depends, as such dynamic observations must, on the skill and form of the observer, should be carried out photographically, and some experiments made in this direction by Mr H. G. Bell strongly confirm this view. Periods also should be taken photographically by a running film in conjunction with a time trace, so as not only to save delay at a time when variation in the temperature and hence in the period would affect the accuracy of the experiment, but also to deal effectively and simply with the problem of self-inductance.

Table 11. Evaluation of logarithmic decrements

Oscillating coil C_2 $C = 1{\cdot}36$ A.				
Scale readings		Amplitudes	Mantissa of logarithm	Differences of six swings
	49·40	—	—	—
45·28		94·68	0·97626	—
	40·10	85·38	0·93136	—
36·89		76·99	0·88643	—
	32·51	69·40	0·84136	—
30·08		62·59	0·79651	—
	26·33	56·41	0·75136	—
24·52		50·85	0·70629	0·26997
	21·32	45·84	0·66124	0·27012
20·00		41·32	0·61616	0·27027
	17·26	37·26	0·57124	0·27012
16·34		33·60	0·52634	0·27017
	13·95	30·29	0·48130	0·27006
Average for 6 swings = 0·27012 Whence λ (common) = 0·04502				

Oscillating coil C_3 $C = 1{\cdot}45$ A.				
Scale readings		Amplitudes	Mantissa of logarithm	Differences of four swings
	44·80	—	—	—
39·11		83·91	0·92381	—
	34·30	73·41	0·86576	—
29·91		64·21	0·80760	—
	26·24	56·15	0·74935	—
22·88		49·12	0·69126	0·23255
	20·09	42·97	0·63317	0·23259
17·49		37·58	0·57496	0·23264
	15·38	32·87	0·51680	0·23255
Average for 4 swings = 0·23258 Whence λ (common) = $0{\cdot}05814_4$				

Large field-coils should be used, giving greater available sensitivity and allowing of more clearance between themselves and the oscillating coil. Further, direct steps should be taken to keep the resistance of the oscillating coil as constant as possible. With suitable field coils, the use of manganin for the oscillator is by no means out of the question.

We are meanwhile investigating practically how close to the linear law of inductance we can approach with a multiple-layered oscillating coil of slightly larger mean radius, as in our main research we were hampered by lack of clearance (owing to the sphere and suspension terminals) and so our oscillating coils were rather on the small side.

If the lower suspension is replaced by a mercury contact, the apparatus makes possible the determination, not only dynamically but also statically, of the rigidity of the main suspending wire, for it enables a constant and easily measurable couple to be applied electrically.

§ 7. ACKNOWLEDGMENTS

We would express our gratitude to Prof. A. Griffiths of Birkbeck College, who has done everything possible to encourage us in our difficulties and to provide us with necessary apparatus. We are also grateful to Mr H. G. Bell and to Mr F. Staley of Birkbeck College, on whose mechanical skill we have relied throughout the investigation, and to Mr R. A. Dobb, who has facilitated our work in many ways. We are indebted to Mr R. Belshaw for calculating the Legendre functions and for checking our other tables of special functions, and to Prof. G. Temple of the Imperial College of Science, who has taken great interest in our work and ever been ready to advise us, and to Dr R. G. Cooke of Birkbeck College who has made suggestions to us for further development on the mathematical side.

APPENDIX: TABLES OF FUNCTIONS FOR CALCULATING THE MUTUAL INDUCTANCE FOR SPECIAL CASES OF NON-COPLANAR CIRCLES.

Table 12 gives the values and their logarithms of Legendre functions $P_n(\cos\theta)$, where n is an odd integer, up to $n = 21$ for the angles 89° to 83° inclusive; and of $P_n(\cos\theta)$ and $P_n'(\cos\theta)$ up to $n = 31$ for the angle 79°. The values up to $n = 7$ are those of Tallqvist*. All higher values were specially calculated for us by Mr R. Belshaw in the first place by recurrence formulae, and checked independently by him and by us by the direct expression

$$P_n(\cos\theta) = \frac{4}{\pi}\,\frac{2.4\ldots 2n}{3.5\ldots(2n+1)}\left[\frac{\cos(n\theta+\phi)}{(2\sin\theta)^{\frac{1}{2}}} + \frac{1^2}{2(2n+3)}\,\frac{\cos(n\theta+3\phi)}{(2\sin\theta)^{\frac{3}{2}}} + \frac{1^2.3^2}{2.4(2n+3)(2n+5)}\,\frac{\cos(n\theta+5\phi)}{(2\sin\theta)^{\frac{5}{2}}} + \ldots\right]$$

where $\phi = \theta/2 - \pi/4$.

Table 13 gives the logarithms of functions used in calculating mutual inductance from the expressions of equations (4), (5) and (8) of this paper. The functions of which the logarithms are tabulated are:

$$B = \frac{2}{n(n+1)}\,\frac{1^2.3^2.5^2\ldots n^2}{2^2.4^2\ldots(n-1)^2},$$

$$C = \frac{2}{n(n+1)}\,\frac{1^3.3^3.5^3\ldots n^3}{2^3.4^3\ldots(n-1)^3},$$

$$D = 1 + (n+2)(n-1),$$

$$E = 1 + 10(n+2)(n-1) + (n+4)(n+2)(n-3)(n-1),$$

$$F = 1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-3)(n-1) + (n+6)(n+4)(n+2)(n-5)(n-3)(n-1).$$

* *Acta Societatis Scientiarum Fennicae*, Helsingfors, **33**, 4 (1905), and **33**, 9 (1906).

Table 12. Values of spherical functions

n	P_n (cos 89)	log P_n (cos 89)	P_n (cos 88)	log P_n (cos 88)
1	+ 0·0174524064	$\bar{2}$·2418553	+ 0·0348994967	$\bar{2}$·5428192
3	− 0·0261653202	$\bar{2}$·4177260	− 0·0522429783	$\bar{2}$·7180279
5	+ 0·0326767619	$\bar{2}$·5142390	+ 0·0650650303	$\bar{2}$·8133476
7	− 0·0380725550	$\bar{2}$·5806120	− 0·0755080389	$\bar{2}$·8779932
9	+ 0·0427577	$\bar{2}$·6310143	+ 0·0843585	$\bar{2}$·9261288
11	− 0·0469330	$\bar{2}$·6714783	− 0·0919990	$\bar{2}$·9637831
13	+ 0·0507147	$\bar{2}$·7051339	+ 0·0986456	$\bar{2}$·9940777
15	− 0·0541767	$\bar{2}$·7338125	− 0·1044325	$\bar{1}$·0188357
17	+ 0·0573691	$\bar{2}$·7586781	+ 0·1094486	$\bar{1}$·0392102
19	− 0·0603276	$\bar{2}$·7805160	− 0·1137562	$\bar{1}$·0559751
21	+ 0·0630786	$\bar{2}$·7998820	+ 0·1174010	$\bar{1}$·0696718
n	P_n (cos 87)	log P_n (cos 87)	P_n (cos 86)	log P_n (cos 86)
1	+ 0·0523359562	$\bar{2}$·7188002	+ 0·0697564737	$\bar{2}$·8435845
3	− 0·0781455571	$\bar{2}$·8929043	− 0·1037861291	$\bar{1}$·0161393
5	+ 0·0968786895	$\bar{2}$·9862283	+ 0·1278363599	$\bar{1}$·1066544
7	− 0·1116796606	$\bar{1}$·0479741	− 0·1459810203	$\bar{1}$·1642964
9	+ 0·1236765	$\bar{1}$·0922872	+ 0·1596458	$\bar{1}$·2031575
11	− 0·1334045	$\bar{1}$·1251705	− 0·1694988	$\bar{1}$·2291666
13	+ 0·1411603	$\bar{1}$·1497125	+ 0·1759198	$\bar{1}$·2453147
15	− 0·1471279	$\bar{1}$·1676950	− 0·1791658	$\bar{1}$·2532551
17	+ 0·1514333	$\bar{1}$·1802214	+ 0·1794429	$\bar{1}$·2539263
19	− 0·1541715	$\bar{1}$·1880041	− 0·1769401	$\bar{1}$·2478263
21	+ 0·1554214	$\bar{1}$·1915108	+ 0·1718480	$\bar{1}$·2351445
n	P_n (cos 85)	log P_n (cos 85)	P_n (cos 84)	log P_n (cos 84)
1	+ 0·0871557427	$\bar{2}$·9402960	+ 0·1045284633	$\bar{1}$·0192346
3	− 0·1290784997	$\bar{1}$·1108539	− 0·1539374478	$\bar{1}$·1873443
5	+ 0·1576637203	$\bar{1}$·1977318	+ 0·1860957742	$\bar{1}$·2697365
7	− 0·1778359544	$\bar{1}$·2500195	− 0·2067077744	$\bar{1}$·3153568
9	+ 0·1912894	$\bar{1}$·2816909	+ 0·2177447	$\bar{1}$·3379476
11	− 0·1988401	$\bar{1}$·2985040	− 0·2202513	$\bar{1}$·3429185
13	+ 0·2010073	$\bar{1}$·3032118	+ 0·2150316	$\bar{1}$·3325023
15	− 0·1982155	$\bar{1}$·2971376	− 0·2028794	$\bar{1}$·3072379
17	+ 0·1908789	$\bar{1}$·2807579	+ 0·1846633	$\bar{1}$·2663806
19	− 0·1794384	$\bar{1}$·2539154	− 0·1613523	$\bar{1}$·2077752
21	+ 0·1643764	$\bar{1}$·2158395	+ 0·1340115	$\bar{1}$·1271421
n	P_n (cos 83)	log P_n (cos 83)		
1	+ 0·1218693434	$\bar{1}$·0858945		
3	− 0·1782789647	$\bar{1}$·2511001		
5	+ 0·2128790434	$\bar{1}$·3281329		
7	− 0·2321080669	$\bar{1}$·3656902		
9	+ 0·2382868	$\bar{1}$·3771000		
11	− 0·2328666	$\bar{1}$·3671072		
13	+ 0·2172037	$\bar{1}$·3368672		
15	− 0·1927956	$\bar{1}$·2850971		
17	+ 0·1613402	$\bar{1}$·2077426		
19	− 0·1247175	$\bar{1}$·0959274		
21	+ 0·0849331	$\bar{2}$·9290770		
n	P_n (cos 79)	log P_n (cos 79)	P_n' (cos 79)	log P_n' (cos 79)
1	+ 0·1908089954	$\bar{1}$·2805988	+ 1·0000000000	Zero
3	− 0·2688460236	$\bar{1}$·4295036	− 1·2269394546	0·0888231
5	+ 0·2989725190	$\bar{1}$·4756313	+ 0·971481534	$\bar{1}$·9874346
7	− 0·2913338271	$\bar{1}$·4643909	− 0·315154223	$\bar{1}$·4985231
9	+ 0·2526096	$\bar{1}$·4024498	− 0·614849037	$\bar{1}$·7887685
11	− 0·1905053	$\bar{1}$·2799071	+ 1·641528	0·2152483
13	+ 0·1141450	$\bar{1}$·0574569	− 2·562565	0·4086749
15	− 0·0334361	$\bar{2}$·5242156	+ 3·183962	0·5029679
17	− 0·0419178	$\bar{2}$·6223985	− 3·354408	0·5256158
19	+ 0·1034638	$\bar{1}$·0147884	+ 2·993787	0·4762209
21	− 0·1449163	$\bar{1}$·1611172	− 2·110615	0·3244090
23	+ 0·1628042	$\bar{1}$·2116656	+ 0·805062	$\bar{1}$·9058293
25	− 0·1567627	$\bar{1}$·1952428	+ 0·743397	$\bar{1}$·8712208
27	+ 0·1294338	$\bar{1}$·1120478	− 2·30214	0·3621317
29	− 0·0859955	$\bar{2}$·9344757	+ 3·62147	0·5588849
31	+ 0·0333865	$\bar{2}$·5235709	− 4·47502	0·6507949

A function A used in equation (7) is defined thus:

$$A = \frac{2}{n(n+1)} \frac{1.3.5 \ldots n}{2.4 \ldots (n-1)},$$

and is easily deduced from the relation

$$\log A = 2 \log B - \log C.$$

In every case n is an odd integer.

Table 13. Logarithms of special functions

n	log B	log C	log D	log E	log F
1	Zero	Zero	Zero	Zero	Zero
3	$\bar{1}$·5740313	$\bar{1}$·7501225	1·0413927	2·0043214	2·9595184
5	$\bar{1}$·3699113	$\bar{1}$·6429126	1·4623980	2·8948697	4·3051148
7	$\bar{1}$·2327381	$\bar{1}$·5726861	1·7403627	3·4649364	5·1756596
9	$\bar{1}$·1289887	$\bar{1}$·5200892	1·9493900	3·8890214	5·8196025
11	$\bar{1}$·0454426	$\bar{1}$·4779359	2·1172713	4·2279124	6·3322427
13	$\bar{2}$·9754694	$\bar{1}$·4427248	2·2576786	4·5105584	6·7588045
15	$\bar{2}$·9152560	$\bar{1}$·4124746	2·3783979	4·7531616	7·1243848
17	$\bar{2}$·8624037	$\bar{1}$·3859512	2·4842998	4·9657519	7·4444134
19	$\bar{2}$·8153037	$\bar{1}$·3623323	2·5786392	5·1549866	7·7290792
21	$\bar{2}$·7728239	$\bar{1}$·3410418	2·6637009	5·3255177	7·9854753
23	$\bar{2}$·7341371	$\bar{1}$·3216602	2·7411516	5·4807268	8·2187430
25	$\bar{2}$·6986204	$\bar{1}$·3038722	2·8122447	5·6231511	8·4327315
27	$\bar{2}$·6657928	$\bar{1}$·2874350	2·8779470	5·7547435	8·6303987
29	$\bar{2}$·6352752	$\bar{1}$·2721574	2·9390198	5·8770400	8·8140677
31	$\bar{2}$·6067637	$\bar{1}$·2578863	2·9960737	5·9912708	8·9855969

DISCUSSION

Prof. A. Griffiths. The work has been carried out under the very great difficulties imposed by the limited accommodation available at Birkbeck College. The method seems to me to be good, and I venture to hope that it will be tested to a higher degree of accuracy at some larger laboratory.

Dr G. Temple. The theory of this paper centres around the series

$$M = Gq \sum_{n=1}^{\infty} \frac{2}{n(n+1)} \left(\frac{s}{r}\right)^{n-1} P_n'(\cos\psi)\, P_n'(\cos\phi)\, P_n(\sin\theta) \quad \ldots\ldots(2),$$

and the derived series which form the coefficients of powers of θ in equations (5) and (8). Dr Nettleton has estimated the sums of these series by the direct calculation and addition of the first sixteen terms, i.e. up to the term for which $n = 31$, n being an odd integer. It is therefore important to know an upper bound to the magnitude of the remainders.

These values can be obtained from the following inequality,*

$$|P_n^m(\cos\theta)| \cdot \sin^m\theta < \frac{\Pi(n+m)}{\Pi(n)} \left\{\frac{4}{n\pi \sin\theta}\right\}^{\frac{1}{2}},$$

* E. W. Hobson, *The Theory of Spherical and Ellipsoidal Harmonics*, §§ 200–202 (Cambridge University Press, 1931). There is a misprint in equation (29) of page 313.

where $0 < \theta < \pi$, $n - m + 1 > 0$, and m is a positive integer or zero. $\Pi(n)$ is Gauss's Π-function which reduces to $n!$ when n is integral. The particular cases of this inequality required in the discussion of the series (2) are

$$| P_n(\cos\theta) | < 2(n\pi \sin\theta)^{-\frac{1}{2}}$$

and

$$| P_n'(\cos\theta) | \cdot \sin\theta < 2(n+1)(n\pi \sin\theta)^{-\frac{1}{2}}.$$

Now, according to Hobson's definition of $P_n^m(\cos\theta)$,*

$$P_n^1(\cos\theta) = -\sin\theta \cdot P_n'(\cos\theta).$$

Hence

$$| P_n^1(\cos\theta) | < 2(n+1)(n\pi)^{-\frac{1}{2}} \sin^{-\frac{1}{2}}\theta.$$

It follows that if a_n is the term of order n in the series (2), then a_n

$$| a_n | < C_n \cdot n^{-\frac{3}{2}} \cdot x^{n-1},$$

where

$$x = s/r, \qquad x$$

and

$$C_n = 8(n+1)n^{-1}\pi^{-\frac{3}{2}} \cdot \sin^{-\frac{5}{2}}\psi \cdot \sin^{-\frac{5}{2}}\phi \cdot \cos^{-\frac{1}{2}}\theta. \qquad C_n$$

Hence the magnitude of the remainder R_N, where R_N

$$R_N = \sum_{n=N}^{\infty} a_n,$$

is less than

$$C_N \sum_{n=N}^{\infty} n^{-\frac{3}{2}} x^{n-1}.$$

It may similarly be shown that the estimation of the remainders in the series which form the coefficients of θ, θ^3 and θ^5 in equation (8) depends upon allied series of the form

$$\sum_{n=N}^{\infty} n^{-s} x^{n-1},$$

where $s = \frac{1}{2}, -\frac{3}{2}$ and $-\frac{7}{2}$.

The order of magnitude of the sums of these series can be inferred from the Eulerian integral

$$\Pi(s-1) = \int_0^{\infty} t^{s-1} e^{-t}\, dt$$

if $s > 0$, or from Hankel's contour integral for the gamma function if $s < 0$. We have that

$$\Pi(s-1) \cdot n^{-s} x^n = \int_0^{\infty} t^{s-1} (xe^{-t})^n\, dt,$$

whence

$$\Pi(s-1) \sum_{n=N}^{\infty} n^{-s} x^n = \int_0^{\infty} t^{s-1} \sum_{n=N}^{\infty} (xe^{-t})^n\, dt$$

$$= \int_0^{\infty} \frac{t^{s-1}(xe^{-t})^N}{1-(xe^{-t})^2}\, dt,$$

since $0 < x < 1$, and the summation is over odd values of n.

Now

$$1 - (xe^{-t})^2 > 1 - x^2.$$

Hence

$$\Pi(s-1) \sum_{n=N}^{\infty} n^{-s} x^n < \int_0^{\infty} \frac{t^{s-1}(xe^{-t})^N}{1-x^2}\, dt$$

$$= \frac{\Pi(s-1)}{1-x^2} \cdot N^{-s} x^N.$$

* Equation (4), page 90.

Therefore $$\sum_{n=N}^{\infty} n^{-s} x^n < \frac{x^N}{(1-x^2) N^s},$$

and the magnitude of the remainder R_N is less than

$$\frac{C_N \cdot x^{N-1}}{(1-x^2) N^{\frac{1}{2}}}.$$

Dr D. Owen. The authors have in this paper presented the Society with the results of a very careful and valuable piece of work. It is interesting to observe how the mutual inductance of a pair of coils can by proper choice of dimensions be made a linear function of their angular position. But the essential feature of the paper lies of course in the transformation, with undoubted gain, of the original Weber damping method. It is amazing, on the face of it, that a method depending on two dynamical observations, namely, of a time of oscillation and a logarithmic decrement, should be in the running for a high place among the means of accurate absolute determination of resistance. In this paper we see what good results may be obtained with no specialized experimental facilities at disposal.

Looking at the figures I am puzzled to discover the records of the times of oscillation with the vibrating coil on closed circuit—only λ, but not T, appears to be given in tables 5, 6 and 10.

Mr J. Guild said that the logarithmic decrement depends on variable factors, such as barometric pressure and the properties of the suspension, which would need to be taken into account for precision measurements.

Dr L. Hartshorn (*communicated*): The work described is very interesting as extending the range of possibilities of the method of measuring electrical resistance by observation of the damping of mechanical vibrations; and the question is revived whether, with the elaboration outlined in the paper, the method could compete with others for the absolute measurement of resistance. From experience of precision measurements of the kind involved, I cannot help feeling that the method of damping suffers from severe disadvantages, even in the improved form described. At the National Physical Laboratory, precision measurements have been made by the Lorenz method and by Albert Campbell's latest method. The Lorenz method requires only a mutual inductance and a speed of rotation, while the Campbell method requires only two mutual inductances, which can be equal, a frequency, and the ratio of two resistances. As against these, the present method requires two logarithmic decrements, a period of oscillation, a mutual inductance, and the ratio of two currents which cannot be observed simultaneously. No deflections have to be observed in either the Lorenz or the Campbell methods: each is a null method, and the mutual inductance involved is that of a rigid system, instead of one with a comparatively delicate suspension. These points are probably sufficient to show that for standardizing work, in which an accuracy of a few parts in 100,000 is essential, the present method could hardly compete with the best available. Probably its greatest asset is the simplicity of the apparatus required for a determination with moderate accuracy, say to a few parts in 10,000.

AUTHORS' reply. We thank Prof. A. Griffiths for his kind remarks and Dr G. Temple for the trouble he has taken in investigating the rapidity of the convergence of the infinite series on which our calculations depend.

In reply to Dr Owen: the question of the possible accuracy of measurement of $(\lambda - \lambda_0)$ strikes admittedly at the root of the method. Unfortunately our oscillating coils, owing to slight warming from the current-bearing field-coils, were usually slowly rising in resistance and consequently λ under the same current showed a slow and continuous fall. By changing from a higher to a lower current we often obtained periods of steady temperature, as will be seen in tables 5 and 6, and under such conditions our logarithmic decrements were usually consistent to 1 part in 5000. That logarithmic decrements are capable of measurement with an accuracy exceeding ours will be seen from the data of Dorn,* who was handicapped by the change of period with arc of oscillation. When a large number of readings are taken during a decrement the value may be estimated with great accuracy, the various methods of estimating graph-slope yielding identical results, as the abscissae are successive integers.

In reply to Mr Guild: while the constancy of external conditions he mentions would be most desirable in future work, we point out that the measurement of λ takes only 3 or 4 min. and the resistance of the oscillating coil is compared both before and after with a standard resistance. The statical part of the experiment may, as was shown in the paper, be performed later.

We entirely agree with the communicated remarks of Dr L. Hartshorn and are under no delusion that the method could ever compete for precision with the methods he alludes to. We have merely improved a very old and interesting method by removing quantities exceptionally difficult to measure and corrections difficult to apply. Nevertheless, in view of the heaviness of the system and the remarkably close approach to true damped-simple-harmonic motion over considerable amplitude, and the fact that the resistance measured may be large, we confess, especially in the light of the results obtained by Dorn, that we hope that some day the method may be pushed to its limits under more favourable conditions on the lines we suggest. Apart altogether from the absolute measurement of resistance, the linear law obtained is of interest and there are, perhaps, other possibilities for such a mutual-inductance dynamometer.

* *Wied. Ann.* **36**, 22, 398 (1889).

THE ELECTROSCOPE CAPACITY BALANCE

By E. S. BROWN, D.Sc., A.C.G.I.

Communicated by Prof. T. H. Laby, October 14, 1931. *Read in title, January* 15, 1932

ABSTRACT. Capacities of a few micromicrofarads are measured by means of two electroscopes connected in series with a source of e.m.f. and shunted by variable condensers. The intersection of the gold leaves lies on a straight line so long as the capacities of the condensers are equal. The unknown capacity is shunted upon one of the condensers, the capacity of which is then adjusted till the sum of the two capacities is equal to the capacity of the remaining condenser.

THIS apparatus was developed by the author for the rapid comparison of small capacities of the order of a few micromicrofarads. Ordinary alternating current from supply mains of any frequency is used, and the accuracy is not affected by any variations of frequency, wave-shape or voltage. The apparatus may be easily and cheaply constructed, most of the requisites being found in any electrical laboratory.

Figure 1 shows a diagram of connections. Two single-leaf gold-leaf electroscopes A and B are connected in series across the secondary of a small step-up transformer giving about 1000 volts. The crossed images of the gold leaves of A and B are optically projected on a vertical screen as shown in figure 2.

The gold-leaf terminals of the two electroscopes are connected together, and the cases of the instruments are connected to the terminals of the transformer. In parallel with electroscope A, which has its case earthed, is connected a screened standard graduated variable condenser K_A having its screen connected to the earthed side of the electroscope A. In parallel with the electroscope B a condenser K_B is connected with its screen connected to the high tension terminal of the transformer. K_B need not be graduated and is employed merely as a ballast condenser.

Any unknown capacity X which is to be measured may be connected in parallel with K_A or disconnected from it at will by means of a low-capacity switch. This may conveniently consist of a fine springy wire which may be caught on the terminal of X or released by touching it with an ebonite rod so that it springs clear of X.

With the connections of figure 1, when the voltages across the two electroscopes are equal, while the total voltage is varied, the locus of the intersections of the crossed images of the leaves is a line shown on the screen in figure 2 and denominated the *equivolts line*. This line may be obtained by connecting the electroscopes in parallel and varying the applied potential-difference. When the image of the intersection of the leaves falls on the equivolts line the voltages applied to the electroscopes must be equal.

If this condition is satisfied when connections are made as in figure 1, the combined capacities of electroscope A and its parallel condenser K_A must be equal to the combined capacities of the electroscope B and its parallel condenser K_B. The intersection of the leaves may then be moved to right or left of the equivolts line by alteration of either condenser, or can be set exactly on the equivolts line. It is then known that the voltages on both sides and hence the capacities on both sides are equal because of their series connection.

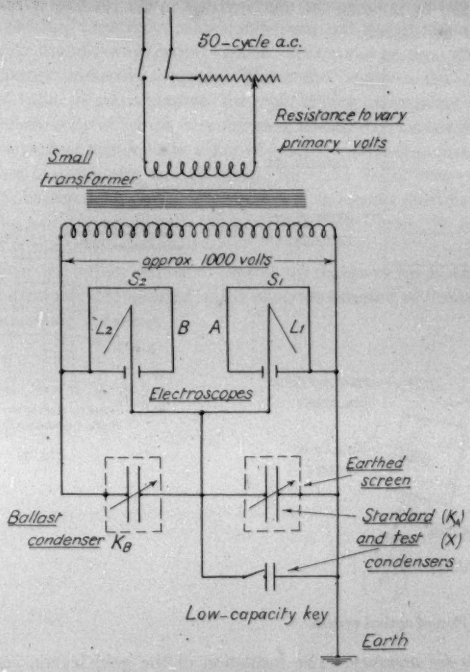


Fig. 1. Connection diagram.

The apparatus may be likened then to a balance with equal arms. The addition or removal, in parallel with the standard K_A, of any small capacity X which is to be measured will disturb the balance, which may be restored by alteration of the variable standard condenser K_A. Thus the value of the small capacity may be read off by the change of reading of the standard required to restore balance.

As suggested by my assistant, Mr E. O. Willoughby, it is easy to obtain a number of cumulative readings of any very small capacity X as follows. X is

connected in parallel with K_A; K_B is adjusted to bring the image of the leaves on to the equivolts line. X is then disconnected and balance is restored by increasing the capacity of K_A by an amount equal to X. Without any alteration of K_A, X is now reconnected and K_B is altered to restore balance again. X having been disconnected once more, it is necessary to further increase the capacity of K_A to restore balance, the increase giving a second value of X. This process may be carried on for any number of steps required. An average value of the capacity X can then be obtained by dividing the total increase in the reading of the standard by the number of steps taken.

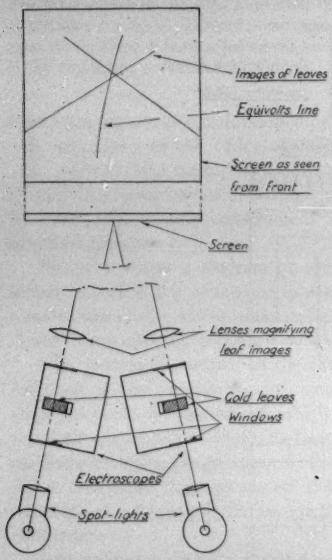


Fig. 2. Plan of optical system.

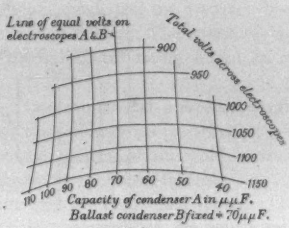


Fig. 3.

Precautions for accuracy. The insulation of the gold leaves, condensers and parts connected electrically thereto must be high to avoid errors due to leakage. Care must be taken to avoid hand and body-capacity effects on any parts of the leaf connections which are unshielded. Such effects may be at once detected by movement of the leaves. In order to get the greatest sensitivity the residual capacity of connecting wires, condensers and electroscopes should be kept as small as possible. It is suggested that as a safety measure a high resistance should be inserted in the lead from the h.t. terminal of the transformer.

The author considers that the apparatus described is of value for instructional purposes. Its use reveals to the experimenter and student the existence of small

inter-capacities which are often unsuspected or ignored, and facilitates their measurement. Many instructive and interesting experiments may be performed; the capacity of small wires, spheres, etc. suspended in air may be measured, and also the variation of these capacities due to distance from the earth or other bodies. The capacity of an average human being standing on an insulation stool about 5 ft. from the ground is found to be about 70 $\mu\mu$F. As the distance to earth has little effect, this value must approximate to the self-capacity of the human body.

The apparatus may be used for measuring the capacity of terminals or small switches, the residual capacity of microcondensers and the small capacities which are of importance in short-wave radio practice. It may also be used for rapid determination of specific inductive capacity. The author suggests that the apparatus may also be of value to manufacturers for rapid testing and sorting of small fixed condensers, the capacity of which is required to lie between certain limits. Application of the condenser terminals to a pair of contacts determines instantly whether the capacity lies between any desired limits; for these limits lines may be drawn in the diagram, between which the intersection of the leaves must lie; and further, if the screen be suitably graduated, the percentage error from the standard value may be instantly read off.

In conclusion, the author wishes to express his thanks to Mr E. O. Willoughby, who drew the diagrams and assisted in the experiments, and to Professor Laby for kindly communicating the paper.

DEMONSTRATIONS

"Apparatus for the measurement of relative humidity." *Demonstration given on December* 4, 1931, *by* Mr J. H. ORCHARD

While an immense amount of research work has been carried out on the problem of humidity-measurement, and many instruments have been devised, it appears to be generally considered there is still room for improvement in the apparatus for the determination of humidity on a commercial basis, and it is hoped that the instrument demonstrated is a step in the right direction. It uses the principle of the wet-and-dry-bulb platinum-resistance thermometer and has two main points of interest. The first is the form of thermometer adopted and the second is the means by which direct readings in percentage of relative humidity are secured without reference to humidity tables.

The thermometer consists of an enamelled and silk-insulated platinum wire, $0{\cdot}002_5$ in. in diameter, adjusted to have a fundamental interval of 40 ohms. This wire is doubled and drawn through copper capillary tubing, the bore of which is only sufficiently large to allow the doubled wire to pass without damage. The two ends of platinum wire are connected to small terminals moulded into a capsule and soldered on to one end of the capillary tubing, the other end of which is soldered up an inch or so beyond the looped end of the platinum wire. A thermometer so constructed has the following advantages: (i) it is hermetically sealed; (ii) it can be coiled into any shape; (iii) it can be copper or nickel-plated, or otherwise protected against corrosion; (iv) the platinum wire is not stressed and yet is entirely protected; (v) there is small heat-mass and large area, and thus a very small time-lag; (vi) the conduction error is extremely small.

To adapt this type of thermometer to the wet bulb of a hygrometer presented some difficulties, but in the model shown an artificial-silk sleeving has been slipped over the copper tubing, and the water is allowed to run down from an overhead supply instead of being drawn up by capillarity.

Immediately the preliminary difficulties had been overcome, this form of thermometer showed great advantages, for the purpose of humidity-measurement, over other forms with which I am familiar. The construction enables any number of wet-bulb thermometers to be made with the certainty that they will be identical as regards lag, self-heating and conduction error. The large area exposed for evaporation ensures that the thermometer is less subject than the majority to the effect of air-velocity, the depression not being affected above velocities of 1 m./sec. as compared with the more generally accepted figure of 3 m./sec. These remarks apply to the thermometer when the tubing is stretched out straight, which is the condition under which the writer has carried out all the tests illustrated on the records exhibited. When the thermometer is coiled up this good feature is naturally sacrificed, as a zone of humidity is formed which must be dispersed by the movement of air if true depression is to be obtained.

For the purpose of demonstration a portable instrument was shown, with a thermometer unit consisting of both wet-and-dry-bulb thermometers on one frame and without a water supply, the sleeving being wetted by submerging the wet bulb in water. The reading is obtained so quickly that under normal atmospheric conditions it is safe to rely on the wick remaining wet for a sufficiently long time. After the test position has been correctly set, to compensate for any variation of voltage in the dry cell, the switch is moved to the stud marked "Dry thermometer" and the large circular dial of the rheostat is set to the reading thus obtained. The switch is then moved to the humidity position and the percentage relative humidity is read directly on the lower of the two scales. The setting of the rheostat has adjusted the sensitivity of the galvanometer to that necessitated by the dry-bulb temperature, so that measurement of the depression of the wet bulb will enable relative humidity to be read direct. In the recording instrument this adjustment of the rheostat is done automatically by means of a relay mechanism.

As the actual contour of the humidity curve varies slightly with the dry-bulb temperature, and the rheostat can only be adjusted to suit one particular contour, an error is necessarily introduced by departure from the master curve selected. This error, however, is not greater than 2 per cent. relative humidity over a range of from 30 to 100° F.

"Some effects produced when liquid jets of low velocity fall on a barrier." *Demonstration given on December* 4, 1931, *by* J. H. BRINKWORTH, A.R.C.S., D.I.C., D.Sc.

So very many investigations, both experimental and theoretical, have been made on liquid jets and their properties that it is, perhaps, rather surprising that the effects which are to be described have not been previously recorded. If the flow from a nozzle is restricted to such an extent that a continuous column of liquid is just obtained, and if this jet is allowed to fall on a barrier, it takes up a beaded form somewhat like a capillary surface of revolution. These beaded jets showing stationary vibrations are, in general, perfectly stable, provided that the barrier is such that the liquid can flow away freely from the region of impact of the jet, i.e. that the barrier is a stream-line surface for the source on its periphery.

By altering the position of the barrier the changes in form of the jet can be readily observed. In all the experiments shown the nozzle was a circular orifice of diameter about 1 cm., the barrier a glass sphere diameter 5 cm., and the flow of the order of 1·5 gm./sec.

It is of interest to compare the shape of these beaded jets in which a flow is maintained with the "strictly theoretical solutions of the configuration of a liquid under the influence of gravity and supported by a solid," i.e. under static-equilibrium conditions. Some of the latter were worked out graphically by Prof. J. Perry

about the year 1874, when he was a student under Lord Kelvin at Glasgow University. Illustrations of some of these original drawings will be found in Kelvin's *Popular Lectures and Addresses*, volume 1.

Figures 1 and 2 (*a*, *b*, *c*), reduced to the same nozzle diameter, illustrate the differences between the outlines of the theoretical static drop and the dynamical jet obtained under the experimental conditions referred to above.

For certain very definite positions of the barrier the jets exhibited in the present demonstration maintain vibrations transverse to the length of the jet. The outlines, figure 3 (*a*, *b*), show the minimum and maximum horizontal amplitudes of vibration in two cases. In other positions of the barrier the jet will show maintained longitudinal vibrations.

The thanks of the author are due to Mr F. C. Daniels for the excellence of figures 2 and 3.

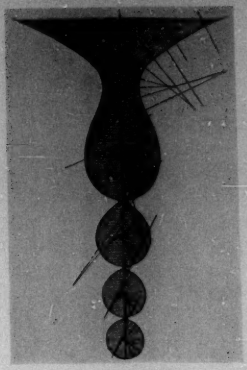


Fig. 1. Static.

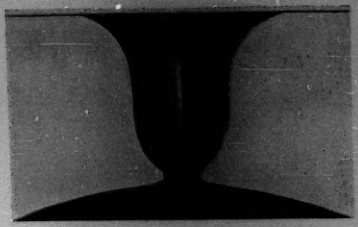


Fig. 2 (*a*).

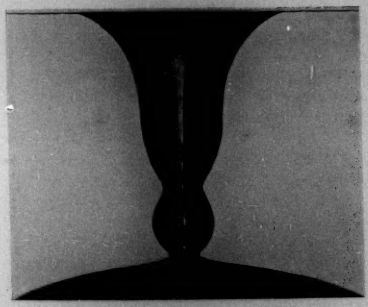


Fig. 2 (*b*).

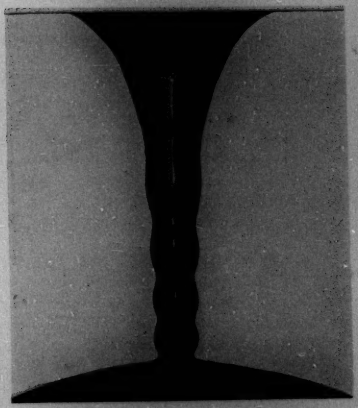


Fig. 2 (*c*).

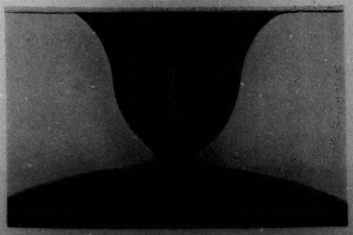


Fig. 3 (*a*).

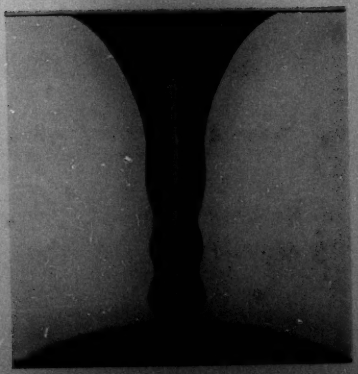


Fig. 3 (*b*).

REVIEWS OF BOOKS

Problems of Modern Physics, by H. A. LORENTZ. Pp. vi + 312. (Boston, U.S.A.: Ginn and Co.) 12*s*. 6*d*.

The volume, published in 1927, is a record of lectures delivered by the late Prof. Lorentz in 1922, and naturally raises the question as to what may be called modern. We know that New College, Oxford, is still new after 550 years, and that the modern girl has been modern for a good while now. However, it seems clear that a time is ultimately reached at which *new* and *modern* cease to be applicable terms, as witness the case of the new woman. To what extent do these examples find their counterpart in physics? Is the subject of Lorentz's book the analogue of the new woman, with the new physics (born in 1925) taking the place of the modern girl? Both, at any rate, have been accused of a principle of indeterminacy, but there the resemblance appears to end.

The problems which Lorentz considers centre round two points, whether an accelerated electron must of necessity radiate, and whether light is corpuscular or a wave-motion.

The first problem has been solved, in the years which have passed since the lectures were delivered, by the expedient of abolishing the accelerated electron itself in the sense understood at that time; but, none the less, the discussion retains its interest.

In treating the second question, Lorentz comes very near to our present solution of the problem, when he refers to the possibility of considering Maxwell's electro-magnetic field as a ghost or shadow field, which serves to guide the photons in their motion. Unfortunately, he gets into difficulties in deciding how to apportion the energy between the field and the photons, and turns away to discuss other possibilities which might lead to solutions of the problem. This part of the work includes a very acute treatment of the classical theory of light as founded on Huyghen's principle, a section which is likely to be of permanent value.

I have said that the lectures group themselves about the discussion of two main problems, but it is natural that in the course of these discussions many short investigations should have to be carried out, and these are always done with great clearness. Their presence makes the book into a valuable reference work, as well as an exceptionally instructive example of how a master sets about solving a problem. Among these investigations are such problems as those of light-scattering, group-velocity, and a discussion as to whether the rotating-mirror method for the velocity of light measures group-velocity or wave-velocity.

Sandwiched between the main discussions, there is a delightfully clear account of the principles of the Einstein relativity theory. This is not a problem of modern physics in the sense of the other two matters dealt with, and (perhaps for that reason) it is likely to be of more permanent value to students.

No physicist who reads this book will waste his time in doing so, but perhaps the examination student had better wait till he has got his degree, since much of the material will not help in passing examinations.

It ought to be mentioned that a book-mark will be essential, because the volume is not divided into chapters, and does not indicate where one lecture ended and the next began. Moreover, the book-mark is required every few pages for another reason. Approximately half the book consists of notes which generally elaborate a mathematical argument merely sketched in the text; these notes are collected together at the end of the volume, making one's progress a somewhat jerky one. This, however, is a minor disadvantage,

and one which readers of the same author's *Theory of Electrons* have long ago learned to deal with.

The final word must be one of thanks to Prof. H. Bateman of the California Institute, without whose editorship it appears that the book might never have been published. He has added a few notes in places, in a noble effort to bridge the gap between 1922 and 1927.

J. H. A.

The Combination of Observations, by DAVID BRUNT. Pp. x + 239. (London: Cambridge University Press.) 12*s*. 6*d*.

Those of us who have, for some fifteen years past, industriously thumbed our copy of Mr Brunt's book know very well that it is one of the most reliable of its kind, written as it is by an author who combines wealth of knowledge with the teacher's instinct and is not afraid to begin at the beginning.

The subject is a fairly stable one, and much alteration in this new edition was neither to be expected nor desired. We find, therefore, the old bill of fare almost unaltered. The law of error; the case of one unknown; weighting; the adjustment of indirect observations containing several unknowns; most probable values and probable errors; criteria of rejection; alternatives to the normal law; correlation; harmonic analysis; the periodogram —these were, and are, the principal topics discussed. The chapter on alternatives to the normal law of error has been amplified, and the chapter on correlation now contains a section dealing with multiple correlation. The section dealing with harmonic analysis has been re-written, and the application of the method to the periodogram has been revised.

These are the principal changes to record, and it will be seen that they increase the usefulness without unduly enlarging the scope of the book, which remains one of the most useful elementary expositions of the orthodox theory in print.

The measure of Mr Brunt's orthodoxy may be gauged from the observation that the name of Campbell does not appear in his index of authors.

A. F.

London and the Advancement of Science, by various authors. Pp. vi + 321. (London: The British Association for the Advancement of Science.)

The British Association—A retrospect, 1831–1931, by O. J. R. HOWARTH, O.B.E., M.A., Secretary to the British Association. Pp. vii + 330, with 21 illustrations. (London: The British Association for the Advancement of Science.)

The first of these two books is the outcome of a happy suggestion by Dr Allan Ferguson that, instead of the usual handbook descriptive of the town and neighbourhood in which the Association is meeting, there should be prepared for the centenary meeting a book giving some indication of the part which London and London residents have played in the advancement of science. The field to be surveyed is truly stupendous and one cannot but admire the courage shown by the members of the Council when they decided to adopt the suggestion. The result is a book which rivals in usefulness and interest any previous publication of the Association. Considering further that the whole project was carried through in less than twelve months, we must proffer to all those concerned in the enterprise our very sincere congratulations.

The story opens with a very unorthodox survey by Dr Ferguson. This is written round the work of Chaucer, who is introduced to us in the unfamiliar garb of astronomer, Gilbert, Harvey, Bacon (an admirable summary here), Wren, John Hill, Henry Cavendish and William Hyde Wollaston. Varied fare, indeed, but one which the Londoner cannot but regard with mixed feelings. John Hill, for whom Dr Ferguson has obtained a testimonial

from Prof. G. T. R. Hill, was a person to provoke our interest rather than our pride. Neither his *Vegetable System* in 26 folio volumes nor his herb garden on the site of Lancaster Gate earned him the esteem of his contemporaries. Christopher Smart labelled him "Pimp! Poet! Paffer! 'Pothecary! Player!," but then Smart's feelings had been hurt. Johnson merely said of him that "he was an ingenious man, but had no veracity." A striking figure, well deserving of 9 columns in the *Dictionary of National Biography*, but not of a tenth of Dr Ferguson's survey.

Cavendish, too, hardly shows the London scientist at his best. Certainly none of his predecessors had excelled him in experimental skill and few did as much to advance knowledge of physical science, but none was more indifferent to any desire the rest of humanity might have to share that knowledge. That his natural eccentricity was much increased by his daily diet of mutton we can well believe; but why give him six pages to the exclusion of men who gave the world the benefit of their talents?

What of that much maligned genius Robert Hooke; Boyle, who lived for 23 years in Pall Mall; Flamsteed, so ill-used (if we may believe his own account) by Newton; Halley, who first persuaded a British government to spend money on a scientific expedition; Samuel Molyneux, whose telescope at Kew revealed the aberration of light; Stephen Gray of Charterhouse; William Watson, once apothecary; John Canton, the Spital Square school-keeper; Stephen Hales, Teddington curate and founder of plant physiology? Perhaps, after all, Dr Ferguson was wise: the list is endless, space was short and time still shorter.

Following the introductory survey are two chapters descriptive of the formation and aims of all the more important scientific societies; an exceptionally interesting historical account of education in London by T. L. Humberstone; a chapter by Sir Frank Heath summarizing the activities of the Government in connexion with scientific research; an account of the development of medicine in London by H. H. Bashford; sections descriptive of the London museums past and present, of the Royal Observatory, Kew Gardens, and the John Innes Horticultural Institution; finally an all too brief *History of the London Makers of Scientific Instruments*. This last chapter contains many interesting facts, new at any rate to the reviewer, but is somewhat marred by the unaccountable absence of any reference to Thomas Tompion of Water Lane, Fleet Street, and his famous pupil George Graham, both buried in the same grave in Westminster Abbey.

There is little space left for notice of the second volume. It is indeed only a reprint of the *Retrospect*, published in 1922, but brought up to date, in particular, by the addition of a chapter describing Down House and its acquisition by the Association. It still contains, by implication, the statement that the late Sir Henry Roscoe was appointed President of the Association at the phenomenally early age of 5 years.

D. O. W.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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THE RAPID DETERMINATION OF MOISTURE IN SEEDS AND OTHER GRANULAR SUBSTANCES

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ABSTRACT. Methods previously available for the determination of the moisture-content of seeds are summarized and discussed. Two new methods are described, the first involving the use of a thermionic oscillator, the second a direct-current galvanometer and a battery. The first method is best suited for use with large-grained seed, whilst the second can be used for all types of seed.

§ 1. INTRODUCTION

THE work described in this paper was undertaken at the request of the Welsh Plant Breeding Station, Aberystwyth. Part of the work of this Station consists of large-scale production of pedigree grass and clover seeds, and since the keeping and germinating properties of seeds are largely dependent on their moisture-content, it is necessary to carry out, each season, a large number of moisture-content determinations. Hitherto these determinations have been carried out by the heat-drying method, which consists in grinding the seed and then heating it to about 98° C. until constant weight is obtained; the moisture-content can then be calculated from the initial and final weight of the seed taken. The disadvantages of this method are (i) that it requires a number of accurate weighings and (ii) that it is incapable of being carried out rapidly.

The aim of the experiments described below was to develop an instrument which would enable the moisture-content of seeds to be determined rapidly by a person who has not received any special training in accurate observation. It is thus necessary that the instrument, in addition to giving a reasonably accurate result rapidly, should be simple in principle, direct-reading, permanent in calibration and free from any complicated preliminary adjustments; in addition, it should preferably be portable.

§ 2. METHODS AVAILABLE FOR MOISTURE-CONTENT MEASUREMENT

The methods used nowadays for moisture-content measurement can be divided into two main classes, electrical and non-electrical. The latter class includes such methods as (i) the heat-drying method, described above; (ii) the oil-distillation method, sometimes called the Brown-Duval method (this consists in soaking the seed in a liquid which is immiscible with water and whose boiling point is higher than that of water; the mixture is then distilled and the moisture-content is determined from the relative proportions of oil and water in the distillate); (iii) the acetylene method. In this the moisture-content of the seeds is calculated from the amount of acetylene generated on the mixing of a known weight of seed with calcium carbide.

Two electrical properties appear to be suitable as a basis for moisture-content measurement—dielectric constant and resistivity; both these quantities should vary with moisture-content. Two electrical methods have been published—the Burton-Pitt method[1] and the Berliner-Rüter "D.K." method[2].

The Burton-Pitt method depends on the change in the d.c. component of the anode current of an oscillating thermionic valve, the change being brought about by inserting a glass vessel filled with seed in the oscillatory-circuit coil. The change in direct anode-current may be caused by (*a*) a change in the effective inductance and resistance of the coil due to eddy currents induced in the seed, or (*b*) a change in the self-capacity of the coil due to change in the dielectric constant of part of its surroundings. Calculation shows that the change in the effective resistance and inductance of a coil, due to the insertion of a core of resistivity equal to that of seed, is very small indeed. In fact, Belz[3], using the sensitive heterodyne-beat method, has shown that this eddy current effect is negligible even at radio frequencies for a highly conducting liquid such as concentrated sulphuric acid, provided that the coil is electrostatically screened from the core. Burton and Pitt apparently did not use an electrostatic screen and hence it is probable that the effect in their case is due to change in dielectric constant.

The Berliner-Rüter method depends directly on variation of dielectric constant with moisture-content. The dielectric constant is measured at radio-frequency by the following method. A valve oscillator is coupled through an intermediate aperiodic circuit to a low-decrement circuit of fixed capacity and fixed inductance. The intermediate circuit contains a sensitive current-indicator, by means of which resonance between the oscillator and the fixed-frequency circuit can be detected. By variation of its capacity the oscillator is initially tuned to the fixed-frequency circuit. A parallel plate condenser containing the seed is then placed in parallel with the oscillatory circuit, whose capacity is then varied until resonance is restored.

These two methods give rapid results, the time required for a moisture-content determination being a minute or so. The Burton-Pitt method suffers from two disadvantages: (i) great sensitivity is difficult to obtain without the use of a d.c. amplifier or an arrangement for balancing out the initial steady anode current;

(ii) the preliminary adjustments to repeat calibration conditions are somewhat complicated. It has the advantage of giving direct readings on a pointer instrument, and can thus be used by unskilled persons. The main disadvantage of the Berliner-Rüter method is that it requires an adjustment for resonance, and is thus not so simple to use as the Burton-Pitt method. The permanency of calibration of the Berliner-Rüter method is probably better than that of the Burton-Pitt, since the former is, to a certain extent, a null method. Another disadvantage of the Berliner-Rüter method is that the sharpness of the resonance-indication diminishes rapidly as the moisture-content increases, owing to the resistance of the seed-vessel becoming more comparable with its reactance. This disadvantage can be countered by increasing the frequency, but this, in turn, increases the disturbing effects of stray capacities. Both methods suffer from the drift effects usually associated with valve oscillators.

It should be noted that all methods described above, with the exception of the heat-drying method, are empirical and thus require calibration by means of samples whose moisture-content is determined by the latter method. Furthermore, a separate calibration is required for each type of seed to be tested. These remarks are equally true as regards the methods developed by the author, which will now be described.

§ 3. THE VALVE-OSCILLATOR METHOD

This method probably involves the dependence of both dielectric constant and resistivity on moisture-content. It had been developed before the work of Berliner and Rüter had been brought to the author's notice, and since it differs from the latter in its mode of action it will be described briefly.

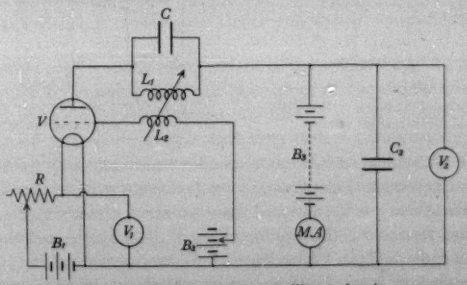

Fig. 1. Diagram of valve-oscillator circuit.

The apparatus consisted of a valve oscillator using magnetic reaction as shown in figure 1. The anode coil L_1 was a closely-wound, single-layer solenoid with 120 turns of double-silk-covered copper wire, s.w.g. 22, on an ebonite former of diameter 8·8 cm. The inductance of L_1 was approximately 800 μH. and its d.c. resistance 1·5 ohms. The grid coil, L_2, consisted of 100 turns of double-silk-covered copper wire, s.w.g. 40, wound on a presspahn former which just slipped over the

coil L_1. C_2 is a 2-μF. condenser used to by-pass radio-frequency currents from the anode battery, B_3, and the d.c. milliammeter MA. Across L_1 was connected a stand consisting of two insulated vertical aluminium strips mounted on an ebonite base; a parallel-plate condenser consisting of two square aluminium plates of side 10 cm. could be inserted between the vertical strips, which were provided with a spring arrangement to give good rubbing contact between the plates and the strips.

The moisture-content determination consisted in measuring the direct anode current indicated by MA, first with the empty condenser and then with the condenser filled with the seed under test. The change i_a in anode current thus obtained was found to be a function of the moisture-content of the seed used.

i_a

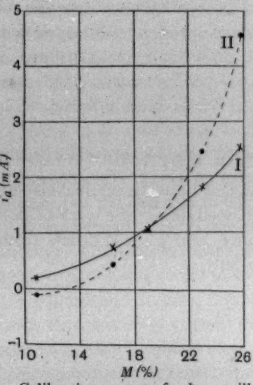

Fig. 2. Calibration curves of valve-oscillator apparatus for cocksfoot seed.

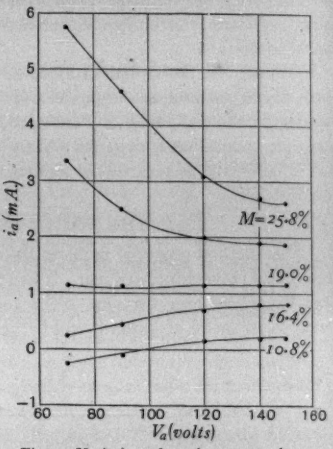

Fig. 3. Variation of anode current change with anode potential.

Figure 2 shows typical calibration curves for cocksfoot seed; moisture-contents M are taken as abscissae, and changes i_a in anode current as ordinates. The voltage V_f across the filament was 3·8 V.; the grid was connected directly to the negative end of the filament. In curve I, the anode voltage V_a was 150 V. and the initial value of the anode current 10·55 mA.; in curve II, V_a was 90 V. and the initial value of the anode current 8·35 mA. The valve used in these experiments was a Marconi-Osram L.S. 5 b. Experiment showed that the sensitivity of the apparatus is greatest when a valve of high amplification-factor is used. It was found, however, that the stability of the apparatus was far from good when a gettered valve, of the type used in broadcast-receiving sets, was employed; far better stability is obtained with a bombarded valve, especially when the valve is run at a filament voltage less than the makers' rating. Under these conditions, the permanency of calibration of the apparatus is likely to be good also.

M, V_f

V_a

The main disadvantage of this method is the number of variables involved; changes in anode and filament voltages especially affect the value of i_a corresponding to a given moisture-content. By suitable choice of these voltages it is possible to arrange matters so as to make the effect of such changes a minimum.

Figure 3 shows the variation of i_a with anode voltage V_a for different moisture-contents M; the filament voltage V_f was maintained constant at 3·8 V.

Figure 4 shows the variation of i_a with V_f for different values of M; V_a was maintained constant at 150 V.

From these diagrams, the slope of the i_a/V_a curves is seen to be a minimum when $V_a = 150$ V.; this, therefore, is the optimum value of V_a. In the case of the i_a/V_f curves, the value of V_f for minimum slope differs for different values of M; when $V = 3{\cdot}8$ V. the slope is a minimum for values of M lying between approximately 10 per cent. and 20 per cent., and since this is the most important range the optimum value of V_f is taken as 3·8 V. These diagrams show that the indications

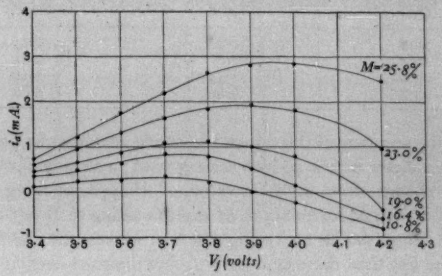

Fig. 4. Variation of anode current change with filament potential difference.

of the instrument can be made independent of the variations of V_a and V_f likely to occur in practice by suitable choice of the values of these quantities. In practice it is easy to dispense with the filament voltmeter V_1 and the anode voltmeter V_2, since it is possible to devise a simple switching system incorporating suitable series resistances which will enable the milliammeter MA to be used for these purposes. It is found, however, that the filament-voltmeter connexion is unnecessary, since the steady anode current I_a with no seed in the condenser is a sufficient index of the filament voltage, provided that the anode voltage has its calibration value. Accordingly, the simplest procedure is to adjust the anode voltage to its calibration value, and then, when conditions have become steady, to adjust the filament rheostat R_1 until I_a is at its calibration value.

I_a

It is interesting to calculate the possible error in the evaluation of moisture-contents by this procedure, assuming that the anode voltage can be set to within 1·5 V., as would be the case when a dry battery is used as the source of anode potential. When $V_f = 3{\cdot}8$ V. and $V_a = 150$ V., $I_a = 10{\cdot}55$ mA.; if V_a is reduced to 140 V., V_f has to be increased to $3{\cdot}82_7$ V. to obtain the same value of I_a. (This value

is found by plotting I_a against V_f, V_a being constant and equal to 140 V.) By drawing a family of curves similar to those in figure 4, but for a value 140 V. of V_a, the value of i_a can be found for different moisture-contents when $V_f = 3{\cdot}82_7$ V. Table 1 shows the values of i_a corresponding to various values of M with two different sets of values of V_a and V_f which give the same value of I_a. These were as follows: (*a*) $V_f = 3{\cdot}8$ V.; $V_a = 150$ V.; $I_a = 10{\cdot}55$ mA. (*b*) $V_f = 3{\cdot}82_7$ V.; $V_a = 140$ V.; $I_a = 10{\cdot}55$ mA.

If i_a be plotted against M, it is seen that the maximum difference in M due to the difference in i_a in the two cases is 0·6 per cent. Thus, for a difference of 1·5 V. in V_a, I_a being set in each case at its calibration value as described above, the maximum difference in M will be 0·09 per cent.

Table 1.

M (%)	i_a under conditions (*a*) (*mA.*)	i_a under conditions (*b*) (*mA.*)
10·8	0·2	0·15
16·4	0·8	0·75
23·0	1·15	1·10
25·8	2·60	2·80

As regards the accuracy of this apparatus, anode currents can be read to within 0·02 mA. when a meter with a 4½-inch scale is used; a change of 0·02 mA. in i_a is equivalent to a change of 0·2 in M for values of M approximating to 10 per cent, and a change of 0·06 in M for values of M approximating to 25 per cent. Owing to unavoidable variations due to packing and similar causes, the real accuracy of the apparatus will be less than these figures; probably moisture-contents can be determined to within 0·6 per cent when the moisture-content is about 10 per cent, and to within 0·2 per cent when the moisture-content is about 25 per cent. It is interesting to note that, with the adjustment procedure described above, the limits of accuracy of the instrument are not exceeded if considerable variations in anode potential occur.

This method was found to work very well with coarse smooth-skinned seed, such as cocksfoot, and it can be used for the routine testing of this type of seed; such seed "packs" consistently without the use of pressure. When the method was tried with timothy seed, difficulties due to inconsistent packing were encountered; this is due to the fact that timothy seed is fine, is not smooth-skinned, and tends to adhere in one loose mass. Pressure is necessary to obtain consistent packing, and the parallel-plate condenser described above is inconvenient in this connexion.

The advantage of this method over the Burton-Pitt method is its greater sensitivity, while it is more suitable for unskilled use than the Berliner-Rüter method, since the indications are obtained on a pointer instrument.

§4. THE DIRECT-CURRENT RESISTANCE METHOD

This method, although designed primarily for fine seed such as timothy seed, gives satisfactory results with coarse seed such as cocksfoot seed or oats, as well as with finely-powdered substances such as flour. The method employed is to measure the ohmic resistance of the substance contained in a suitable vessel; with the vessel employed and the moisture-contents encountered in practice, this resistance is of the order of a few megohms. It is unnecessary to use alternating current for testing, since experiment shows that consistent results can be obtained with direct current. Two convenient methods can be used for testing: (i) measurement of the current due to a battery of 40–100 V. with a microammeter; (ii) a megger method.

As was pointed out above, the main difficulty in the design of a moisture-tester to give satisfactory results with seed similar to timothy seed is the difficulty of ensuring consistent packing. This can be overcome by use of the testing-vessel shown in figure 5 *a*. This consists of a brass tube *A*, closed at its lower end by an

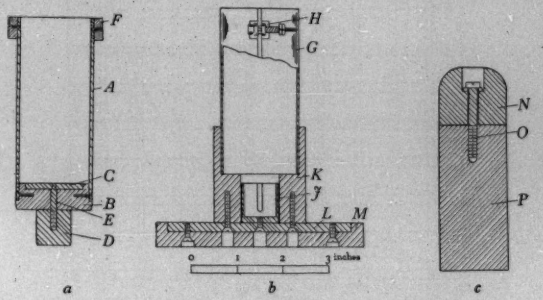

Fig. 5. *a*, Seed vessel; *b*, seed-vessel holder; *c*, plunger.

ebonite plug *B*, into which is sunk a brass disc *C*. This disc is fixed to the solid brass rod *D* by means of the screwed brass rod *E*. For convenience in handling, the upper end of the vessel is fitted with an ebonite ring *F*. The two electrodes used in the resistance-measurement are the cylinder *A* and the disc *C*. A convenient stand for the seed vessel is shown in figure 5 *b*. The cylinder *A* of figure 5 *a* fits into the brass cylinder *G*, which is split along its length and is fitted with the spring arrangement *H* to ensure good rubbing contact. The rod *D* of figure 5 *a* similarly fits into the brass cylinder *J*, which is split into four sections along part of its length. *G* is fixed into the ebonite base *K* which serves to insulate it from *J*. *J* is fixed by a screw to the brass plate *L*, which is embedded in the ebonite disc *M*. Connexion may thus be made between the external circuit and the electrodes through *G* and *L*. Figure 5 *c* shows the plunger used for applying pressure to the seed; it consists of a lead weight *N* fixed to the ebonite cylinder *P* by means of the screw *O*.

The advantages of using a seed vessel of the form shown is two-fold. In the

first place, consistent packing can be obtained by filling the vessel with the seed under test, pressing the plunger as hard as possible by hand and then releasing the pressure. In the second place, the resistance of the vessel when it contains seed packed as described above is independent of the quantity of seed present, provided that the level of the seed exceeds a certain limit determined by the dimensions of the vessel. This is due to the fact that the lines of flow of current between the disc and cylinder are closest in the lower part of the vessel, and thus it is the seed contained within this part that contributes most to the resistance between the two electrodes. To investigate this point further, dilute zinc sulphate solution was placed in the seed vessel, and the resistance between the two electrodes was measured by the ordinary a.c. bridge method for various heights of electrolyte. Figure 6 represents
h graphically the results obtained. The abscissae represent the height h of the liquid

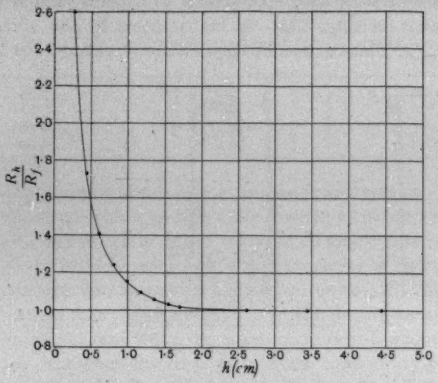

Fig. 6. Variation of resistance of seed with depth of medium.

R_f in the vessel in cm.; the ordinates represent the ratio R_h/R_f where R_f is the resistance
R_h of the vessel when filled with the electrolyte, and R_h the resistance when filled to a height h. It is seen that R_h is independent of h provided that the latter is greater than 2·6 cm., whilst the value of R_h when h is 2 cm. is only 1 per cent greater than its limiting value. Experiment showed that when the vessel was initially filled with timothy seed and the seed then packed as described above, the height of the seed in the vessel was about 2·5 cm.; deliberate variation of the amount of seed initially introduced was then tried, and it was found that in no case likely to occur in practice would h be less than 2·3 cm. It is thus seen that variations in the quantity of seed introduced can be neglected—an important feature when the instrument has to be used by unskilled observers, and when the seeds under test tend to adhere together.

The two d.c. resistance methods mentioned above were investigated and found to be satisfactory. Since the battery and microammeter method is considerably cheaper than the megger method, the former was finally adopted. One fact should,

however, be mentioned; it was found that the resistance between the electrodes with packed seed depends on the applied voltage and on the previous history of the seed. Figure 7 shows the variation with applied voltage V, of the resistance R (in megohms) of timothy seed of approximately 14 per cent moisture-content; this sample had not been previously tested in the moisture-tester. The full curve was obtained on increasing V step by step to 370 V., and the broken curve on decreasing V step by step from this value. If the same sample of seed were again subjected to an applied voltage less than 370 V., its resistance would be appreciably different from either of the values shown on the curves. It is difficult to suggest an explanation for this phenomenon; it appears to be due to some kind of polarization within the seed. Any attempt at an explanation is complicated by the colloidal nature of

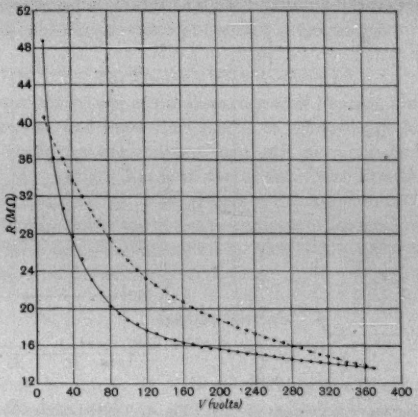

Fig. 7. Variation of seed resistance with applied potential.

the seed, since electro-osmosis and cataphoresis will be involved. This dependence of resistance on applied voltage does not in any way vitiate the method, provided that a constant voltage be used for calibration and testing; this can easily be arranged when using the battery and microammeter, by incorporating a switch to enable the microammeter to function as a voltmeter and thus to check the battery voltage. If a megger method be adopted, an instrument incorporating a constant-voltage generator should be used.

In its final form, the apparatus was connected up as shown in figure 8, in which A represents the seed vessel, and G a microammeter permanently shunted so as to give nearly full-scale deflection with the wettest seed likely to be encountered in practice. B is a dry battery whose voltage can be adjusted in steps of $1\frac{1}{2}$ V., whilst S is a three-pole three-position switch of the type used in wireless sets. R_1 is the usual series resistance for converting a microammeter into a voltmeter; R_2 is a shunt resistance brought into action when the microammeter is used as a voltmeter. The

purpose of R_2 is to reduce the sensitivity of the microammeter so as to obtain a reasonable value for R_1, which would otherwise be of the order of 2 to 3 megohms. When the switch S is in the right-hand position, G is in series with A, R_1 and B; the apparatus is then ready for seed-testing. The central position of the switch is an off position. When the switch is to the left, A is short-circuited while G is shunted by the resistance R_2 and is in series with the resistance R_1; it then functions as a voltmeter. In the instrument constructed the battery voltage used for testing was 45 V.; the resistance R_1 was 100,000 ohms, whilst G was an Onwood pointer galvanometer shunted so as to give full-scale deflection for about 500 μA. It will be noticed that the resistance R_1 is permanently in circuit; for seeds of low moisture-content (10–15 per cent) its presence is immaterial; with seeds of medium moisture-content (15–30 per cent) it tends to make the calibration curve less steep; with very

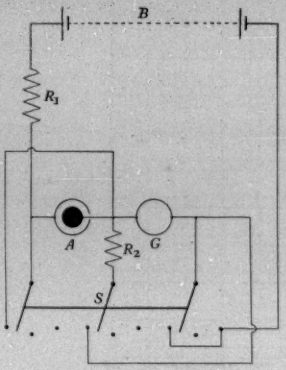

Fig. 8. Diagram of connexions of resistance method apparatus.

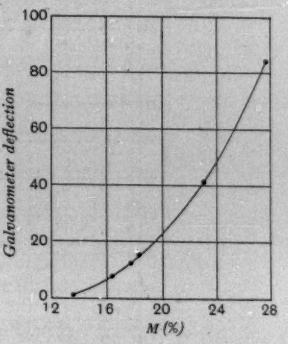

Fig. 9. Typical calibration curve of resistance method apparatus for timothy seed.

wet seeds (30 per cent and over) it makes the slope of the calibration curve small, since the resistance of R_1 is large in comparison with that of the seed vessel under these conditions. This last effect is not detrimental, since seeds containing more than 30 per cent of moisture are seldom encountered.

Figure 9 shows the calibration curve of the instrument for timothy seed; galvanometer deflections are taken as ordinates and moisture-contents M as abscissae.

To using the instrument, the following procedure is followed. (i) The switch S is turned to the left-hand position and the battery voltage adjusted to its calibration value. (ii) The switch S is turned to the central position and the seed vessel filled and packed as described above. The seed vessel is then inserted in its stand, and, when the microammeter pointer has become steady, the switch is rotated to the right-hand position. (iii) The moisture-content of the seed is found by reference to the calibration table giving the moisture-contents corresponding to each division on the galvanometer scale.

This apparatus has now been in use for some months and has been found to work satisfactorily; the accuracy of the instrument is such that it will give readings on any part of its scale which differ by less than 0·1 in moisture-content.

Its chief advantages are (i) the simplicity of principle and construction; (ii) the ease and rapidity of the moisture-estimation, readings being obtained with a pointer; (iii) the preliminary adjustment to reproduce calibration conditions is simple and, moreover, is not frequently required; (iv) the permanency of calibration; (v) the sensitivity is adequate, and can be varied to suit different conditions by varying the testing voltage or, alternatively, by varying the sensitivity of the microammeter.

It is proposed to extend the use of the instrument to the measurement of the moisture-content of grasses and root crops. In the former case the grass will be cut up by passing it through a miniature chaff-cutter; in the latter case it is proposed to use two insulated spikes inserted in the root as electrodes.

§ 5. ACKNOWLEDGMENTS

The work was carried out at the Department of Physics, University College of Wales, Aberystwyth, and the writer wishes to acknowledge his indebtedness to Prof. Gwilym Owen for his constant help and encouragement, to Prof. R. G. Stapledon and Mr Gwilym Evans of the Welsh Plant Breeding Station for the various seeds used, and to Prof. T. W. Fagan of the Agricultural Chemistry Department for the determination of the moisture-contents of the seeds used for calibration.

The apparatus is being manufactured by Messrs A. Gallenkamp and Co., Ltd., Technico House, Sun Street, London, E.C. 2.

REFERENCES

(1) E. F. Burton and A. Pitt. *Phil. Mag.* **5**, 939 (1928); *Canadian J. of Research*, **1**, 155 (1929).

(2) E. Berliner and R. Rüter. *Z. f. das ges. Mühlenwesen*, **5**, 168 (1929); **6**, 1 (1929); *Kolloid-Zeit.* **47**, 251 (1929).

(3) M. H. Belz. *Proc. Camb. Phil. Soc.* **21**, 52 (1922); *Phil. Mag.* **44**, 479 (1922).

DISCUSSION

Dr Ezer Griffiths. The paper will be of interest to those concerned with the determination of the moisture-content of materials such as wool, cotton, etc. The author might with advantage state what concentration of a salt solution gives a resistance comparable with that of the seeds tested. One would also like to know more of the mechanism of the electrical conduction and the mode of distribution of the flow lines in an individual seed.

A practical detail which needs consideration is the state of the surface of the narrow annular ring of ebonite separating the two electrodes. If moisture collects on this surface then troubles due to surface leakage would arise.

Mr G. R. Stanbury. Mr Davies's paper is very interesting to us on account of some similar experiments we have been carrying out on the rapid estimation of the moisture-content of textile fibres. Professor Burton left his apparatus with the

Wool Industries Research Association for this purpose when he was over here eighteen months ago, and it was exhibited at the Physical Society's Exhibition in 1931. We agree with Mr Davies regarding certain of the disadvantages of the apparatus, but we fail to see in what way the valve-oscillator method described in § 3 is any improvement, as the circuit used is identical with that of the Burton-Pitt instrument. The only apparent difference is that the moisture-containing material is introduced into the condenser instead of between the coils as described by Burton and Pitt. This alternative had, however, already been provided for in the apparatus loaned to us, but it was not found to give any more reliable results. We disagree with the statement that with the Burton-Pitt instrument it is difficult to obtain great sensitivity, since we have obtained plate-current changes of equal order to those given in figure 2 for similar conditions. The sensitivity depends purely on the values of the various electrical quantities involved. The main disadvantage from our point of view is the difficulty of repeating calibration conditions, and the fact that the calibration curves vary with almost every conceivable type or quality of wool and the method of packing. The direct-current-resistance method offers greater promise and the method adopted for overcoming the necessity for accurate measurement of samples is very ingenious. In experiments carried out on single wool fibres we have noticed the same hysteresis effect as shown in figure 7.

Has the author's attention been called to another direct-current-resistance method for grain which is being marketed by the C. J. Tagliabue Mfg. Co., Brooklyn? In this method a certain quantity of the grain is allowed to pass between two rotating rollers which act as electrodes, the average deflection of the galvanometer pointer being a measure of the moisture-content of the grain.

Mr D. K. McCleery. I should like to ask what interval elapsed between the application of the battery and the reading of the current, and also whether any variation of the current with time was noticed. It is the usual commercial practice in testing dielectrics to observe the resistance after the potential has been on for one minute, but everyone knows that this is purely arbitrary though it serves as a rough basis of comparison. To wait until the absorption current has died away, leaving only the leakage current, would require infinite patience in most cases; hence the necessity for a standard time-interval. In the present case the method is described as "rapid," so it is inferred that this interval was not great.

The curve shown in figure 7 is similar to the results obtained by Evershed* in his tests of cotton and paper under various conditions of humidity. The application of the superposition principle of Hopkinson† to such a case is complicated owing to this characteristic. We may suppose that the current through the seed under a potential V applied at $t = 0$ is expressed by

$$i = V^n \phi(t) \qquad \text{.....(1),}$$

$n, \phi(t)$ where n and $\phi(t)$ are experimentally determined.

r If the potential is applied in r small steps each equal to V/r and acting at times

* S. Evershed, *J. Inst. E. E.* **52**, 51 (1914). † J. Hopkinson, *Original Papers*, Nos. 18, 19.

0, t_1, t_2 ... t_{r-1}, instead of all at once then, by the superposition principle, the current t_r
i is given by

$$i = (V/r)^n \{\phi(t) + \phi(t - t_1) + \phi(t - t_2) + \ldots + \phi(t - t_{r-1})\} \quad \ldots\ldots(2).$$

Now if the function $\phi(t)$ decreases with time, as in the case of a normal dielectric, we should expect the current in (2) to be greater than in (1); or, in other words, the resistance after time t to be less when the potential is stepped up than when it is applied completely at $t = 0$.

When the potential is reduced to the level V', at time t_r for example, the apparent resistance will be higher than if it had been raised to V' at $t = 0$, and if it is reduced step by step then by the above argument the resistance will *a fortiori* be higher. This may be the explanation of the fact that the broken curve in figure 7 lies above the full curve. In conclusion I should like to urge that a constant time-interval is no less important than a constant voltage.

Mr J. H. Coste. The author, in his interesting paper, is obviously speaking from the point of view of a physicist in classifying methods of determining moisture as electrical and non-electrical. As a chemist I should feel inclined to describe them as direct and indirect. Some time ago I devoted much attention to this apparently very simple determination in relation mainly to fuel. The international committee of which I was vice-president found that it was advantageous to dry at 105–110° C. rather than at about 98° C. as mentioned by Mr Davies (the temperature is often much lower in parts of a water oven), and we were inclined to prefer for precise work drying in an inert atmosphere and absorbing the water in a weighed tube containing a desiccant, or the very slow process of drying in a vacuum. The xylene method (Brown and Duval), in which the water distilled off with the solvent is measured, is a good one, as also is the acetylene or carbide method.

The author has described two methods of determining physical properties of seeds, the variation of which properties he associates with variations of water-content. The object of his work is fairly described in § 2 of the paper and clearly the more *ad hoc* methods which they are intended to replace do not lend themselves to mass production by unskilled labour.

The precision required of moisture-determinations in a seed-testing establishment is probably not high, but in mentioning the sensitiveness of his apparatus the author is not describing its accuracy, and I think he would add to the value of the paper if he gave a table of comparative determinations of moisture in different kinds of seeds by a recognized *ad hoc* method and by, say, his second method which seems to be the more generally useful.

In figure 7 he shows how the resistance of an aggregate of seed varies with applied potential. One is inclined to wonder whether the (admittedly very small) heating effect of the applied current on the seed-aggregate tested causes either evaporation of water which is partly reabsorbed as the voltage is decreased, or whether the increase which it must produce in the degree of ionisation of the salts in the seed causes the observed reduction in the resistance. The electrolysis caused by a direct current also introduces some complication.

In attempting to apply the second method to roots it will be necessary to remember that what is really being measured is the resistance of a dilute solution of salts, as these contribute mainly to the conductance of such vegetable substances. The specific conducting of pure water is very low—Kohlrausch and Heydweiller found it to be $0{\cdot}04 \times 10^{-6}$ ohms^{-1}-cm.$^{-1}$, and ordinary tap water owes its relatively high conductivity to the dissolved salts.

I cannot consider the theoretical basis of either process as very well established, but I am prepared to believe that both may be very useful for the purpose for which Mr Davies designed them, provided periodical check determinations are made by a recognized method.

Author's reply. In reply to Dr Ezer Griffiths: The resistance of the vessel when filled with seed is of the order of megohms, and it is of the order of 50 ohms when filled with distilled water to which a few cubic centimetres of normal zinc sulphate solution has been added.

No case of deposition of moisture on the ebonite insulation between disc and cylinder in the seed-vessel has yet been brought to my notice, although several hundreds of tests have been made with the instrument. Deposition of moisture in this way is easily detected by making an observation with the empty seed vessel, in which case a deflection indicates faulty insulation. In the instructions for use accompanying the instrument it is recommended that this insulation test be frequently applied, especially when testing very wet samples. If the insulation be found faulty in this respect it can easily be rectified by rubbing with a silk rag moistened with methylated spirits, and then drying.

In reply to Mr Stanbury: I was unaware of the fact that the Burton-Pitt apparatus incorporated an arrangement for introducing the moisture-containing material into the condenser; there is no mention of this in either of the two papers mentioned above, neither was it apparent in the instrument shown at the Physical Society Exhibition in 1931. I agree that a sensitivity comparable with that shown for my first apparatus is obtainable with the Burton-Pitt apparatus, but under these conditions the latter is very unstable. The data shown in figure 2 were obtained with a perfectly stable apparatus, the optimum working conditions being determined by the experiments described on pages 234, 235; under these conditions it is possible to repeat calibration conditions exactly. Greater sensitivity than that shown in figure 2 is obtainable with my apparatus, but always at the cost of stability.

It is interesting to note that a hysteresis effect is also obtained with single wool fibres and this property may well be characteristic of organic material.

I was unaware of any details concerning the Tagliabue Moisture Meter when my paper was written; a complete detailed account does not appear to be available. It is stated, however, that the grain-meter is "similar in principle to the lumber moisture-meter," and the latter is definitely a valve method.

In reply to Mr McCleery: The procedure adopted in the direct-current-resistance method was to insert the packed seed vessel in the apparatus and then switch on the testing voltage; the galvanometer was read as soon as the pointer had come

to rest. The galvanometer used was of the Onwood type and this necessarily takes some time to come to rest; with the range of moisture-contents covered in my experiments and with the testing voltage used, the galvanometer reading then remained constant, so that the need for a constant time-interval for testing was eliminated. With the higher voltages used in the experiments of figure 7, the time effect was noticeable, and this is why a comparatively low voltage (40–60 V.) is used for testing. This time effect was also encountered in some experiments where abnormally wet seeds were used.

Mr McCleery's application of Hopkinson's principle of superposition is interesting and deserves more detailed investigation, particularly as to the values of n and ϕ; it certainly does give a possible explanation of the hysteresis effect of figure 7 in terms of phenomena observed with a steady testing voltage.

In reply to Mr J. H. Coste: As was pointed out in this communication, the absolute determination of moisture-content is by no means a simple experiment, and considerable discrepancies are shown by methods such as the Brown-Duval and the oven method, which are our closest approximation to absolute methods. The difficulty is enhanced in the case of seeds, since chemical decomposition occurs if their temperature is raised above 98° C. or so. The acetylene method is good in principle, but suffers from the defect that it can only be used for purely pulverized substances such as flour; it is quite inapplicable to seeds, mainly because of the hard husk which surrounds the seed nucleus.

As to the accuracy and permanency of calibration of my direct-current-resistance method, the apparatus contains only one variable factor, the battery voltage; and variations in this are immediately detectable. Tests show that the permanency of calibration is better than 0·1 per cent in moisture-content. Mr Coste's remarks as to the explanation of the hysteresis effect of figure 7 merit further investigation, although the heating effect must necessarily be very small with currents of the order of 100 μA. The methods described have admittedly no theoretical basis apart from the fact that the electrical conductivity of a seed aggregate is definitely a function of the moisture-content of this aggregate. It may be added that the methods described—especially the second method—fulfil their function admirably.

THE COLLISIONAL FRICTION EXPERIENCED BY VIBRATING ELECTRONS IN IONIZED AIR

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ABSTRACT. The variation of the radio-frequency conductivity of ionized air with pressure has been studied experimentally at frequencies of the order of $10^9 \sim$. From the measurements of the critical pressure at which such conductivity is a maximum the magnitude of the collisional frictional forces experienced by vibrating electrons has been estimated.

§ 1. INTRODUCTORY

Previous laboratory investigations of the radio-frequency properties of ionized air have been largely confined to the study of the dielectric constant of such a medium and, in particular, to the confirmation of the theory that the dielectric constant can, under certain conditions, be less than unity. In such experiments radio-frequencies up to $10^8 \sim$ have been used.

The experimental investigations here described were carried out with a higher frequency range, up to $10^9 \sim$, and, although evidence of a dielectric constant less than unity has been observed incidentally, the observations are concerned chiefly with the radio-frequency resistance of the medium and with the variation of such resistance with pressure. Using ultra-short waves of wave-length 40 to 100 cm., produced by the method of Barkhausen and Kurz*, the existence of a maximum conductivity at a certain critical pressure for any given frequency has been demonstrated, and from the measurements of these critical pressures the magnitudes of the frictional forces experienced by the vibrating electrons have been deduced.

§ 2. THE RADIO-FREQUENCY CONDUCTIVITY OF AN IONIZED MEDIUM

The theory of the propagation of electromagnetic waves through an ionized medium was given by H. A. Lorentz† as early as 1909. From the results of Lorentz we can deduce, with only a slight modification, the expression for the radio-frequency conductivity of the medium which we use here.

m, e Consider an electron, of mass m and charge e, vibrating in a gaseous medium
E, E_0, pt under the influence of an alternating electric force $E = E_0 \sin pt$. Lorentz writes the equation of motion of such an electron as

$$m\frac{d^2x}{dt^2} + g\frac{dx}{dt} + fx = e\,(E + \tfrac{1}{3}P) \qquad \text{.....(1).}$$

* *Phys. Z.* **21**, 1 (1920).

† *Theory of Electrons*, p. 132 (Leipsig, 1909).

Here x represents the displacement and t the time; g is the frictional force suffered by the electron per unit velocity and f the restoring force per unit displacement. P is the polarization of the medium and is equal to Nex, where N is the number of electrons per cm^3 x, t, g f P, N

The symbols retaining the same significance, it is easy to show, from a comparison of the equations of Lorentz and the equations relating to the propagation of electromagnetic waves through a conducting dielectric, that we can regard the ionized medium as possessing a dielectric constant K and a conductivity σ given respectively by K, σ

$$K = 1 + \alpha/(\alpha^2 + \beta^2) \qquad \text{.....(2),}$$

and
$$\sigma = p\beta/(\alpha^2 + \beta^2) \qquad \text{.....(3),}$$

where
$$\alpha = f/Ne^2 - mp^2/Ne^2 - \tfrac{1}{3} \qquad \text{.....(4),} \quad \alpha$$

and
$$\beta = pg/Ne^2 \qquad \text{.....(5).} \quad \beta$$

It is the expression for the high-frequency conductivity σ with which we are here concerned. We note from (3), (4) and (5) that this may be written

$$\sigma = \frac{Ne^2 g}{\{(f/p - mp - Ne^2/3p)^2 + g^2\}} \qquad \text{.....(6).}$$

Now Gutton*, from a very extensive series of researches, has come to the conclusion that electrons in an ionized gas are subjected to quasi-elastic forces giving rise to the existence of a resonance frequency which depends on the ionic concentration. Some experiments† carried out in this laboratory, while confirming the experimental results of Gutton, have, however, led to the conclusion that his deductions from his results are erroneous. All the quasi-resonance effects are to be explained, both qualitatively and quantitatively, as due to the fact that the dielectric constant of the ionized gas under the experimental conditions assumes negative values. We therefore shall assume the restoring force coefficient, f, to be zero, so that (6) becomes

$$\sigma = \frac{Ne^2 g}{(mp + Ne^2/3p)^2 + g^2} \qquad \text{.....(7).}$$

The assumption that the restoring force acting on the vibrating electron is equal to zero raises a question relative to the magnitude of the frictional coefficient g. Lorentz, to whom the recognition of the effects of collisional friction is due, has shown that, for the case of bound electrons, we have

$$g = 2m/\tau \qquad \text{.....(8),}$$

where τ is the average time between two collisions of an electron with the gas molecules. We believe, however, that the reasoning used by Lorentz‡ for bound electrons does not apply without alteration to the case of free-electrons. The following treatment is a revision of Lorentz's calculation assuming the charges to be free. τ

Consider an electron vibrating under the influence of a periodic electric force and subjected to gaseous collisions. We suppose that there is no real friction but

* See, for example, *Ann. de Physique*, 13, 62 (1930).

† An account of these experiments will shortly be published by Dr J. Goodier and one of the authors.

‡ See H. A. Lorentz, *Theory of Electrons*, p. 309.

that the vibrations of the electrons are over and over again disturbed by impacts at irregular intervals. The equation of motion of the electron is

j
$$m\frac{d^2x}{dt^2} = eE\epsilon^{jpt} \qquad \text{.....(9),}$$

v whence
$$\frac{dx}{dt} = v = -j\frac{Ee}{mp}\epsilon^{jpt} + C \qquad \text{.....(10).}$$

If we consider (10) as applying to a large number of electrons for all of which the t_1 last collisional interruption of their motion took place at some time t_1, the mean value of their velocity at that instant will be zero, so that from (10) we have

$\overline{C}$
$$\overline{C} = j\frac{Ee}{mp}\epsilon^{jpt_1} \qquad \text{.....(11).}$$

To each electron of this group we can therefore assign a velocity given by

ϑ
$$v = -j\frac{Ee}{mp}(\epsilon^{jpt} - \epsilon^{jpt_1}) = -j\frac{Ee}{mp}(1 - \epsilon^{-jp\vartheta})\epsilon^{jpt} \qquad \text{.....(12),}$$

where
$$\vartheta = t - t_1 \qquad \text{.....(13).}$$

Following Lorentz, we may now say that since the collisions succeed each other irregularly we may reckon that the number of particles for which the interval lies N between ϑ and $\vartheta + d\vartheta$ is $(N/\tau)\,\epsilon^{-\vartheta/\tau}\,d\vartheta$, where N is the total number of particles τ considered and τ is the average time between two collisions.

Multiplying (12) by $(N/\tau)\,\epsilon^{-\vartheta/\tau}\,d\vartheta$, integrating from $\vartheta = 0$ to $\vartheta = \infty$, and dividing $\overline{v}$ by N we find that the mean velocity $\overline{v}$ of all the electrons is given by

$$\overline{v} = \frac{Ee}{m(jp + 1/\tau)}\epsilon^{jpt} \qquad \text{.....(14).}$$

Now if we assume a frictional coefficient for a vibrating electron we obtain the following equation for the velocity,

$$\overline{v} = \frac{Ee}{jpm + g}\epsilon^{jpt} \qquad \text{.....(15),}$$

so that for free electrons we obtain from a comparison of (14) and (15)

$$g = m/\tau \qquad \text{.....(16).}$$

We see, therefore, that since τ is inversely proportional to the gas pressure, g varies directly with the pressure.

Returning now to consideration of (7), we note that, if N is kept constant and g varied by an alteration of pressure, the conductivity is a maximum when

$$g = mp + Ne^2/3p \qquad \text{.....(17).}$$

Under the conditions of our experiments, however, the ionization was so small that the magnitude of the polarization term $Ne^2/3p$ was small compared with mp*, so that we write, simply,

$$g = mp \qquad \text{.....(18)}$$

as the condition for maximum conductivity.

* This statement is not strictly true for the higher values of tube current used, but the fact that, even under those conditions, the value of g was not found to vary with N, leads us to believe that polarization effects were still negligible.

We see therefore that if the value of N is kept constant and g varied, by variation of the gas pressure, there will be a maximum conductivity for the ionized medium at a certain critical pressure. At that pressure the value of g is given by mp. Our object, in the experiments described below, was to demonstrate the existence of this maximum conductivity and thereby estimate the value of g with the aid of (18). The ionized medium consisted of a low-pressure discharge, the high-frequency conductivity of which was studied by a method similar to that used by Van der Pol* in his study of the variation of conductivity along a glow discharge.

These experiments were supplemented by an exploring-electrode analysis of the discharge in question. This analysis was necessary for two reasons. In the first place, before we can associate any meaning with the observed variation of conductivity with pressure it is necessary to be certain of the constancy, or at least the law of variation, of N, the number of electrons per cm^3 with pressure over the pressure range used. Secondly, in order to be able to compare the experimentally-determined values of g with the theoretical value m/τ it is necessary to know the thermal velocity v of the electrons, from which, knowing the mean free path, the value of τ can be deduced. The use of a probe-electrode for the determination of electron concentration and electron velocity has been described by Langmuir and Mott-Smith†.

§ 3. EXPERIMENTAL ARRANGEMENTS

The discharge tube used (see figure 1) was 30 cm. long and 3 cm. in diameter, and fitted with nickel electrodes 2 cm. in diameter and 10 cm. apart. The electrodes were held in position on to a glass rod by means of small nickel clips welded on to them. A cold cathode discharge was maintained between them by means of a motor-generator, the discharge current being maintained constant by the inclusion of a saturated diode in series with the discharge. The discharge tube was set up vertically between two condenser plates KK which constituted the terminal capacity of a pair of Lecher wires, figure 2. An exploring electrode was also fused into the side of the discharge tube. This consisted of a short length of phosphor-bronze wire 0·0076 cm. in diameter which was welded on to a short nickel rod. The whole was sheathed in a glass tube which was drawn down to a small bore at one end, through which 0·3 cm. of the phosphor-bronze wire was allowed to protrude. Care was taken that the wire did not touch the glass at the point where it emerged from the sheath. The whole electrode system could be moved in relation to the condenser plates or to the exploring electrodes.

The condenser plates were of thin copper foil 4 cm. long and 3 cm. wide and were situated symmetrically with regard to the discharge tube, 3·5 cm. apart, and the electrode system was adjusted so that the Faraday dark space was between the plates. The Lecher system consisted of two copper wires of 16 s.w.g., 1 m. long, and 5 cm. apart. Instead of the usual wire bridge for resonance-adjustments a copper reflecting plate (P in figure 2), 30 cm. square, was used. This was fitted with sleeves so that it could be made to slide along the Lecher wires.

* *Phil. Mag.* **38**, 352 (1919). † *Gen. Elec. Rev.* **27**, 449 (1924).

Forced vibrations were produced in the Lecher-wire system by means of local ultra-short-wave triode generators. A wave-length range from 40 to 90 cm. was used.

The amplitude of the forced vibrations maintained in the Lecher-wire system was measured in arbitrary units by means of a galena crystal detector D, figure 2, and a dead-beat moving-coil galvanometer. The detector circuit was coupled to the

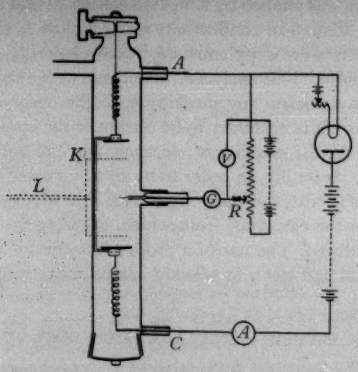

Fig. 1.

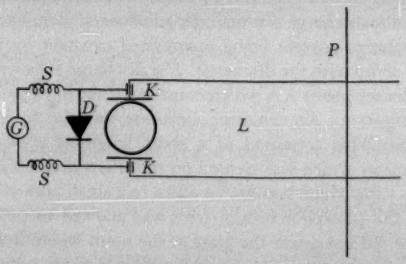

Fig. 2.

Lecher-wire system by means of two small condensers consisting of short brass tubes insulated from the Lecher wires by means of glass tubing.

The presence of a conducting dielectric, such as ionized air, between the condenser plates reduced the amplitude of the forced vibrations in the Lecher-wire system by an amount which was dependent on the conductivity. We have not attempted to relate conductivity to galvanometer-deflection but have merely assumed, as seems reasonable, that an increase of conductivity brought about a reduction of galvanometer-deflection.

§4. EXPERIMENTAL PROCEDURE

To study the relation between conductivity and pressure, the resonance deflection was first obtained for no ionization in the discharge tube. The discharge was then started for the minimum pressure in the range to be worked, and the system was then tuned again and the deflection noted. The pressure was allowed to increase in steps, and corresponding readings of pressure and resonance galvanometer-deflections were noted. Such observations were repeated for other wave-lengths and it was noted that the pressure for minimum deflection and therefore for maximum conductivity varied with the wave-length.

The values of electron-concentration N were determined by the now well-known methods of measuring the current to the probe as a function of the potential between the probe and the anode of the discharge tube. The measurements of N yielded the very convenient result that, even though the pressure of the discharge was varied, the maintenance of constant tube-current by the use of the diode ensured that N remained constant through the range of pressures used.

§5. EXPERIMENTAL RESULTS

In figure 3 is shown a typical series of results illustrating the relation between resonance amplitude in the Lecher-wire system and gas-pressure for two values of tube current. Since the galvanometer deflection decreases with increase of con-

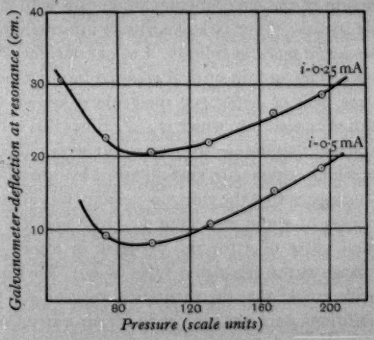

Fig. 3. Results relating to a wave-length of 80 cm.

ductivity of the ionized medium, the curves clearly demonstrate the existence of a maximum conductivity at a certain pressure, as predicted by theory.

In the table are collected the results for a range of wave-lengths. In the fifth column are given the values of g calculated from (18). The constancy of the last column indicates that g is proportional to the pressure for the range considered.

Table.

Wave-length λ (*cm.*)	p ($\sim \times 10^9$)	Pressure for maximum σ		g ($10^{-18} \times$ *dynes per unit velocity*)	A calculated as g/P (*e.m.u.* $\times 10^{-20}$)
		(*mm. of Hg*)	(*Microbars, P*)		
88	2·14	0·08	105	$1·9 \times 10^{-18}$	1·83
60	3·14	0·11	153	$2·8 \times 10^{-18}$	1·83
54	3·49	0·12	167	$3·1 \times 10^{-18}$	1·89
48	3·93	0·15	193	$3·5 \times 10^{-18}$	1·81

§ 6. DISCUSSION OF RESULTS

The results described above confirm the theory in so far as a maximum conductivity has been found at a certain pressure for a given frequency of the electromotive force. This result has possibly some significance in connexion with the absorption of wireless waves during their process of reflection by upper-atmospheric ionization; for, since the attenuation factor of wireless waves is proportional to σ, we see that for waves of a given frequency there will always be some particular height in the atmosphere at which a given ionization-content will produce maximum absorption.

The values of the frictional coefficient g which have been deduced from the observations show a proportionality with the gas-pressure. This is to be expected from the relation $g = m/\tau$, since τ varies inversely as the pressure.

An exact comparison of the experimental value of g with the theoretical value m/τ is hardly possible in view of our lack of accurate information concerning τ. But a rough comparison may be made as follows. Let us consider the case in which the wave-length used was 88 cm. and the critical pressure found to be 0·08 mm. For this pressure the mean free path of a nitrogen molecule is known to be 0·080 cm., so that the mean free path of an electron, which is $4\sqrt{2}$ times this value, is 0·45 cm. Now the probe analysis indicates that the electron-velocity corresponds to a fall of potential of about 6 V., a value which agrees with that obtained by other investigators for this type of discharge. This means that the electron-velocity was about $1·5 \times 10^8$ cm./sec., in which case the values of τ and m/τ were 3×10^{-9} and 3×10^{-19} respectively. Now the experimental value of g for the pressure in question was $1·9 \times 10^{-18}$, a value 6·3 times as large as the calculated value of m/τ. We do not think that this discrepancy necessarily proves that the theoretical relation $g = m/\tau$ is not valid, as our experimental conditions were not such as to favour a strict comparison of g and m/τ. The electron velocity was doubtless not constant across the cross-section of the tube. Also the positive ions in the sheaths next to the glass walls may have played a part in determining the variations of conductivity, in which case the mean free path would be smaller and the value of m/τ correspondingly larger than in the case of the electrons. It is, however, possible to obtain another estimate of m/τ in the following way. According to Langmuir and Compton the direct current
μ mobility μ of electrons in a discharge is given by

$$\mu = 0·85 e\lambda/mv \qquad \text{.....(19).}$$

Now the drift current i per unit area is given by

$$i = Ne\mu\bar{E} \qquad \text{.....(20),}$$

where $\bar{E}$ is the electric force applied. Therefore from (19) and (20) we have

$$mv/\lambda = m/\tau = 0{\cdot}85\,Ne^2\bar{E}/i \qquad \text{.....(21).}$$

For the body of the discharge we are considering, the value of $\bar{E}$ at a pressure of 0·08 mm. was found, from probe measurements of the space potential, to be about 1 V./cm. The current-density was known to be $1{\cdot}6 \times 10^{-4}$ A./cm.^{2} Also N for these conditions of current and pressure had been found to be $1{\cdot}5 \times 10^9$, so that on substitution in (21) we find that m/τ is 2×10^{-18}. This is very nearly the value of g obtained experimentally at the pressure in question.

Summarizing therefore we see that in the example chosen the experimental value of g was $1{\cdot}9 \times 10^{-18}$, while m/τ was estimated as $0{\cdot}3 \times 10^{-18}$ and 2×10^{-18} by two different methods. The agreement is, we think, as good as could be expected in view of the heterogeneous nature of the discharge and the fact that probe measurements could be made only at one point in a cross-section.

Before concluding this discussion one or two points of interest in connexion with the dielectric constant of an ionized medium may be mentioned. Various observers have found it difficult to demonstrate the existence of a dielectric constant less than unity. Van der Pol*, who was the first to examine this effect, used wave-lengths of 3 m. and only for very small electron concentrations found the dielectric constant diminished. For greater concentrations the apparent dielectric constant was greater than unity. Frl. Szekely†, working with wave-lengths between 242 and 58 m. found a dielectric constant greater than unity at all concentrations. In this connexion it should be noticed that the theoretical expression for the dielectric constant of an ionized medium does not permit values greater than unity, whatever the ionic concentration may be. As has been shown by Appleton and Childs‡, the increased dielectric constant, when measurements are made with internal electrodes, is due to the influence of the ionic sheaths which form round the electrodes.

When very short waves and external electrodes are used, as in the present experiments, the difficulty mentioned above disappears and it is very easy to demonstrate the existence of a dielectric constant less than unity. In figure 4 is shown the relation between Lecher-wire tuning and tube current for a wave-length of 80 cm. It will be seen that, even for the highest tube current we could use, the Lecher-wire length had to be increased to restore resonance conditions indicating that the capacity of the terminal condenser was reduced by the presence of the ionized medium. The interesting variation in this curve at A, which suggests some species of resonance effect, was repeatedly found, and is not believed to be spurious. It is possibly connected with the free period of the plasmoidal oscillations discussed by Tonks and Langmuir, since the relation between the wave-length and the electron concentration N at which the effect occurs is such as may be predicted from their theory. Since the wave-length λ was 80 cm. and since a tube current

* *Thesis, University of Utrecht* (1920).

† *Ann. der Phys.* **5**, 3, 112 (1929).

‡ *Phil. Mag.* **10**, 969 (1930).

of 450 μA. was known to correspond to an electron concentration of $1{\cdot}62 \times 10^9$, we have $N\lambda^2 = 1{\cdot}04 \times 10^{13}$, while for resonance with plasmoidal oscillations for a cylindrical plasma between plane electrodes we infer from the theoretical treatment of Tonks and Langmuir that $N\lambda^2$ lies between $1{\cdot}06 \times 10^{13}$ and $2{\cdot}12 \times 10^{13}$ according to the relative dimensions of the condenser plates and the diameter of the tube.

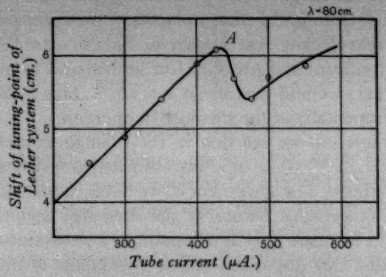

Fig. 4. Results relating to a wave-length of 80 cm.

For a wave-length of 80 cm. it will be seen that there is no sign of the quasi-resonance effects obtained by Gutton for the range of tube currents used. We believe that this was due to the fact that currents were not large enough to give the effect with this short wave-length. In order to check this point experiments were carried out with a wave-length of 370 cm. and the quasi-resonance effect was found. The electron concentration required to produce this effect was such that $N\lambda^2 = 1{\cdot}0 \times 10^{14}$. This agrees with the results of other experiments carried out in this laboratory which show that the value of $N\lambda^2$ is always greater for the resonance effect of Gutton than for the resonance due to plasmoidal oscillations. We believe, therefore, that the two effects are quite distinct.

§ 7. ACKNOWLEDGMENT

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DISCUSSION

The PRESIDENT asked how the amplitude of the oscillation executed by the electrons compared with the free path.

AUTHORS' reply. Knowing the electric fields applied and the frequency of their alternation we can calculate the amplitude of the electronic vibrations. Neglecting friction, this is found to be about 2×10^{-4} cm., which is a small quantity compared with the mean free path.

SURFACE HEATING BY NEUTRALIZED POSITIVE RAYS BEFORE AND AFTER RETURN TO NORMAL STATE

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ABSTRACT. A nearly parallel beam of positive rays emerges through a cylindrical aperture in a cathode, and enters a space where the vacuum is kept high enough for gas collisions to be negligible. Experiments are carried out to decide whether the impact of neutralised atoms in the beam is able to heat a solid target to the same extent before and after their spontaneous return from excited to normal atomic state. This is done by investigating the disturbance of a bridge balance between two filaments accurately aligned across the beam at different distances along its path. With the aid of control experiments, and after elimination in the bridge of all effects common to the two wires, an excess heating is shown to occur in that filament which receives the larger proportion of atoms in excited states. The variation of this excess heating with the potential driving the discharge is found to be quantitatively in agreement with the variation of concentration of excited atoms at the different distances along the beam: this latter is calculated from Wien's experiments on the exponential decay of excited states along the path of neutralized positive rays.

§ 1. INTRODUCTION

When gas molecules strike a solid surface in a high vacuum, many kinds of energy exchange between the phases can take place, ranging from the very small transfer of kinetic energy accompanying specular reflection of molecules at a clean wire[1], to the violent destruction of an electrode surface by ionic bombardment, or the intense heating of a gas-solid interface at which a highly exothermic chemical reaction is taking place. In some previous studies I have described experiments illustrating two particular aspects of these energy exchanges: (*a*) the varying efficiency of thermal exchange when the solid is covered with different non-reacting layers of adsorbed gas[2], and (*b*) some conditions governing the liberation of energy when a hydrogen atom from the gas strikes and recombines with a hydrogen atom adsorbed on the solid[3]. An important though usually inaccessible variable in such investigations, independent of the kinetic energy of the impinging molecule, must be its state of internal energy, as specified by the quantum levels of its electrons. Some consequences of abnormal energy content of this kind are well known. For instance the concept of molecules which are activated, and thereby enabled to react at a surface, is familiar in chemistry. Recently several workers have shown that atoms in high quantum states can also cause the emission of electrons from solids on which they impinge, an energy

exchange which Oliphant[4] has investigated in detail. In a previous communication to this Society[5] I have described experiments in which mercury atoms, after excitation by absorption of certain radiations, cause liberation of gas from surfaces with which they collide. In view of such results it is desirable to devise direct thermal means of detecting any difference which may exist in the energy transferred by a gas molecule to a solid surface, according as the former is (*a*) in its normal state of minimum internal energy, or (*b*) possessed of excitation energy consequent on an electron having been raised to, or having been captured into, an outer orbit.

In the present paper we approach this problem by demonstrating a difference between the heating of a wire filament by impinging atoms before and after their spontaneous transition from higher to lower states of excitation; we use apparatus whose design aims at keeping constant all thermal exchanges between gas and solid phases, except those dependent on progressive de-excitation in the former.

Means for doing this are available from the classical experiments of Wien and his school, who studied optically a beam of positive rays entering a high vacuum, and traced the progress in spontaneous return of atoms to normal and other definite states. In our experiment we interrupt such a beam by two similar filaments, placed so that the principal difference between them is that one receives a considerable proportion of atoms before their return to normal and the other receives almost exclusively those whose return has been completed. We find that, after allowing for geometrical factors, the former filament acquires more heat from the rays than the latter, and that when control conditions are varied this excess varies in accordance with a law deducible from Wien's measured constant for the "decay" of excited states in a beam of neutral atoms. We conclude that either (*a*) excited atoms can communicate some of their internal energy as heat to a solid, or alternatively (*b*) possession of that energy renders them more than normally efficient in communicating to a target their kinetic energy or other source of heat such as energy of recombination.

We proceed to a quantitative treatment of such positive ray phenomena as constitute the mechanism of our experiment.

§ 2. THE EXPONENTIAL DECAY OF THE RATE OF ENERGY EMISSION FROM NEUTRALISED RAYS

When a beam of positive rays is allowed to escape into a highly evacuated chamber through a perforated cathode, it can be shown that a considerable proportion of the particles in the emerging stream are neutral. These have captured electrons close to the edge or side of the perforation, and may pass through more than one stage of excitation subsequently, but continue the high-velocity flight begun under the potential which had accelerated them when they were charged. Wien and others have shown that finite intervals of time elapse before these atoms regain their normal state of minimum internal energy; and since these intervals are commonly of the order of 10^{-8} sec., and the speed of the rays in an ordinary tube is of the order of 10^{8} cm./sec., the relative concentration of atoms in normal

and in still excited states will vary enormously over a length of a few centimetres along the path of the beam of rays. This variation is even roughly verifiable by eye, the luminosity accompanying de-excitation being intense just behind the cathode, decreasing over a short distance, and vanishing within a few centimetres, so long as the vacuum is high enough to eliminate the gas collisions which would allow the remaining ions to generate luminosity further along the tube. Such reorganization of the neutralized atom may take place by more than one process, but involves necessarily a loss of the difference between its excited energy E_1 and its normal energy E_2: there occurs either (*a*) the emission of a spectral line whose frequency is given by

E_1 E_2

$$\nu = (E_1 - E_2)/h; \qquad \nu$$

or (*b*) a super-elastic collision in which $E_1 - E_2$ reappears as dissociation energy or kinetic energy in some molecule struck by the atom.

If, by the use of very fast pumps, the gas pressure behind the cathode is maintained much lower than that required to carry the discharge in front of the cathode, gas collisions may be neglected and the spontaneous process (*a*) predominates until the rays strike a solid surface. This spontaneous transition between quantum states has been shown to follow probability laws, which have been investigated in detail by Wien and his school. These laws are such that if the average time τ of duration of an excited state is given by

τ

$$\tau = 1/2\alpha, \qquad \alpha$$

while the rays travel from the point of excitation along an x-axis with velocity v, the concentration N of atoms in still excited states at any point x is given by

v N, x

$$N_x/N_0 = e^{-\alpha x} \qquad \alpha$$

$$= e^{-2\alpha x/v} \qquad \text{.....(1).}$$

By knowing the value of v and observing the decrement of the luminosity accompanying de-excitation for any given spectral line, Wien determined α and hence τ for various states of various atoms, using methods which he described to this Society in his Guthrie lecture in 1925.

The value of τ seems to be normally of one or other of two distinct orders of magnitude: (*a*) for many states of hydrogen and other common gases its value is between 10^{-7} and 10^{-9} sec.; (*b*) certain states of helium, mercury, and others are metastable, and terminate spontaneously only after 10^{-2} sec. or longer periods. In the experiments of Oliphant and others on the behaviour of excited atoms, use was made of metastable atoms which could travel long distances without de-excitation, but since we are here seeking a rapid variation of N_x, hydrogen is not unsuitable. The energy difference between atomic states of hydrogen most relevant to the present case is given by

$$E_1 - E_2 = 10{\cdot}15 \text{ electron-volts}$$

$$\doteq 235{,}000 \text{ cal./gm.-mol.}$$

This characterises transition from a 2-quantum to a 1-quantum state: by measurement of the exponential decay of Lyman lines Wien[6] determined for this transition

$$2\alpha = 14{\cdot}8 \times 10^7 \text{ sec.}^{-1} \qquad \text{.....(2).}$$

x_1, x_2 In the present experiments we are concerned with the impact of the rays on targets placed at distances x_1, x_2, where

$$\left.\begin{aligned} x_1 &= 0{\cdot}9 \text{ cm.} \\ x_2 &= 2{\cdot}9 \text{ cm.} \end{aligned}\right\} \qquad \text{.....(3).}$$

These distances are measured from the front face of the cathode, very close to which the majority of the atoms concerned start their transition by one of several possible mechanisms which need not be detailed here.

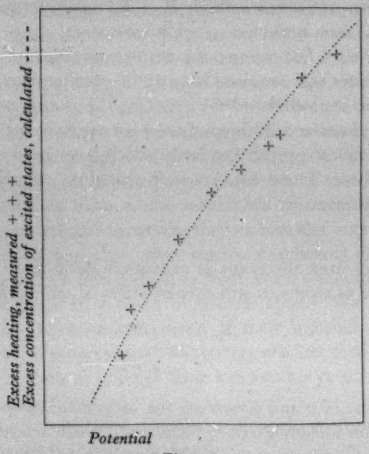

Fig. 1.

v would most accurately be obtained from the maximum of a curve of spectral-intensity-distribution across the Doppler-displaced line emitted, but to avoid violent disintegration of the delicate targets used it was necessary to keep the positive-ray current below the limit for accurate spectroscopic measurement. An average value of v can be approximately determined from

$$\tfrac{1}{2}mv^2 = e\overline{V} \qquad \text{.....(4),}$$

$e, m, \overline{V}$ where e and m are charge and mass of the proton and $\overline{V}$ is the effective potential corresponding to the velocity for a given Doppler effect. In a paper read some V years ago before this Society I discussed the relation of $\overline{V}$ to V, the potential measured across the running tube; it was seen that

$$\overline{V} = \tfrac{1}{2}V \qquad \text{.....(5)}$$

can represent, at not too high potential, a fair approximation which we shall adopt here.

From equations (1, 2, 3, 4, 5) we calculate the proportion of atoms which still retain their excitation energy at the points x_1 and x_2 behind the cathode, for certain values of the accelerating potential measured across the tube.

V (volts):	2500	3000	4000	5000	6000	7000	8000
$e^{-2\alpha x_1/v}$:	0·067	·085	·118	·148	·174	·200	·223
$e^{-2\alpha x_2/v}$:	0·0003	·0004	·001	·002	·004	·005	·008

The dotted line in figure 1 represents the calculated variation of

$$[e^{-2\alpha x_1/v} - e^{-2\alpha x_2/v}]$$

with V.

This provides a means of interpreting any difference that our experiments may reveal between the extent to which targets at x_1 and x_2 are able to acquire heat by impact of atoms; since, if such difference is in fact due to difference in states of excitation, it should vary with V in the same manner as the above calculated quantity.

§ 3. APPARATUS

Figure 2 indicates the way by which it was arranged that hydrogen should flow continuously through the system, maintaining a gas-pressure sufficient to

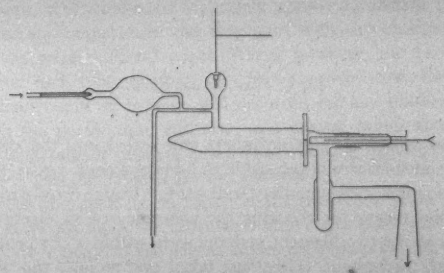

Fig. 2.

carry a discharge in the front portion of the tube while the region behind the cathode was continuously evacuated to about 10^{-5} mm. in order to eliminate gas collisions and so to allow the transitions between atomic states to be spontaneous except when atoms were striking the filaments. Steady admission of gas took place through a fine leak consisting of glass tubing drawn down on to asbestos fibre along its axis; this was found to be more constant in its conductance of hydrogen than the usual palladium valves, and, so far as could be determined spectroscopically, efficient as an impurity filter. The length and gauge of wire for

any required rate of flow was determined by preliminary experiments. An efficient liquid-air trap between the 3-stage steel vapour pump and the high-vacuum end of the tube was necessary, in view of the disintegration of tungsten filaments if accessible to mercury vapour, and was made large enough not to impede the rate of pumping seriously. The source of high potential was a large coil and rectifier: this is at a disadvantage compared with high-tension direct current, since the potential accelerating the ions fluctuates enormously around the value read on any instrument; but I was encouraged not to abandon the work for lack of direct current by the fact that some of the most accurate measurements of the exponential decay of hydrogen rays were carried out by Atkinson[7] with only an induction coil.

The rays were able to pass through the cathode by a cylindrical aperture, accurately bored with a fine drill, giving a conical beam of very small angle. At the distances x_1 and x_2, given above, were tungsten filaments perpendicular to the axis of the beam and to each other. These were rigidly supported on thick leads

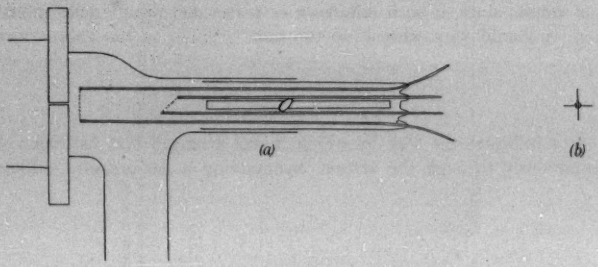

Fig. 3. Filament-mounting.

gripped between concentric glass cylinders and sealed into the system in such a way that no waxed surface was accessible to bombardment, figure 3 (*a*). The leads were carried by low-resistance mercury contacts to a bridge fitted with milliammeter and potentiometer means for checking the constancy of its electrical conditions. The resistances of the two filament systems were within 2 per cent of each other and remained so throughout. Care was taken not to heat the wires except in hydrogen, and to clean by glowing.

Since the positive-ray current was kept very small to minimise secular variations by disintegration of the filaments, their change of resistance due to heating by the impact of the rays was always small. Any differential heating of one in excess of the other was therefore a second-order quantity, and could be quite accurately studied in terms of the direct deflections of a reflecting galvanometer as the bridge balance between the two filaments was disturbed.

The constructional detail requiring most care was the alignment of the filaments relative to the beam of rays they were to interrupt. Unless they intercepted precisely known cross-sections of the beam it would not be possible to assume that

any disturbance of bridge-balance was due to the greater or lesser proportion of excited atoms striking one or the other. To check this accuracy of alignment a small mirror was mounted on the filament-supports, figure 3 (*a*), and a beam of light was projected along the axis of the discharge tube from the anode end: during final adjustment and sealings, and at any subsequent examination, the small circular spot of illumination provided by the cathode perforation could be observed in this mirror, and could be seen crossed accurately by the two filaments, as in figure 3 (*b*).

The following dimensions were adopted, and when transferred to a large-scale diagram were utilized to determine the extent to which the filament at x_1 screened that at x_2 from impact of the rays:

Diameter of cathode perforation:	0·075 ± ·003 cm.
Length of cathode perforation:	0·6 cm.
Diameter of filament wire:	0·015 cm.
Length of filament wire:	> 1 cm.

Hence at distances 0·9 and 2·9 cm. behind the front face of the cathode the filament-length ensured that each extended more than across the beam, whose divergence is calculable from the above dimensions. It is found that the calculated screening of x_2 by x_1 amounts to approximately 9 per cent.

§ 4. RESULTS

Each filament, when balanced separately against a constant resistance in the bridge, changed its temperature and resistance with any change of gas-pressure according to the normal Pirani effect. When, however, the two filaments were balanced against each other so that, to a first approximation, their Pirani effects cancelled, a disturbance of balance still occurred for any unidirectional net flow of gas through the perforation, and also, to a greater extent and in the opposite direction, for a stream of positive rays. The following experiments were carried out in the use of this disturbance for our purpose:

(*a*) To check the geometrical factor in the difference between the impact of rays on front and back filaments, calculated above as a 9 per cent screening effect, measurements were made of the galvanometer deflections accompanying the efflux of rarefied gas through the perforation into a high vacuum without any discharge. The beam of *atomstrahlen* thus striking the filaments was found to cool the filament at x_1 approximately 10 per cent more than that at x_2. (*b*) During discharge the stream of positive and neutralized rays was found to heat the filament at x_1 more than at x_2: the excess heating effect varied up to about 80 per cent according to the potential accelerating the rays, with comparatively small variation in the positive-ray current.

The measurements (*a*) and (*b*) were only susceptible of very limited accuracy, since they involved measuring the change of resistance of each wire separately, as well as the difference between the changes of resistance of the two when balanced. The latter we have referred to as a second-order effect compared with the former

as a first-order effect. It is only for the second-order effect that the bridge method serves to eliminate the great variety of undesired complications enumerated in § 5. Accordingly we place no weight on these preliminary measurements (*a*) and (*b*), except to prove that the excess heating of the front filament could not be accounted for merely by a geometrical effect of spread of beam, screening, or inaccuracy of alignment.

Hence there remains a large excess of heating by impacts on the front wire, which (*a*) and (*b*) show must be attributed to changes other than geometrical occurring in the beam between x_1 and x_2. We ascribe this to the excess concentration of still excited atoms, since such excess is both known to exist and is the only difference, other than geometrical, known to exist between x_1 and x_2.

(*c*) The quantitative correlation of impact heating with this concentration of normal and excited states was carried out by measuring the second-order effect, in terms of galvanometer deflections, for a sequence of different values of the potential across the tube. All those determinations whose reliability was checked by dependence on more than a single reading are plotted, in arbitrary units, upon the graph (+) of figure 1. This observed variation of the excess energy given by gas impacts to the front wire runs parallel, within the accuracy of the experiments, to the line already drawn to represent the calculated excess of atoms in the excited state. This parallelism only becomes a congruence if an additive constant is included; but owing to the necessary roughness of our method of obtaining v it is not safe to use this constant to correct x and so redetermine the point at which the rays were neutralized. In the absence of a more precisely known accelerating potential than is afforded by coil and electrostatic voltmeter, the investigation does not serve to do more than show the general agreement between the growth of excess heating and the growth of excess concentration of excited atoms.

§ 5. DISCUSSION OF RESULTS

Any interpretation of this heating accompanying de-excitation must be made in full consideration of the great variety of causes by which the temperature and resistance of a wire exposed to a gas may be changed. The following are the more important sources from which change of filament resistance may be expected under given conditions of the supply to it of electrical energy.

A. Fluctuations in the gaseous phase: (1) At low pressure, change of total gas pressure, or of partial pressure of differentially conducting components, heats or cools the wire. (2) Molecules of high potential energy may diffuse away from the wire, as in Langmuir's experiments with atomic hydrogen.

B. Secular alterations in the wire: (3) The accommodation coefficient, or fraction denoting efficiency of exchange of kinetic energy between gas and solid, is altered if the solid undergoes oxidation or other covering of the surface. (4) The area accessible to heat-conducting gas molecules is altered by recrystallisation or other secular changes in surface structure. (5) The volume resistivity of the wire alters if gas dissolves in or combines with the solid structure.

C. Temporary interaction of gas with wire: (6) Heat of adsorption may be

liberated. (7) Heat of reaction may be liberated if exothermic combination takes place between gas and solid, or between gas and adsorbed layer. (8) High-speed ions give up part of their kinetic energy to the wire on impact. (9) Impinging molecules in other than normal states may (possibly) communicate part of their internal energy to the wire as heat.

Processes (3), (4), (5) emphasise the importance of the precaution previously referred to in the treatment of filaments for this type of experiment. In a state of pressure equilibrium, or during isotropic changes of pressure, all causes will affect each of the two filaments equally, and so their effects are eliminated by the bridge arrangement used.

During non-isotropic pressure change, such as the efflux from the cathode perforation, the equality of effect on the two filaments is disturbed by the screening of one wire by the other. This was calculated in our case as 9 per cent, and measured, with respect to process (1) of the above list, as approximately 10 per cent in the rough first-order investigation.

The further excess heating of the front filament, which can only be ascribed to the greater concentration of excited atoms at the front filament, may be considered to act through more than one of the above-enumerated processes. There are two alternatives:

(*a*) The energy of de-excitation, representing the difference between the atomic state immediately after electron-capture or excitation and that after reorganisation of the atom into its normal state has been completed, may itself be partially transferred to vibrational energy of the atoms in the solid and reappear as heat, as in process (9). This would be analogous to the better known super-elastic collisions in the gaseous phase.

(*b*) Atoms before returning to normal state may be more efficient than normal atoms in liberating energy at the surface by processes (6), (7), or (8). If the forces of attraction between a neutral hydrogen atom and a tungsten surface depend on the electronic configuration of the former, the chance of adsorption and of recombination with already adsorbed atoms may be very different at the two wires, according as they receive greater or lesser proportions of atoms still in excited states. Similarly with regard to process (8), the transfer of kinetic energy from a fast atom, moving with a speed given by the accelerating potential when it was charged, depends on the mechanism of the collision: reflection coefficients and accommodation coefficients may be different for normal and excited atoms.

The relative amounts of energy available for transfer from impinging atom to solid are as follows:

(i) Kinetic energy in process (8).

$$\tfrac{1}{2}mv^2 = e\overline{V}$$

$$= 3{\cdot}2 \times 10^{-9} \text{ ergs per atom at 2000 volts.}$$

(ii) Heat of recombination in process (7).

$$100{,}000 \text{ cal./gm.-mol} = 6{\cdot}8 \times 10^{-12} \text{ ergs per atom}$$

$$= 4{\cdot}3 \text{ electron-volts.}$$

(iii) Heat of adsorption in process (6). Probably 10,000–20,000 cal./gm.-mol.

(iv) Energy emitted in de-excitation from nth orbit.

If $n = 2$, $E_1 - E_2 = 1{\cdot}6 \times 10^{-11}$ ergs per atom
$\doteqdot 10$ electron volts.

If $n > 2$, $E_1 - E_2 > 10$ and $< 13{\cdot}6$ electron-volts.

It is clear from these data that kinetic energy of impact presents far the greater share of theoretically available energy at the gas-solid interface. This feature of the present investigation may be contrasted with the heating of a tungsten wire discussed in a previous paper[8]; in that case the kinetic energy of molecules was solely that of thermal agitation instead of being derived from large accelerating potentials, and the energy available was mainly the heat of recombination.

But any actual heating of the gas-solid interface depends not only on availability of energy but on efficiency of its transfer: the former we have stated as known, but little is known about the latter, so that it is impossible, in the present rudimentary stage of the kinetics of the interface, to resolve the above alternatives (*a*) and (*b*). The fact that the available energy is very largely kinetic, together with the fact that we find the excess heating due to de-excitation to be as great as 80 per cent, imposes, however, the following conditions on any interpretation of the alternatives:

(A) If the excess heating arises from a direct transfer of energy of de-excitation, the efficiency of the underlying physical mechanism must be large, compared with a very small efficiency of kinetic-energy transfer. It should be noticed that the high efficiencies sometimes attributed to the latter are generally taken from measurements of perpendicular incidence on a plane, and not, as here, on a convex surface of small radius, where impact is mainly oblique and probably inefficient as regards the exchange of energy.

(B) If the excess heating is an indirect consequence of excitation, the latter increasing the efficiency of such energy transfer as takes place by (i), (ii), (iii), the magnitude of (i) compared with the others indicates a greater likelihood that the mechanism of excess heating is an enhancement of the accommodation coefficient, or efficiency of transfer of purely kinetic energy at impact, for excited as compared with normal atoms. This would carry with it a modification of adsorbability and chemical activity, but the actual heating observed more probably represents part of the kinetic energy itself transferred with increased facility.

§ 6. CONCLUSION

The experiments establish that two filaments situated at different points along a beam of positive rays are differentially heated by the impinging beam. The control and screening experiments, and the use of a bridge method, limit the purely geometrical contribution to a verifiable 10 per cent difference, whereas the actual difference between the heating of the filaments rises to 80 per cent. The variation of the effect with the potential is shown to be quantitatively in accord with the

variation in the excess of excited atoms at one wire, calculable from Wien's experiments on the progressive return of neutral rays to normal state. We conclude that the greater part of the excess heating of one of the wires is controlled by the proportion of atoms which have not yet undergone spontaneous de-excitation. Hence the possession of energy of excitation in a neutralized atom enables it either to transfer some of that energy to the solid target, or else to increase the efficiency with which it transfers kinetic energy of impact. In either of these alternatives the beam of neutralized positive rays would exert an excess heating on the wire nearer to its origin, and this excess would vary with potential as we found it to vary, in accordance with the decay curves of excited states discovered by Wien.

§ 7. ACKNOWLEDGMENTS

I am indebted to Prof. S. W. J. Smith, F.R.S., whose kindness and generosity make possible this work, and also to Mr G. O. Harrison for his constructional skill and originality.

REFERENCES

(1) *Proc. R.S.* **129**, 146 (1930).
(2) *Proc. R.S.* **128**, 432 (1930)
(3) *Proc. R.S.* **132**, 67 (1931).
(4) *Proc. R.S.* **124**, 228 (1929).
(5) *Proc. Phys. Soc.* **42**, 490 (1930).
(6) *Ann. d. Phys.* **83**, 1 (1927).
(7) *Proc. R.S.* **116**, 81 (1927).
(8) *Proc. R.S.* **132**, 80 (1931).

DISCUSSION

Prof. E. V. Appleton. I imagine that the atomic stream issuing from the small orifice must contain some charged ions as well as the two varieties of neutralized atoms. It is known from the work of Holst and Oesterhuis, and also from the more recent experiments of Oliphant, that such ions can liberate secondary electrons from metal surfaces. The thermal profit-and-loss account of this process, in so far as the metal target is concerned, will depend on whether the emitted electrons are returned to the metal or whether they are collected by some other conductor which is maintained at a positive potential with respect to the target. I do not think that this effect could possibly upset Dr Johnson's conclusions (which seem to me very satisfactorily established) concerning the differential heating of the two filaments, since, with his symmetrical electrical circuit, the effects on both filaments would probably be identical. But if the experiments are pushed to a quantitatively further stage it would possibly have to be taken into account.

AUTHOR'S reply: Undoubtedly there will be some positive ions left in the stream behind the cathode, though Oliphant has shown that their number may be small in some cases; the cooling of filaments when impact causes electron-emission will only occur if the surroundings are at high enough potential to draw off a considerable electron stream. If this occurs here it will affect both filaments alike, since hardly any change in the number of ions can take place between the two filaments at such low gas pressures. Neutralization needs a collision and will be very rare in the space of a couple of centimetres, whereas return of neutral atoms to ground state is spontaneous and needs no gas. Hence the effect mentioned will be among those eliminated by the bridge, as common to both wires.

ELECTROLYTIC WATER-TRANSPORT AND IONIC TRANSPORT NUMBERS

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ABSTRACT. This paper gives the results of determinations of the electrolytic water-transport per faraday in the case of copper sulphate solutions of concentrations ranging from normal to 1·8-normal. The parchment diaphragm method has been employed; and difficulties experienced owing to the effect of diaphragm-polarization have been surmounted by adopting a special experimental procedure. The electrolytic water-transport per faraday remains constant with increasing concentration of copper sulphate up to normal, as the author has shown in a former paper, but it decreases regularly with further increase in concentration. The probable factors contributing to this decrease are considered. The sequence in the values of the electrolytic water-transport per faraday corresponds with that in the values of the slope of the curve connecting the Hittorf transport number with the electrolyte concentration, but the corresponding values are not in numerical agreement as required by an expression proposed by Riesenfeld and Reinhold. This discrepancy is investigated by means of a more accurate expression derived by the author. The determinations of the electrolytic water-transport are employed to correct the values of the experimental Hittorf transport number.

§ 1. INTRODUCTION

The author, in a former paper*, has examined the relationship between the electrolytic water-transport and the transport numbers of the anion in the case of aqueous solutions of copper sulphate of normal and lower concentrations. It was found that the relationship could be expressed in terms of the slopes of the curves connecting the Hittorf transport number and the true transport number respectively with the electrolyte concentration; and there appeared to be a correspondence between the slope of the former curve and the electrolytic water-transport per faraday, both quantities remaining constant with increasing concentration of copper sulphate to normal.

At concentrations exceeding normal, the slope of the curve representing the Hittorf transport number decreases regularly with increasing concentration; and it seemed desirable to ascertain whether the electrolytic water-transport per faraday suffers a similar decrease. The present paper gives the results of an investigation into this question, the parchment diaphragm method being employed for the direct measurement of the electrolytic water-transport. The use of strong copper sulphate solutions, i.e. solutions of concentration greater than normal, was found to introduce difficulties owing to the effects of diaphragm-polarization, but an experimental procedure has been devised enabling satisfactory measurements to be made with solutions of concentration not exceeding 1·8-normal.

* H. C. Hepburn, *Proc. Phys. Soc.* **43**, 524 (1931).

§ 2. EXPERIMENTAL

The apparatus employed, which was similar to that described by the author in the former paper, consisted of two electrode chambers (one open to the air and the other closed) separated by a diaphragm of parchment paper. The liquid-transport across the diaphragm, when an electric field was applied at the electrodes, was determined, as in the previous work, by observing the movement of the liquid thread in a capillary tube attached to the closed electrode chamber.

The apparatus was immersed in an electrically heated water-bath with thermostatic control, the temperature of the bath being maintained within the limits $18 \pm 0{\cdot}1^{\circ}$ C. Moreover, to avoid thermometric effects, it was arranged that no heat should be supplied to the tank immediately before or during the course of the flow-measurements.

In the author's earlier measurements, which related to copper sulphate solutions of normal and lower concentrations, a potential difference of 20 V. was applied at the electrodes. The application of this voltage in the case of the more concentrated solutions of the present work was found to give rise to a considerable heating effect, the rate of liquid transport, as indicated by measurements made over consecutive lengths of the capillary tube, being less regular than in the case of the lower concentrations. It was therefore decided to adopt a lower working voltage.

Preliminary measurements with a potential difference of 4 V. applied at the electrodes showed that the current, after maintaining a constant value for a short period, slowly fell to a lower value, e.g. from 0·095 A. to 0·060 A. in the case of 1·4-normal copper sulphate solution. The period intervening between the establishment of the potential difference at the electrodes and the observed decrease in current value was found to decrease with increasing concentration. On reversing the direction of the current, the initial value was restored and was maintained for a period considerably in excess of the initial period.

In order to confirm that the effect described was not due to incomplete washing of the diaphragm, a further quantity of solution was allowed to flow through the diaphragm under a hydrostatic pressure head, and the measurement was repeated; a similar result was, however, obtained. It was evident, moreover, that the effect could not be ascribed to the copper electrodes employed, since no decrease in current value was observed when the diaphragm was removed from the apparatus. It appears probable from these results that the effect may be attributed to the establishment at the diaphragm of a polarizing counter-electromotive force which produces an additional apparent resistance in the circuit. In the analogous case of measurement of the conductivity of a collodion membrane in equilibrium with an electrolyte solution, A. A. Green, A. A. Weich and L. Michaelis* have avoided the production of a similar polarization effect by the use of an alternating current.

A similar diaphragm-polarization effect, obtained in the case of the electrolysis of a dilute solution divided by means of a diaphragm of collodion or of pig's bladder or the like, has been explained by A. Bethe and T. Toropoff† in terms of

* *J. Gen. Physiol.* **12**, 473 (1929).

† *Z. Phys. Chem.* **89**, 597 (1915).

the displacements in the ionic concentrations at the diaphragm boundary surfaces. Such concentration changes might be expected to produce a progressive building up of the polarization potential from the moment of establishing the external electric field; but the process, in the initial stages, appears to be a slow one in view of the sensibly steady values obtained for the current through the diaphragm in electro-endosmotic measurements with dilute solutions*. A similar action appears to operate in the case of the copper sulphate solutions employed in the present work, the slow beginning, or virtual time-lag, in the polarization process again being apparent. The effect of destroying the initial polarization potential by reversing the direction of the applied current is apparently to prolong the virtual time-lag in the establishment of a reversed polarization potential.

The decrease in current value referred to on page 268 was accompanied by a decrease in the magnitude of the liquid-transport; and it was only practicable, during the short period the current remained steady, to obtain an approximate measure of the liquid-transport. The current after reversal, however, maintained a steady value for a sufficient time to enable a measurement of liquid-transport to be made over 0·5 cm. of the capillary tube, but the reappearance of the polarization effect precluded satisfactory measurements in the case of solutions whose concentration exceeded 1·8-normal.

The rate of liquid-transport after current-reversal was found to be practically uniform whilst the current maintained a steady value and was in satisfactory agreement with the approximate initial measurement. The progressive displacements in the ionic concentrations at the diaphragm boundary surfaces, referred to above, thus appear to have no material effect on the present measurements.

Determinations of liquid-transport, following the procedure indicated in the preceding paragraphs, were carried out with the solutions in order of increasing concentration; and adjustment was made, as described in the former paper, for gravitational movement and for the thermometric action due to the heating effect of the current. The general procedure, preliminary to the measurements of liquid transport, was similar to that adopted in the former paper; and the initial direction of the applied electric field was selected so that the measured liquid-transport through the diaphragm, i.e. the liquid-transport after reversal of the applied field, was directed towards the closed electrode-chamber.

The direction of liquid-transport throughout the range of concentrations investigated was from anode to cathode.

§ 3. RESULTS AND CONCLUSIONS

The experimental results are given in table 1. $1/T_m$ (reciprocal seconds) is the reciprocal of the time of liquid-transport per cm. of the capillary tube, based upon measurements made over a length of 0·5 cm. of the tube, less the correction for the gravitational movement. The liquid-transport v_f in cm.^{3}/faraday is calculated from the expression $[(1/T_m)(1/I_m)(0{\cdot}00973)(96{,}540)] - [6{\cdot}8]$, I_m being the mean current through the diaphragm in amperes, and 0·00973 the volume of the capillary

T_m

v_f

I_m

* H. C. Hepburn, *loc. cit.*

in $cm.^3/cm.$ The deduction of 6·8 $cm.^3$ represents the correction for the heating effect of the current. The values given for the specific conductivity λ have been obtained by large-scale graphical interpolation of the data of Kohlrausch.

Table 1.

Concentration of $CuSO_4$ (gm.-equiv./litre)	Mean current I_m (A.)	Specific conductivity λ (recip. ohms)	I_m/λ	Reciprocal $1/T_m$ of time of flow per cm.	Liquid transport v_f ($cm.^3$/faraday)
1·0	—	—	—	—	30·6*
1·2	·0854	·0292	2·92	·003383	30·4
1·4	·0947	·0323	2·93	·003500	27·9
1·6	·1010	·0351	2·88	·003520	25·9
1·8	·1110	·0378	2·94	·003556	23·3

* Mean of two determinations with 6 V. applied at the electrodes. See H. C. Hepburn, *loc. cit.*

The electrolytic water-transport per faraday and its dependence on the electrolyte concentration. It will be observed from table 1 that the mean current I_m through the diaphragm is proportional, within close limits, to the specific conductivity λ of the solution. Control experiments showed also that the current value was only slightly increased when the parchment diaphragm was removed from the apparatus. The interposition of the parchment partition in the current path thus appears, during the short periods of measurement here adopted, to produce little change in the electrolytic transference of ions from one electrode chamber to the other.

The water-transport associated with this electrolytic transference of ions is given directly by v_f, table 1, since the specific (electro-endosmotic) effect of the parchment diaphragm may be neglected in the case of the high concentrations here considered†. The values of v_f decrease regularly with increasing concentration above normal, the rate of decrease from concentration 1·2-normal following a linear law. A similar relation to concentration has been found by J. Baborovsky and J. Velesek‡ in the case of solutions of certain alkali chlorides.

It was suggested in the former paper that the constant values obtained for v_f, the electrolytic water-transport per faraday, in the case of copper sulphate solutions of normal and lower concentrations, were probably the result of two antagonistic actions, (*a*) a decrease in the transport number of the cation with increasing concentration, and (*b*) a concurrent increase in the number of mols of water associated with one gram-equivalent of ions; the major effect under this head, since the resultant water-transport was directed towards the cathode, relating to the cations. The rate of decrease of v_f with increasing concentration from normal suggests, however, that the associated water referred to under (*b*) also diminishes as the saturation point is approached. This effect is possibly attributable to changes in the state of molecular aggregation of the solvent, e.g. breaking down of double

† See H. C. Hepburn, *loc. cit.*

‡ *Chem. Listy*, **22**, 265 (1928).

water molecules into single molecules, with decrease in the number of mols of water chemically bound to the ions*. Changes in the magnitudes of the inter-ionic forces and of the forces acting between the ions and the surrounding water molecules, with consequent variation in the number of water molecules carried along by the ions by means of viscous drag†, may also be of importance.

The values of v_f in table 1 relating to concentrations normal and 1·2-normal, viz. 30·6 cm.^{3} and 30·4 cm.^{3} respectively, are somewhat higher than the mean (29·3 cm.^{3}) of the "constant" values obtained in the former work over the concentration range 0·4-normal to normal. The variations involved, however, which appear to be within the limit of error in the former determinations of the current heating correction, have no material effect on the values of the true transport number‡, e.g. the value in the case of normal copper sulphate solution is changed from 0·666 to 0·664.

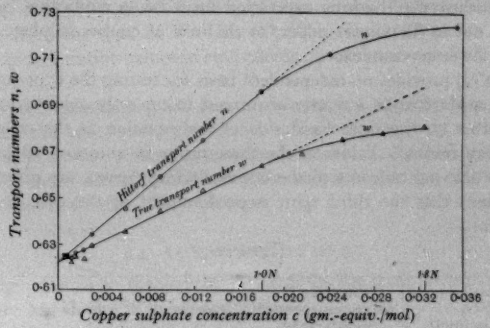

Fig. 1. Transport numbers plotted against concentration.

The relation between the electrolytic water-transport per faraday and the ionic transport numbers. The following equation, which connects the electrolytic water transport x in mols per faraday with the slope of the curve obtained by plotting the Hittorf transport number n against the electrolyte concentration c in equivalents per mol, has been proposed by E. H. Riesenfeld and B. Reinhold§, viz. x n, c

$$x = dn/dc \qquad \ldots\ldots(1).$$

The n/c curve appropriate to the case of copper sulphate solutions is given in figure 1. The values of n have been obtained from the data of H. Metalka‖ (concentrations 0·0212-normal to 0·1639-normal) and of W. Hittorf¶ (concentrations 0·327-normal to 1·96-normal)**. It will be observed that dn/dc remains constant

* See N. V. Sidgwick, *The Electronic Theory of Valency*, pp. 185 *et seq.* (1927).

† See H. C. Hepburn, *loc. cit.* p. 533. ‡ See page 272.

§ B. Reinhold, *Z. Phys. Chem.* **66**, 672 (1909).

‖ H. Jahn and collaborators, *Z. Phys. Chem.* **37**, 673 (1901).

¶ W. Hittorf, *Pogg. Ann.* **89**, 177 (1853).

** No data appear to be available for copper sulphate solutions of concentrations outside the range 0·0212-normal to 1·96-normal.

with increasing concentration to normal, but decreases regularly with further increase in concentration. There is a corresponding sequence in the values of x, the electrolytic water-transport per faraday, but the corresponding values of the two quantities are not in numerical agreement (see table 2), and dn/dc decreases more rapidly than x with increasing concentration from normal.

In order to investigate these discrepancies, it is convenient to examine the differential form of equation (1), arranged as an expression for x, viz.

$$x = dn/dc - dw/dc - c\,(dx/dc) \qquad \ldots\ldots(2).$$

It is easily seen that the simplified form proposed by E. H. Riesenfeld and
w B. Reinhold, viz. $x = dn/dc$, requires that w, the true transport number, and x should be independent of concentration. This condition is not fulfilled in the case of copper sulphate solutions. If x is alone independent of concentration, the term $c\,(dx/dc)$ vanishes, the resulting expression for x being applicable, as the author has pointed out in his former paper, to the case of copper sulphate solutions of normal and lower concentrations.

Equation (2) provides no independent basis for testing the experimental values of x, but it appears to be a matter of interest to trace the relative importance of the three terms on the right hand side of the equation as the concentration is increased from normal. Values of the three terms in question, which have been obtained by drawing tangents to the corresponding curves, are given in table 2. It will be seen that the third term is predominant in the case of the highest concentration.

Table 2.

Concentration c of $CuSO_4$ (gm.-equiv./mol)	Electrolytic water-transport x* (mol/faraday)	dn/dc	dw/dc	$c\,(dx/dc)$
·01806 (1·0 N)	1·70	4·1	2·4	0·0
·02166 (1·2 N)	1·69	2·6	1·4	− 0·5
·02528 (1·4 N)	1·55	1·9	1·2	− 0·9
·02890 (1·6 N)	1·44	1·2	0·9	− 1·1
·03256 (1·8 N)	1·29	0·5	0·4	− 1·2

* $x = v_f/18{\cdot}02$ (see table 1 for values of v_f).

Determination of true transport number. The following relation between the electrolytic water-transport and the ionic transport numbers, appropriate to the case of copper sulphate solutions, has been deduced by E. H. Riesenfeld and B. Reinhold†, viz.

$$w = n - cx \qquad \ldots\ldots(3).$$

Here w is the true transport number, n the experimental Hittorf transport number, c the electrolyte concentration in equivalents per mol, and x the electrolytic water-transport in mols per faraday. The symbols w, n refer to the anion.

The values given in table 3 for the true transport number w, in the case of

† *Loc. cit.*

copper sulphate solutions of concentrations ranging from normal to 1·8-normal, have been computed by means of this equation from the present experimental results for the electrolytic water-transport per faraday.

Table 3.

Concentration c of $CuSO_4$ (gm.-equiv./mol)	Hittorf transport number n	True transport number w
·01806 (1·0 N)	·695	·664
·02166 (1·2 N)	·706	·669
·02528 (1·4 N)	·714	·675
·02890 (1·6 N)	·720	·678
·03256 (1·8 N)	·723	·681

The values of the Hittorf number n have been obtained by large-scale graphical interpolation from the data of W. Hittorf*. The solution-densities required to compute the concentration values in equivalents per mol were specially determined by the author.

It will be seen from table 3 and from the figure that both the Hittorf transport number n and the true transport number w increase regularly with increasing concentration. The rates of increase, however (see table 2), diminish with increasing concentration; and it seems probable that the values of n and w at concentrations exceeding 1·8-normal suffer little variation.

§ 4. ACKNOWLEDGMENT

The author desires to thank Prof. A. Griffiths for facilities provided and for the interest he has shown in this investigation.

* *Loc. cit.*

ON THE MAGNETIC SUSCEPTIBILITIES OF SOME NICKEL COMPOUNDS

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ABSTRACT. The susceptibilities of a number of compounds of nickel in the solid state have been measured at room temperature. The atomic susceptibility of combined nickel is found to remain constant, except in the cases of the anhydrous cyanide and of dioxime compounds. Some of the latter are diamagnetic.

§ 1. INTRODUCTION

Some time ago it was suggested to me by Prof. C. H. Lees that the para- and diamagnetic effects in the molecule of a nickel compound might be capable of being distinguished by a study of the temperature susceptibility relation, and that in particular it might be possible, in the extreme case of a combination of the nickel atom with a strongly diamagnetic acidic radical, to find one effect or the other predominating, according to temperature, so that at a certain critical temperature the sign of the susceptibility would be reversed.

An investigation on these lines has been begun with a series of measurements of the susceptibilities of various nickel compounds at room temperatures, in order to detect any in which the diamagnetism of the acidic radical contributes substantially to the net susceptibility, and the present paper gives an account of these measurements.

§ 2. METHOD

The method used was of the non-uniform-field type, the electromagnet being fitted with specially designed pole-pieces to give a field of such configuration that considerably more latitude than usual was possible in setting different specimens to the same part of the field—this setting constituting the greatest difficulty and principal source of error in the usual non-uniform-field method. The majority of the measurements were made with an apparatus which has already been described*.

For subsequent work on more weakly magnetic substances (cyanides and dioximes) it was necessary to increase the sensitivity of the apparatus. This could have been effected quite simply by fitting a suspension of smaller restoring couple in the torsion balance, but advantage was taken of the opportunity to rebuild the apparatus with several improvements in matters of detail.

* *Proc. Phys. Soc.* **42**, 251 (1930).

The field. As a result of a more extended investigation of the problem of pole-piece design* it became possible to produce a field in which the force exerted on a small para- or diamagnetic body remained constant over a much wider region, so that even the rough setting to a standard position used with the earlier apparatus became unnecessary. The way in which the force exerted on the specimen varied with position in the interpole gap is shown by the curve of figure 1. The pole-pieces used were those described as giving the distribution of figure 10 (*d*) in the paper just referred to, but they were used with a slightly smaller gap in order to flatten out that curve to the more suitable form shown in the accompanying figure 1.

The torsion balance. In place of the simple glass T-tube originally used to contain the torsion balance, a somewhat more elaborate arrangement was built up in order to make the balance more accessible. Glass tubes *G*, figure 2, were cemented

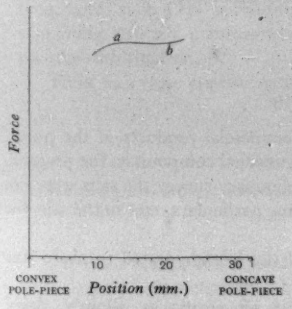

Fig. 1. Distribution of force across the interpole gap.

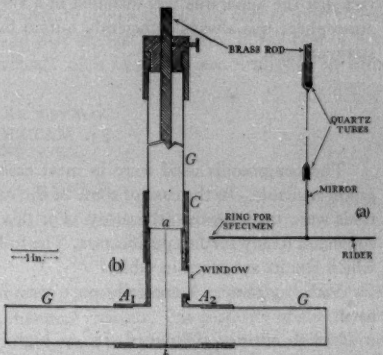

Fig. 2. The torsion balance. (*a*) The suspended system; (*b*) the T-tube.

into a brass T-piece split into two parts A_1, A_2 by a cut along *ab*, over which was screwed a collar *C* carrying the vertical glass tube. The whole was held firmly together when *C* was screwed up, but could be dismantled readily by unscrewing. The two ends of the horizontal tube were closed, as before, by paper caps with glass windows, and a torsion head was fitted to the upper end of the vertical arm.

The balance itself consisted of a rod of specially prepared glass of low susceptibility, to the centre of which was attached a support of copper wire carrying also a galvanometer mirror. The upper end of this wire was cemented into a short length of quartz tube to which the suspending quartz fibre was fused. The upper end of the fibre was similarly fused to a quartz tube which was cemented to a brass rod passing through the torsion head. The specimen was carried on a small wire loop at the end of the arm, and small differences in the weights of different specimens were balanced by a light rider.

* *Proc. Phys. Soc.* **43**, 383 (1931).

Deflections of the system were observed by reflection of a beam of light to a scale at a distance of 2 m. The height of the scale was such that the light spot fell upon it when the rider had been adjusted to bring the specimen to the correct height, i.e. to the axis of symmetry of the field. The diameter of the T-tube, and its position in the field, were so chosen that the specimen was confined to the region *ab*, figure 1, of the field. Thus the only adjustment necessary after a specimen had been placed in the balance was to shift the rider until the spot of light appeared on the scale, showing that the specimen occupied a position on the axis of symmetry of the field.

Since the apparatus consists essentially of an electrically isolated system suspended within a glass tube, electrostatic forces between the fixed and suspended systems might be expected, but were never detected. This was doubtless due to the fact that the apparatus was installed in a very damp basement laboratory, and the atmosphere was always sufficiently humid to prevent the accumulation of electrostatic charges.

§ 3. MATERIALS

The compounds used were in most cases commercial products of the purest grade available. In the case of some of the more unusual compounds, the preparations were made in the laboratory. For this preliminary survey the salts were not subjected to any further purification. The following particulars refer to the salts for which results are given in table 1.

Nickel sulphate. A specially pure sample of the heptahydrated sulphate was used.

Nickel citrate. Obtained in a high state of purity as slightly dried $Ni_3(C_6H_5O_7)_2.14H_2O$, for which the theoretical percentage of nickel is 21·8. The actual percentage was found by analysis to be 22·1.

Nickel potassium sulphate. This was also slightly dehydrated, the percentage of nickel being found to be 14·7. The theoretical value for $NiSO_4K_2SO_4.7H_2O$ is 14·2 per cent.

Nickel benzoate. $(C_6H_5COO)_2Ni.3H_2O$, checked by analysis.

Nickel formate. Certified by the manufacturers as 78 per cent anhydrous $Ni(HCOO)_2$. This corresponds to a composition intermediate between a di- and a trihydrate.

Nickel cyanide. A light green powder, found by analysis to contain 40·32 per cent nickel and about 25 per cent water. This corresponds closely to a composition $Ni(CN)_2.2H_2O$.

Nickel borate. Found by analysis to contain 32·9 per cent NiO and 36·4 per cent B_2O_3. This does not correspond to any exact composition, and it is probable that the product consisted of a mixture of borates, of which a number are known.

Nickel ammonio-sulphate. A bright blue powder, found by analysis to contain 21·2 per cent nickel. A blue compound having the composition $NiSO_4.4NH_3.2H_2O$

is known; it contains 22·7 per cent Ni, and the closeness of the theoretical figure to the observed percentage suggests that the preparation consisted mainly of this compound.

§ 4. EXPRESSION OF RESULTS

Suppose a compound to consist of x per cent nickel, y per cent of an acidic radical, and $(100 - x + y)$ per cent water, whose mass susceptibilities in the combined state are χ_x, χ_y, and χ_w respectively. Then, provided that an additive law is followed, the mass susceptibility χ of the compound will be given by

x

y

χ_x, χ_y, χ_w

$$100\chi = x\chi_x + y\chi_y + (100 - x - y)\chi_w \qquad \text{.....(1).}$$

Investigations on simple salts have already shown that in the case of most simple compounds such a law is followed, and that the last two terms, in the case of salts containing strongly paramagnetic radicals such as nickel, are negligible in comparison with the first.

Thus, as a close approximation, we may write

$$100\chi = x\chi_x,$$

$$\chi_x = 100\chi/x,$$

or, for the gram-atomic susceptibility, χ_A, of the combined nickel,

$$\chi_A = 5869\chi/x \qquad \text{.....(2).}$$

From the results of the measurements, χ_A was evaluated according to expression (2), x having been determined by a chemical analysis. The majority of simple salts gave a nearly uniform value for χ_A, the gram-atomic susceptibility of combined nickel plus the small correcting terms for acidic radical and water. When a divergent value was obtained, the divergence indicated either (*a*) that the contribution of the acidic radical or of the combined water was not negligible in comparison with that of the combined nickel; or (*b*) that the constituents entered into such intimate union that the additive law did not hold; or (*c*) that the effective susceptibility of the combined nickel had a new value.

The magnetic moments have also been expressed approximately in terms of Weiss magnetons by assuming a Curie law ($\chi = C/T$) to be followed. Since, however, solid salts more usually follow a Curie-Weiss law,

$$\chi = C/(T - \theta),$$

these p values* have but little value as such; but the results are useful inasmuch as the fact that the values deduced, with one exception, lie between 15·9 and 16·3 (see table 1) may be taken as an indication that the values of θ for the substances investigated are small.

p

* p is defined as the moment of the gram-molecule, expressed as a multiple of the Weiss unit, 1126 c.g.s.

The observations gave relative susceptibilities. For the purpose of this work they are converted into absolute susceptibilities with sufficient accuracy by assuming Jackson's value for the susceptibility of nickel sulphate—viz. $16{\cdot}0 \times 10^{-6}$ at 292·2° K.*

§ 5. RESULTS

Table 1 gives results for a number of materials which were known to be of reliable purity, and the compositions of which were known with accuracy, as has already been explained.

Table 1.

Salt	Formula	$\chi \cdot 10^6$	Ni (%)	$\chi_A \cdot 10^4$	p
Nickel formate	$Ni(HCOO)_2 + x.H_2O$	23·3	30·8	44·4	16·0
Nickel cyanide	$Ni(CN)_2.2H_2O$	21·0	40·3	30·6	13·3
Nickel borate	?	19·4	25·8	44·0	15·9
Nickel citrate	$Ni_3(C_6H_5O_7)_2 . 14H_2O$	17·5	22·1	46·5	16·3
Nickel sulphate	$NiSO_4 . 7H_2O$	16·0	20·9	44·9	16·1
Nickel ammonio-sulphate	$NiSO_4 . 4NH_3 . 2H_2O$?	15·9	21·2	44·0	15·9
Nickel benzoate	$Ni(C_6H_5COO)_2 . 3H_2O$	12·7	16·5	45·0	16·1
Nickel potassium sulphate	$NiSO_4K_2SO_4 . 7H_2O$	11·1	14·7	44·3	16·0

Although this list includes compounds of various types—mainly with fairly complex organic radicals, the deduced atomic susceptibility of the combined nickel remains nearly constant with a single exception in the case of the cyanide. We thus have strong evidence that the susceptibility of these compounds is due almost entirely to the nickel. The cyanide is dealt with in the next paragraph. With the single exception of the citrate, the compounds listed are all salts of dibasic acids. The citrate, with three nickel atoms to the molecule, gives a value for the atomic susceptibility which is definitely higher than that deduced from measurements on the other salts. The phosphate, also with three nickel atoms to the molecule, gives a similar value, but in this case the result is more uncertain: see table 2.

In table 2, results are given for some compounds of more doubtful composition.

Table 2.

Salt	Formula assumed	$\chi \cdot 10^6$	Ni (%)	$\chi_A \cdot 10^4$
Nickel phosphate	$Ni_3(PO_4)_2 . 7H_2O$	28·2	35·8	46·2
Nickel acetate	$Ni(C_2H_3O_2)_2 . 4H_2O$	17·8	23·6	44·3
Nickel ammonium sulphate	$NiSO_4(NH_4)_2SO_4 . 6H_2O$	11·0	14·9	43·3
Nickel cyanide	—	15·9	—	—
Nickel ferricyanide	—	28·4	—	—
Nickel ferrocyanide	—	11·9	—	—
Nickel fluoride	—	25·9	—	—
Nickel oxalate	—	18·9	—	—
Nickel carbonate	—	32·9	—	—
Nickel chromate	—	30·3	—	—

* *Phil. Trans.* A, **224**, 1 (1923).

In the case of the first three compounds, the formula given was known to be correct apart from some uncertainty as to the exact water-content. The remainder were probably in most cases mixtures of different hydrates, or of normal and basic salts.

Nickel cyanide. The results given in table 1 definitely indicate some anomaly in the susceptibility of the cyanide, and a more detailed investigation of this compound was therefore decided upon. Since the observation of this anomaly*, Bose† has called attention to the low susceptibility of nickel cyanide, and suggests that in the completely anhydrous state the compound will be found to be diamagnetic.

In the first instance, three samples of commercial "pure" nickel cyanide have been investigated. On dehydrating at 180 to 200° C., each sample is reduced to the brick-coloured anhydrous cyanide, with a considerable reduction in susceptibility. The susceptibility of the dehydrated salt appears to be a constant, characteristic of each sample; thus the cyanide for which results are given in table 1 always lost 25 per cent in weight, the susceptibility falling to about 15×10^{-6}; that quoted in table 2 lost 40 per cent, with a reduction of the susceptibility to 8×10^{-6}. The third sample, dehydrated at from 180 to 200° C., and cooled over phosphorus pentoxide, gave the lowest susceptibility so far observed, namely $+3 \times 10^{-6}$.

The observations are not yet sufficiently complete to permit definite conclusions to be drawn, but the results are not incompatible with the theory that the pure anhydrous salt is diamagnetic, since the normal method of preparation of the compound is very liable to introduce impurities, chiefly double cyanides. The results would be explained if the different specimens contained different proportions of a paramagnetic impurity.

Experiments are now in progress with nickel cyanide so prepared as to prevent the introduction of such impurities.

Observations on the ferri- and ferrocyanide are listed in table 2; it was difficult to obtain consistent results, especially in the case of the ferricyanide, and the values given must be regarded as approximate only.

Dioxime compounds. The chemical analysis which had to be carried out during the work involved the preparation of nickel dimethylglyoxime in a high state of purity, and measurements of the susceptibility of this, one of the most complex compounds available, were made. It was found to be feebly diamagnetic. The similar nickel α-benzil dioxime compound was likewise found to be diamagnetic, but one prepared from methylpropyl ketone was slightly paramagnetic. The susceptibility in each case was of the order 10^{-7}; more precise measurements will be made when the apparatus has been modified to permit of a study of susceptibilities at various temperatures.

These are the only nickel compounds which the work has so far shown to be diamagnetic. It is interesting to note that, of the simple salts examined, the brick-coloured anhydrous cyanide is practically the only one not possessing the green or yellow colour characteristic of most nickel salts, and it is the only one with an anomalous susceptibility. On hydration, the compound becomes green, and the

* The measurements were made in 1928.

† *Nature*, **125**, 708 (1930).

susceptibility approaches a normal value. The composition of the red chromate was not known with sufficient accuracy to allow the atomic susceptibility to be deduced. The more complex red and orange dioxime compounds also show anomalous susceptibilities.

§ 6. ACKNOWLEDGMENTS

This work has been carried out at East London College, with the aid of grants from H.M. Department of Scientific and Industrial Research. My best thanks are tendered to Prof. C. H. Lees and Prof. H. R. Robinson for their advice and assistance.

ELECTRONS AND LIGHT QUANTA

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ABSTRACT. The question whether electrons moving with or against a stream of photons are accelerated or retarded by the momentum of the latter is considered. A study based on known physical constants suggests that the magnitude of such an effect should be measurable in the case of strong sunlight and *a fortiori* of X-rays, but an experimental test has yielded negative results. The conclusion is reached that either photons do not exist as defined mass particles during the transmission of light-energy, or else that the apparent area covered by photons in a beam of radiation or by electrons in an electronic current is very small in comparison with the cross-section of the beam or current.

§ 1. INTRODUCTION

It is well known that whilst many optical phenomena can be explained by a wave theory of Light in which the energy is distributed without discontinuity over the wave front, there are other phenomena such as photo-electric effects, the ionization of gases, and also the distribution of energy along the spectrum that seem to demand a quantum theory in which radiant energy is assumed to be emitted or absorbed by atoms in certain definite units.

The effects of interference and diffraction are perfectly explainable in terms of a wave theory in which the oscillating vectors of the light wave, which in accordance with Maxwell's theory are taken to be respectively electric and magnetic forces or fluxes, are at right angles to each other, in step as regards phase, and both normal to the direction of propagation of the wave. The wave-energy or light-intensity is then capable of infinitely gradual variation, and varies as the square of the amplitude.

On the other hand, all those electro-optic phenomena in which electrons enter or leave atoms in exchange for radiation are not explainable, apparently, unless we assume that the light energy is absorbed and emitted in quanta such that the energy of each quantum or photon is numerically equal to the product $h\nu$ where ν is the wave frequency and h is Planck's constant of action, the best value of which is $6{\cdot}5543 \times 10^{-27}$ erg-sec.

Moreover, the pure undulatory theory has never been able to explain the distribution of energy in the spectrum and the fact that it reaches a maximum value for some wave-length intermediate between very long and very short waves in the case of the radiation of a black body in a state of incandescence. The theory of light quanta alone has given an explanation of this fact and Max Planck has deduced a well-known formula for the energy density of radiation of a black body expressed

in terms of wave-length or frequency. On this theory the energy-density of radiation can only increase or decrease by steps or finite units and the intensity is expressed by the number of light-quanta per cm^3.

Einstein put forward in 1905 the suggestion that the energy of radiation whilst being transmitted is concentrated in certain packets or quanta each of energy $h\nu$ and that these travel as such through space with the velocity of light. According to this view a beam of light more resembles a shower of rain than a pure wave motion. Just as the rain consists of discrete drops of water so the light is thought to consist of drops of energy called photons. But this view is perfectly inconsistent with the known facts of interference and diffraction.

G. I. Taylor and also G. P. Thomson have shown that even when light is so feeble that 2000 hours of exposure of a sensitive photographic plate was necessary to secure with it a photograph, it was possible with such feeble light to photograph diffraction-patterns quite sharply which never could be seen directly by the eye since the intensity was so low that only about 1 quantum or photon was contained in 10,000 cm^3.* It is impossible to understand how interference can take place unless at least two quanta arrive simultaneously at the same point by different paths.

Hence an eminent physicist, the late Dr H. A. Lorentz, has said, "It must I think be taken for granted that the quanta can have no individual and permanent existence in the ether, that they cannot be regarded as accumulations of energy in certain minute spaces flying about with the speed of light." In other words, the quanta do not travel through space as such and cannot be regarded as constant spatial entities like electrons or protons.

Seeing then that neither a pure wave theory nor a pure quantum theory of light will explain all known optical phenomena, a compromise has been effected by the assumption that both waves and photons exist, but that the photons carry the energy and the waves guide the photons so that the number of photons per cm^3 is proportional to the energy of the light. It may be objected that a wave conveying no energy is an impossible conception, but we might evade this difficulty by the assumption that the energy of the true wave is very small compared with the energy concentrated in the photons. If then a beam of light can in any way be regarded as a stream of photons or light quanta each having energy and mass, and if the wave energy is electromagnetic in nature, the question arises whether a stream of electrons moving along a beam of light would have their velocity increased or diminished according as they moved with or against the direction of propagation of the light.

§ 2. THEORETICAL CONSIDERATIONS

As will be mentioned presently this question has been tested experimentally, but as a first step it will be of advantage to inquire what theory has to say about the matter.

* See J. H. Jeans, *Report on Radiation and the Quantum theory*, p. 80 (Phys. Soc. 1924).

Planck's formula for the energy-density E_ν of radiation of a black body lying between the frequency limits ν and $\nu + \delta\nu$ is

$$E_\nu = u_\nu d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} d\nu \qquad \text{.....(1)},$$

where $c = 3 \times 10^{10}$ cm./sec. and is the velocity of light, $h = 6{\cdot}55 \times 10^{-27}$, T is the absolute temperature of the black radiator, and k is the gas constant per molecule or $1{\cdot}37 \times 10^{-16}$. Suppose then we apply this formula to the most intense light available, namely brilliant tropical sunlight.

The solar photosphere may be taken to be a black-body radiator at a temperature of about 6000° K. The temperature varies from 6760° K. at the centre of the solar disc to 5400° K. at the edge. The wave-length λ_m corresponding to the maximum energy-density of the solar spectrum is 4700×10^{-8} cm. by observation*.

By Wien's law the product of the absolute temperature of the radiator and the wave-length of maximum energy-density must be equal to 0·2940, or $T\lambda_m$, so that $T = 0{\cdot}294/\lambda_m$. Hence $T = 6254°$ K. which is within the limits of observation.

We have next to settle the value of $d\nu$ numerically. Let us take a range of frequency from about two octaves below the red end of the spectrum, namely $100 \times 10^{12} \sim$, to about an octave beyond the violet end or to $1000 \times 10^{12} \sim$. The difference between these extreme values is 900×10^{12}. If we suppose this spectrum cut up into a million slices, each slice may be considered to be approximately constant in frequency and hence $d\nu = 9 \times 10^8 = 10^9$ nearly.

Supposing then we take ten equidistant values of the frequency ν from 10^{14} to 10^{15} covering the thermal spectrum, and assuming that $d\nu = 10^9$, calculate by Planck's formula (1) the corresponding energy-density for each of these frequencies. I find by trial that when the value of T is taken as 6000° K., so that $kT = 8{\cdot}22 \times 10^{-13}$, the frequency of the maximum energy-density of sunlight would fall at a frequency of 3×10^{14} instead of at 6×10^{14} as it actually does.

Hence I infer that for temperatures as high as 6000° K. the temperature T enters into the exponential function in Planck's formula at a power slightly higher than the first power. I find by trial that to comply with Wien's law we must take kT in Planck's formula to have a value of 13×10^{-13} for bright sunlight. Inserting this value and then calculating the energy-density of radiation for the stated frequencies we have the results given in table 1. It will be seen that the maximum energy-density then falls, as it should do for sunlight, at a frequency of 6×10^{14}.

Also since the energy of each photon or quantum is $h\nu$, if we divide out Planck's expression for the energy-density by $h\nu$ we shall have the value as given in the last column of table 1 for the number of photons or quanta per cm.^{3} for each frequency, on the assumption that these quanta or photons actually exist as such in the transmitted radiation. The values of the energy and photon-density for sunlight are plotted out in terms of the frequency in figure 1, and are taken

* See *Handbuch der Astrophysik*, **4**, 30.

from the calculated values at each frequency as given in table 1. It will be seen that the energy-density values rise to a maximum at the proper frequency and that the photon-density also rises to a maximum but not at the same frequency.

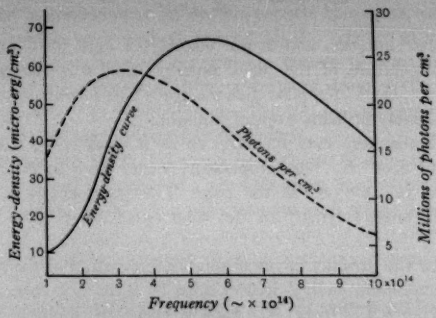
Energy-density (micro-erg/cm³)
Millions of photons per cm³
Energy-density curve
Photons per cm³
Frequency (~ × 10¹⁴)
10 20 30 40 50 60 70
5 10 15 20 25 30
1 2 3 4 5 6 7 8 9 10×10¹⁴

Fig. 1. Planck's energy-density curve for sunlight.

Table 1. Energy-density and photon-density for sunlight.

$T = 6000°$ K., $kT = 13 \times 10^{-13}$, $d\nu = 10^9$, $h = 6{\cdot}55 \times 10^{-27}$ in Planck's formula.

Frequency of radiation ν $\sim \times 10^{12}$	ν^3 $\sim^3 \times 10^{42}$	$h\nu$ (c.g.s. units $\times 10^{-13}$)	$h\nu/kT$	$\epsilon^{h\nu/kT}$ or x	Energy-density (ergs/cm³ $\times 10^{-6}$)	Photons per cm³ ($\times 10^6$)
100	1	6·55	0·5	1·648	9	13·8
200	8	13·1	1·0	2·718	20	21·4
300	27	19·65	1·508	4·518	46	23·4
400	64	26·2	2·01	7·463	59·4	22·6
500	128	32·75	2·52	12·431	65·6	20·0
600	216	39·3	3·02	20·492	66·7	17·0
700	343	45·9	3·53	34·125	62·1	13·5
800	512	52·4	4·03	56·264	55·5	10·6
900	729	59·0	4·54	93·705	47·2	8·0
1000	1000	65·5	5·04	154·471	39·2	16·0
				Mean value:	$47{\cdot}87 \times 10^{-6}$	$15{\cdot}7 \times 10^6$

It will be seen from table 1 that the average energy-density of sunlight is near to 48 micro-ergs per cm³ and the average number of photons is near to 15 million per cm³. This value, obtained as the mean of 10 equidistant ordinates, must be somewhat less than the true mean value.

Maxwell has shown* that the number which denotes the energy-density of radiation denotes also the pressure of radiation per square unit. In the experiments on light-pressure made in 1903 by Nichols and Hull† these observers found that

* *Treatise on Electricity and Magnetism*, 11, 401 (2nd ed.).

† *Proc. Amer. Acad. of Arts and Sci.* 38, 539 (1903).

the light-pressure did agree with the energy-density to within 1 per cent, and that this light-pressure for sunlight would be about 50 microdynes/cm^2.

Again, we have numerous measurements of the solar constant, that is the heat in calories per minute produced when full sunlight falls normally on 1 cm^2 of a black surface. This varies from time to time, but a good average value is 1·94 cal./cm^2-min. Translating this into ergs per second we find it to be 1·36 million ergs/sec. Hence dividing by 3×10^{10} we have the mean energy-density of sunlight as 45 micro-ergs/cm^3, which is in fair agreement with our calculated value of 48 micro-ergs from Planck's formula. This value derived from the solar constant may however also be too low, as correction for atmospheric absorption is difficult.

Furthermore we can check this result by the observed values of Stefan's constant. Stefan's law is that the radiation per cm^2 of a black body is proportional to the fourth power of the absolute temperature multiplied by a constant, and the mean of 12 determinations of this constant σ as given by Fritz Reiche* is $5{\cdot}8 \times 10^{-5}$ σ ergs/cm^2-sec.

The solar surface-temperature is 6000° K. and hence $T^4 = 13 \times 10^{14}$ and $\sigma = 5{\cdot}8 \times 10^{-5}$. Therefore $\sigma T^4 = 75 \times 10^9$. Now the radius of the earth's orbit is $14{,}945 \times 10^9$ cm. and the radius of the sun is $69{\cdot}5 \times 10^9$ cm. The ratio is 215 to 1 and the square of that ratio is 46,225. The radiation from the sun at its surface is thus 75×10^9 erg/cm^2-sec. and dividing this by 46,225 and again by 3×10^{10} we have for the mean energy-density of solar radiation at mean earth distance 54 micro-ergs, which is not far from the results obtained from Planck's equation and from those of the measurements of light-pressure.

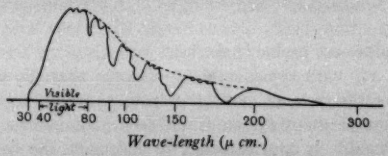

Fig. 2. Bolometer curve for bright sunlight with energy-density plotted in terms of wave-length.

A fourth method by which we can obtain a value for the mean energy-density of sunlight is the integration of the bolometer curve of sunlight. When a bolometer wire is passed along a solar spectrum the readings at every stage give us an ordinate of the radiation-energy curve, but owing to the absorption bands in the spectrum this curve has irregular dips in it; see figure 2. The integration of Planck's equation gives us the area included by the envelope or dotted line, but the determination of the solar constant or integral thermal value gives us the area included by the bolometer curve.

The curve in figure 2 is an observed bolometer curve for sunlight. Dr H. M. Barlow determined for me the ratio between the areas of the envelope (dotted)

* *The Quantum Theory*, p. 129.

curve and of the bolometer curve with an Amsler's planimeter and found it to be 1·135:1. The integration of Planck's equation is easily effected by expanding it in an exponential series.

Then since $$\int x^3 e^{mx}\,dx = e^{mx}\left(\frac{x^3}{m} - \frac{3x^2}{m^2} + \frac{6x}{m^3} - \frac{6}{m^4}\right),$$

it is easy to prove that

$$\int_0^\infty \frac{8\pi h}{c^3}\left(\frac{\nu^3}{e^{h\nu/kT} - 1}\right) d\nu = \frac{6A}{B^4}\,1{\cdot}082,$$

where $A = 8\pi h/c^3$ and $B = h/kT$.

When the proper numerical values have been put in, the numerical value of the integral is found to be 62·32. But this is the value of the envelope area. Dividing by 1·135 gives us 54·9, or say 55 micro-ergs/cm.^{3} as the mean energy-density.

Hence the value (55 micro-ergs) obtained by the corrected integration of the Planck equation agrees quite closely with that obtained from the Stefan constant, and the four values are as in table 2.

Table 2. Values of mean energy-density of sunlight (micro-ergs/cm.^{3}).

From the mean of 10 equidistant ordinates of the Planck curve ...	48
From the solar constant	45
From the Stefan constant	54
From the integration of the Planck equation	55

The last two values are probably the best.

Hence we can say with considerable confidence that the energy per cm.^{3} of bright sunlight at earth distance is not far from 54 micro-ergs. Bright sunlight gives an illumination of about 13,000 candle-feet at normal incidence on a white surface, and this enables us approximately to determine the radiation-density for other beams of light and also the average number of photons per cm.^{3}, if they actually exist as discrete entities in a beam of light, and we can then calculate the total mass and momentum of these photons and compare it with the mass and momentum of any possible number of electrons which may be set moving along the beam.

e By the relativity theory a stream of photons having total energy e should have a total mass e/c^2 and a total momentum e/c where $c = 3 \times 10^{10}$. Hence for bright sunlight the total photon mass per cm.^{3} should be $6/10^{26}$ gm. and the total momentum $18/10^{16}$ c.g.s. units or moms.

It is convenient to have a word to express a momentum equal to that of 1 gm. moving with a velocity of 1 centimetre per second. I suggest (with diffidence) the *mom* word *mom* for this unit of momentum. The momentum communicated to a black surface per cm.^{2} per sec. by bright sunlight is close to 54 micromoms, and the power expended on it is 162×10^4 ergs/sec.

Let us compare these last figures with those of the energy and momentum of an electron current of 1 mA./cm.^{2} flowing *in vacuo* under a voltage-drop of 1 V./cm. This current is equal to 6×10^{15} electrons per second each having a mass of 9×10^{-28} gm. or a total mass of 54×10^{-13} gm. The power expended per cm.^{2} is 10^4 ergs/sec. and hence the velocity of the electrons is given by $v = \frac{2}{3} \times 10^8$ cm./sec. nearly, and the total momentum cm.^{2} is 337 micromoms, nearly.

§ 3. EXPERIMENTAL TEST

These figures showed that it might possibly be worth while to try whether electrons were hindered or helped in moving against or with a beam of bright sunlight by the momentum of the photons if they exist in such beam.

A preliminary experiment with an arc lamp having shown no result, it was decided to go to the opposite extreme and make use of a very powerful beam of X-rays, the energy of each photon in it being about 10,000 times greater than that of a photon in bright sunlight. The following apparatus was then prepared to test this supposition.

The Director of the General Electric Company's Research Laboratory at Wembley near London was so good as to make for me a special form of Fleming thermionic valve. This valve had a dull-emitter filament which gave quite sensible electron-emission when the filament was not visibly incandescent. It gave this emission when a voltage of 2 volts was placed on the filament terminals, and the filament current was then 0·9 A. The valve had two grid anodes of rectangular shape formed of metal wire fixed on either side. Each grid anode was formed of 18 vertical wires and 27 horizontal wires and had an area of 9 cm.^{2} Each anode was 1 cm. from the filament between them (see figure 3).

Hence when the filament was made active and a positive voltage was put on the anodes an electron current flowed to them. This electron current flowed from the filament in opposite directions to the two grid anodes. If then a beam of radiation was passed transversely across the valve the electrons on one side of the filament moved against the radiation and on the other side with it. A powerful radiation was provided by an X-ray water-cooled hot-cathode tube called a Müller structure tube, which was kindly lent for this purpose by the Chemical Department of University College, London.

The cathode of the X-ray tube was heated with a current of 4·5 A. at 12 V. The potential-difference of cathode and anode was created by 60,000 V. alternating at 50 ~, and the electron current was 21 mA. This tube gave an intense X-ray beam coming out through a circular hole about 1 cm. in diameter, which beam was sent across the thermionic valve transversely to the mesh anodes.

In order that the thermionic valve might be shielded from the electron field of the X-ray tube it was enclosed in an earthed metal case, so arranged that the valve could be rotated quickly to bring either anode nearest to the X-ray tube (see figure 4).

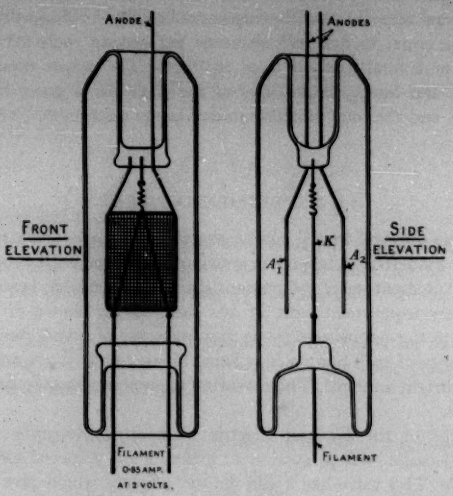

Fig. 3. Special two-anode Fleming thermionic valve with dull-emitter filament.

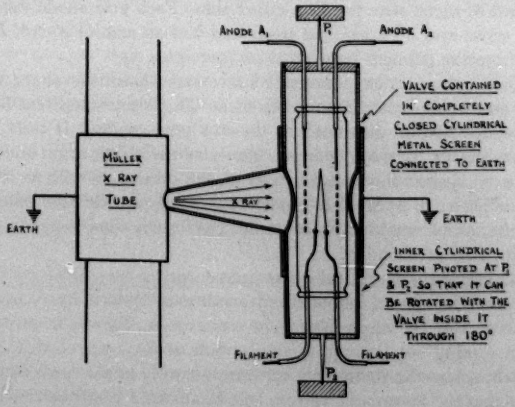
ANODE A_1
P_1
ANODE A_2
VALVE CONTAINED IN COMPLETELY CLOSED CYLINDRICAL METAL SCREEN CONNECTED TO EARTH
MÜLLER X RAY TUBE
X RAY
EARTH
EARTH
INNER CYLINDRICAL SCREEN PIVOTED AT P_1 & P_2 SO THAT IT CAN BE ROTATED WITH THE VALVE INSIDE IT THROUGH 180°
FILAMENT
FILAMENT
P_2

Fig. 4. Two-anode thermionic valve with earthed metal screen round it.

The two grid anodes of the valve were connected to two circuits joined respectively to the two coils of a two-coil Kelvin differential mirror galvanometer of great sensitiveness, and resistances R_1, R_2 were interposed to regulate the currents as shown in figure 5. The anode voltage was supplied by a battery in the common circuit. With an anode voltage of 4 volts the currents through the circuits of the differential galvanometer were each $7{\cdot}4 \times 10^{-6}$ ampere. This then is the electron current to each grid anode of area 9 cm^2, the current-density being 0·82 μA./cm^2.

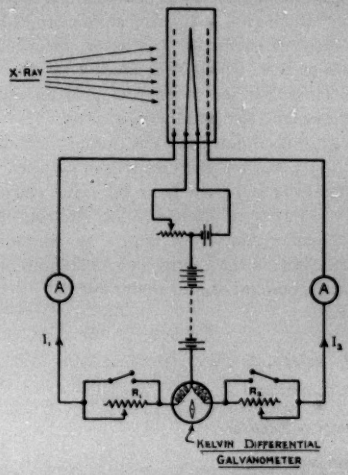

Fig. 5. Arrangement of circuits with two-anode valve for testing the effect of an X-ray beam upon a stream of electrons.

The experiment then consisted in noting whether the incidence of the X-ray beam produced any steady variation from equality in the two electron streams to the two anodes of the valve. No permanent change was however observed in these two electron currents under the continuous action of the X-ray beam. The galvanometer could have detected any change in each electron current which was greater than $3{\cdot}5 \times 10^{-9}$ A.; that is, it could have indicated a change of 1 part in 2000 or 0·05 per cent of the anode electron-currents to the grids. But no change as great as this was found. Hence to that extent the X-ray beam does not alter the electron stream. The same negative result was found when the thermionic current in the valve was given other and greater values.

We can make a rough estimate of the relative energy and total momentum of the electron stream and the X-ray beam as follows. We cannot estimate the energy-density of the X-ray beam by any calorimetric measurements as in the case of sunlight, but we may assume as an approximation that every electron which comes

out of the hot cathode of the X-ray tube and strikes the anticathode generates one photon or quantum and that this is driven against the grid anodes of the valve.

Now the cathode current of the X-ray tube was 21 mA., equal then to 126×10^{15} photons per second striking the grid anode of area 9 cm.², or to 14×10^{15} photons per sec. per cm.² The frequency of this X-ray is roughly 10,000 times that of visible light, and hence the energy of each X-ray photon would be about $10{,}000 \times 36 \times 10^{-13}$ erg by table 1, to 36×10^{-9} erg. The total energy of all the photons would then be 504×10^{6} ergs per cm.² of anode. The total momentum is obtained by dividing this last number by 3×10^{10} which gives 168×10^{-4} mom or 16,800 micromoms per cm.²

When an anode voltage of 4 volts was put on the thermionic valve the current to each grid anode was 7·4 μA., which is equivalent to $44{\cdot}4 \times 10^{12}$ electrons per sec., or to 5×10^{12} electrons per sec. per cm.² of grid surface. The mass of each electron, neglecting the small relativity increase, is 9×10^{-28} gm. and hence the total electron mass is 45×10^{-16} gm./cm.² The power given to each anode is then nearly 300 ergs/sec. or 33·3 ergs/sec. per cm.² of grid surface.

Hence if m is the total electron mass and v the r.m.s. electron-velocity we have $\frac{1}{2}mv^2 = 33{\cdot}3$ ergs or $v = 1{\cdot}25 \times 10^{8}$ cm./sec. The electron-momentum expended per second against each cm.² of grid surface is 56×10^{-8} mom or 0·56 micromom.

Accordingly we can compare the energy and momentum of the above electron stream and photon stream per cm.² of grid as in table 3.

Table 3.

Imparted per second	By electron stream	By X-ray beam
Energy (ergs per cm.² of grid)	33·3	504×10^{6}
Momentum (micromoms per cm.² of grid) ...	0·56	16,800

Hence even making the widest allowance for errors or rough approximation we see that the energy and momentum of the photons in the X-ray beam must have been enormously greater than that of the electron stream flowing to each grid anode; yet they did not affect that current even to the extent of 0·05 per cent.

On the other hand it may not be altogether a question of total momentum but of the chances of a collision between an electron and a photon, which may depend on their relative size and number. In the present case we estimated that 5×10^{12} electrons struck each grid anode per sec. per cm.² and that their velocity was 125×10^{8} cm./sec. Hence there must be 4×10^{4} electrons per cm.³ Taking the square of the cube root of this number, viz. 1169, shows us that in this case there are about 12 electrons per mm.² of cross-section of the current.

Again we estimated that 14×10^{15} photons moving at 3×10^{10} cm./sec. would strike every cm.² of grid area. Hence 466,000 photons should exist in every cm.³ This gives about 6000 per cm.² of section or 60 photons per mm.² The chance of any considerable number of photons or X-ray quanta colliding with an electron

and reducing or decreasing its momentum must depend on the actual and relative size of electrons and quanta, and this is indeterminate.

On certain assumptions we may consider that the diameter of an electron is 10^{-13} cm. and its apparent area 10^{-26} cm^2, but we know nothing about the possible size of light quanta. The absence of any effect upon the electron speed produced by the X-ray beam may be due to the very small probability of a photon hitting an electron, and not to the entire absence of photons as mass particles. The fact that when radiation energy enters or leaves an atom it does so in certain quanta of defined amount, $h\nu$, does not necessarily imply that these quanta travel through space as defined and discrete particles of energy, maintaining so to speak their identity during travel. If for instance water were being put into a canal in whole bucketsful at a time in one place and drawn out in whole bucketsful at another place the water would not travel along the general mass of the canal water as bucketsful.

Again, even if the quanta or photons have any objective existence in space the probability of a collision with electrons may be a question of dimensions. Thus if we were shooting with a shot-gun at a covey of birds covering a certain area of the sky the chance of bringing down a bird would be greater the larger the bird and the larger the shot.

In the well-known Compton effect, A. H. Compton found that light quanta or photons could apparently rebound from electrons tethered in atoms and that the ordinary dynamical laws of impact were obeyed*. In the case of the experiment above described with the thermionic valve, the electrons were free and distributed widely over a large cross-section area. Hence the two cases are not identical and from the fact of the Compton effect we cannot infer that there are necessarily free photons in an X-ray beam or in a beam of light†.

§ 4. CONCLUSION

The conclusion which seems legitimate from the negative results of the above described experiments is either that the light quanta or photons do not exist as defined mass particles during transmission of light energy, or else that the total apparent area covered by the photons in the cross-section of a beam of radiation must be relatively very small compared with that cross-section, and the same also for a current of electrons flowing with or against that beam. Or to put it in another way, the probability of a photon and an electron occupying the same area at the same time is very small, since the ideas now current in connexion with the wave-mechanics theory of matter forbid us to think of the electron or of the photon as having any definite size.

* A. H. Compton, *Phys. Rev.* **21**, 207, 483 (1923).

† W. M. Hicks, "The Nucleus as Radiator." *Phil. Mag.* **8** (1929).

§ 5. ACKNOWLEDGMENT

Lastly I must record my thanks to Professor W. C. Clinton, B.Sc., for affording me facilities for these experiments in the Electrical Engineering Laboratories of University College, London, and also to Professor E. G. Coker, F.R.S., for kindly allowing facilities for conducting them in a room in his laboratory and particularly to Dr H. M. Barlow for carrying out all my suggestions for these experiments with great skill and care.

DISCUSSION

The PRESIDENT read the following extract from the minute-book:

"A meeting of the Society was held in the Physical Laboratory, South Kensington Museum at 3 p.m. on Saturday, March 21, 1874,—Dr Gladstone in the Chair. There were about 35 members present.

"The Chairman gave a brief description of the objects and organization of the Society and noticed the very favourable circumstances under which this Society originated as compared with those attending the origin of its parent the Royal Society. He announced that ninety-nine gentlemen had already expressed their desire to join the Society as original members.

"J. A. Fleming, B.Sc., read a paper on the new contact theory of the galvanic cell. Professor F. Guthrie exhibited experiments illustrating the distribution of a galvanic current on entering and leaving a conducting medium."

Dr W. N. BOND. The author mentions the difficulty of measuring the amount of energy conveyed by the X-ray beam; and he makes a rough estimate by assuming that each electron which strikes the anti-cathode of the X-ray tube generates one photon or quantum, which then passes through the thermionic valve. Calorimetric measurements of the energy conveyed by an X-ray beam have, however, been made by several physicists. In calorimetric experiments that Prof. J. A. Crowther and I carried out*, the conditions seem to have been comparable with those described in the present paper. We used a Shearer tube with molybdenum anti-cathode, working on a transformer. When the peak value of the applied voltage was 55,000, the current about 3 mA., and the mean wave-length 0·6 Å., energy was conveyed by an X-ray beam of solid angle 0·06 at the rate of a calorie in two hours. This would imply that the rate of supply of energy by the X-ray beam was only about 1/10,000 of that estimated (table 3). If the present experiments were used to place an upper limit to the size of the photon, this correction would have to be applied.

Dr L. SIMONS. In connexion with this work it is appropriate to say that Hughes and Jauncey† endeavoured to detect collisions between photons by directing two focussed beams of light towards one another. Observations for scattering effects were made at the point of impact of the two beams, with negative results. The

* *Phil. Mag.* **6**, 401 (1928).

† *Phys. Rev.* **36**, 773 (1930).

conclusion was reached that the cross-section area of a photon was not greater than $10^{-10}\lambda^2$.

Dr D. Owen. This is certainly a highly interesting experiment on a subject of importance in regard to present-day theories of photons and the electron. It has a special interest also in that the electronic valve is used by its distinguished inventor as an instrument of research into the recondite relations between electricity and light. From the negative results obtained, Sir Ambrose concludes that either light quanta do not exist, or that the chances of collision of a light quantum or photon with an electron, when beams of these traverse one another, must be very small. The former conclusion would seem to be already disposed of by the fact of the Compton effect, the existence of which has been amply demonstrated not only as regards the change of frequency of the scattered radiation, but also (from the work of C. T. R. Wilson and others on cloud tracks) as regards the velocity of the recoil electrons. It therefore seems that experiments of the type described in the paper should be regarded as means of determining, or at least of assigning a lower limit to, the chances of collision of a photon and an electron. From the results in table 3 the ratio of the momentum per cm.^{3} of photons in the X-ray beam and of electrons between grid and anode of the thermionic valve is 16,800/0·56, that is, 30,000. The least detectable change of momentum of the electron beam is found to be 1/2000. The product of these factors is 60×10^6. This figure, however, appears to be a considerable over-estimate in two respects. First, the assumption that every electron impinging on the anti-cathode of the X-ray tube generates a photon of wave-length 0·5 Å. gives an efficiency of conversion of electrical energy into X-ray energy which is obviously far too great, working out indeed to be practically unity, whereas a figure of 1/1000 is probably nearer the mark. Next, the calculation assumes that all the photons liberated at the anti-cathode of the X-ray tube find their way to the anode of the valve, whereas it is only the fraction within the solid angle subtended by the anodes at the anti-cathode that are operative, and from the dimensions of the apparatus this fraction appears to be about 1/150. Thus the figure of 60×10^6 is reduced to about 400. If it is now assumed that every encounter of photon with electron is head-on, the negative result of the experiments would be accounted for if fewer than one photon out of 200 makes such a complete collision in its passage through the electrons within the valve. Such a conclusion is not in the least surprising when it is remembered how small is the absorption coefficient for X-rays of even a gas at atmospheric pressure, and that whereas here some 10^{19} gas molecules are present per cm.^{3}, in the grid-to-anode space of the valve the number of electrons per cm.^{3} is only 40,000.

The difficulties attending the theoretical aspect of experiments of this nature are illustrated by a quotation from Sir James Jeans's *Report on Radiation and the Quantum Theory*: "A free electron cannot accept a quantum of radiation under any circumstances whatever.... The simplest structure which can absorb a quantum of energy is a complete atom." (2nd ed., p. 83.) So that, after all, the experiment was foredoomed to be hopelessly negative!

AUTHOR'S reply. A negative result in an experiment is never so interesting as a positive result; nevertheless, the negative result may not be without value if the conditions of the experiment or assumptions made in deductions from it are carefully stated; and this I endeavoured to do in the present paper.

I am glad to be corrected by Dr W. N. Bond on the point of the calorimetric measurement of X-ray energy, and I regret that I had overlooked the paper by Prof. J. A. Crowther and himself. I pointed out, however, that even if the widest allowance is made and if only 1 photon is assumed per 1000 incident electrons in the X-ray tube, the excess of photonic momentum in the thermionic valve is very great. I quite admit that my means of determining any change in the electron current of the thermionic valve by the X-rays was not extremely refined; yet, even in face of the weighty opinion of Dr Owen and the deductions from the Compton effect, the fact remains that no one has given any sufficient proof that a photonic theory of light will explain interference and diffraction effects or the manner in which waves without energy can guide photons with energy so as to produce these effects.

If time had permitted I should have endeavoured to show from certain facts in radio-telegraphy that a pure wave theory may yet explain how it is that only a very few atoms of a gas are ionized by an X-ray beam, a fact which has always been one of the supports of a photon theory. That, however, is a long story.

THE SPHERICAL SHELL METHOD OF DETERMINING THE THERMAL CONDUCTIVITY OF A THERMAL INSULATOR

By S. E. GREEN

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ABSTRACT. The spherical shell method of determining thermal conductivity has been applied to a thermal insulator in a system of comparatively small dimensions, the outer diameter of each of the shells employed being roughly 10 cm. Sulphur was selected as the material under test, and the spherical shells were cast in a brass mould. The supply of heat to the centre of the system was maintained electrically, a power-supply of 2 watts producing a temperature-drop of approximately 10° C. across the specimen. The range of temperature covered by the experiments was from 5° C. to 85° C. The results obtained are compared with those derived for sulphur by other observers.

§ 1. INTRODUCTION

The method of determining the thermal conductivity of a substance by using the material in the form of a spherical shell and observing the steady difference between the temperatures of the surfaces due to a constant flow of heat from the centre has been employed by a number of investigators. Using this method for a good conductor Laws, Bishop and McJunkin [1] experimented on a cast-iron shell having an outer diameter of 6 in. and a thickness of 1·4 in., while Bishop [2] afterwards applied the method to the determination of the thermal conductivity of lead, using a specimen of roughly similar dimensions. In the case of a number of thermal insulators the spherical shape was adopted by Nusselt [3], who filled with the material under test the space between two concentric metal spheres, the inner one being of copper and the outer of zinc. The diameters of these spheres were 15 cm. and 65 cm. respectively. The temperature at a number of points within the specimen being determined by means of thermocouples, care was necessary to ensure that the leads of each couple lay in the isothermal surface for some centimetres from the junction. Neglect of this precaution was shown to give rise to large errors. Among the substances tested were such loose materials as sawdust, cotton, charcoal and asbestos.

The experiments outlined below were conducted in an attempt to determine the thermal conductivity of a poor conductor by the use of a spherical shell of relatively small dimensions, the inner and outer diameters being roughly 5 cm. and 10 cm. respectively.

Over other methods which involve only measurements made under the conditions of thermal equilibrium, with the consequence that the density and specific

heat of the specimen do not enter into the calculation, the spherical shell method possesses advantages from the theoretical point of view. The uniform distribution of the lines of heat-flow leads to an easily determined shape-factor, while there is the absence of a correction such as that which arises from the heat-loss at the edges of a flat plate. The attempt to realize these advantages as far as possible in practice in the case of a small shell of badly conducting material necessitates particular attention to some experimental details. In the first place, the passage of the necessary leads through the shell to supply heat to the inner surface and to register its temperature produces a heat-loss which is the more important the smaller the conductivity of the specimen. Further, the smaller the dimensions of the shell the more important does accurate concentricity of the inner and outer surfaces become.

In the present investigation sulphur was chosen as the substance under test. Most thermal insulators being composite materials, sulphur has the advantage that as an element it is not susceptible to variations in the values of its physical constants from specimen to specimen. Its melting point (115° C.) was conveniently low for making castings, while the solid state was retained well above the maximum temperature, 80° C., at which the thermostat controlling the temperatures of the observations would operate satisfactorily. From this point of view sulphur was preferable to waxes, to which the method is equally applicable.

§ 2. OUTLINE OF THE METHOD

Each shell of sulphur was cast between two concentric spherical surfaces provided by the outer surface of a sphere of brass A, figure 1, and the inner surface of a spherical shell B, also of brass. A steady source of heat was supplied by the passage of an electric current through a 6-volt lamp enclosed within the inner sphere, the interspace between the lamp and this sphere being filled with mercury. The vertical aperture in the casting which permitted the insertion of the source of heat was plugged with sulphur. The system was immersed in water to within a few millimetres of the top of the neck of the outer brass casing, the temperature of the water being maintained constant by an electrically controlled thermostatic device. For a given power-supply at the centre of the system the corresponding steady difference between the temperatures of the inner and outer sulphur surfaces was measured by means of thermocouples. One of these was placed in a glass tube suspended in the water bath, and the other in an aperture let into the inner brass sphere. Under the conditions of experiment the falls of temperature across the metal shells were small and calculable. The inner brass sphere was made thick (1·5 cm.) in order that it might render uniform the temperature of the inner surface of the specimen. If W_1 be the power-supply, θ_1 the difference of temperature attained in the steady state, and K_1 and K the apparent and true thermal conductivities respectively of the material under test, then

W_1, θ_1
K_1, K

$$W_1/\mathcal{J} = K_1 S\theta_1 = KS\theta_1 + H \qquad \ldots\ldots(1),$$

where S is the shape-factor of the shell and H represents the heat lost by conduction along the inner thermocouple wires and the power leads. For this loss a correction

S, H

was applied. The orders of power-supply and current used were 2 W. and 0·4 A. respectively, the temperature-intervals across the specimen varying from 7° C. to 17° C.

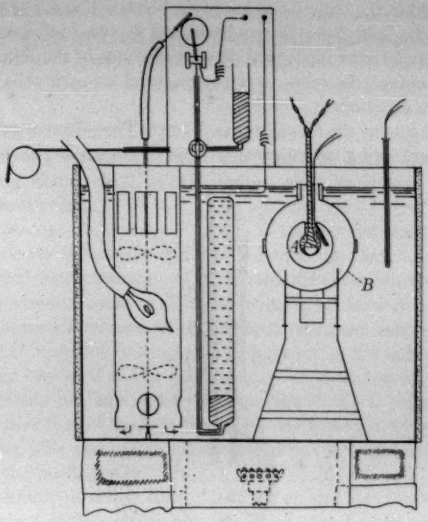

Fig. 1. Schematic view of the apparatus.

§ 3. THE CASTING OF A SHELL

The mould. The mould system was entirely of brass. The outer casing consisted of two hemispherical shells *A* and *B*, figure 2, which screwed together horizontally. The inner surfaces were accurately worked to the same radius and when the two sections were in position the lower rim *r* of the upper shell terminated in the horizontal diametral plane of the inside spherical surface. The lower shell carried a stout cylindrical boss provided with flats to facilitate gripping in a vice. Into the neck *C* at the top of the upper hemisphere screwed a tube *D*, its axis coinciding with the vertical diameter of the inner spherical surface.

The inner sphere also was made in two parts *G* and *H* which screwed together horizontally, in this case the outer surface of each section being accurately worked to the same radius. The rim *s* at the base of the upper portion *G* terminated in the horizontal diametral plane of the outer spherical surface formed when the two parts were screwed together. Into the top of the upper section *G* screwed a tube *K* coaxially with the vertical radius. When the mould system was centred as shown in the figure the inner tube overlapped the outer by some 3 cm., the protruding portion being a hexagonal head of slightly smaller diameter than the main cylindrical

section. The head E of the outer tube D was hexagonal also. Each of the upper hemispheres screwed to a shoulder on the corresponding tube, thereby ensuring that insertion of the tube was always made to the same depth. The inner system could be suspended within the outer by means of six bolts such as b, b screwed up to grip the inner tube, the bolts being in two horizontal groups 120° apart in each. Two well-fitting collars l, l were made to slide over the ends of the inner tube into the annular space between the tubes, a projecting head on each collar facilitating its withdrawal when necessary.

Dimensional data of the mould are as follows. The effective spherical surfaces had radii of 2·8 cm. and 5 cm. respectively, the corresponding shells being 1·5 cm.

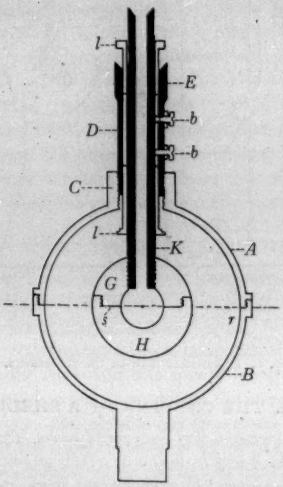

Fig. 2. The mould system.

and 0·3 cm. in thickness. The length and diameter of the inner tube were 17 cm. and 1·9 cm. respectively. The outer diameter of tube D was 2·6 cm. and the annulus between the tubes 0·22 cm. in depth.

Prior to the setting up and adjustment of the mould the walls of this annulus were treated with castor oil to impede the adhesion of sulphur to them. The oil also facilitated the extraction of the collars when necessary. The operation of rendering concentric the surfaces between which the casting was to be made was carried out in the following way. The mould system as shown in figure 2, but with the lower hemispheres B and H removed, was inverted and clamped by the head E on the outer tube, the bolts lightly gripping the inner tube. The collars being in position, motion of the centre of the inner spherical surface was confined to the vertical centre-line common to the two spherical surfaces. Thus only in the vertical

direction was adjustment of the inner surface required. The elevation of the inner tube was adjusted until a straight edge placed across the rim *r* of the outer shell just touched the rim *s* of the inner sphere. By tightening the bolts the inner tube was then locked in position. The inner hemisphere was unscrewed to permit the withdrawal of the collar and was then replaced. The remaining hemispheres were screwed into position to complete the spherical surfaces. The system was removed from the vice and the upper collar extracted. The adjustment of the mould was now complete.

The tube system served a dual purpose. It provided means whereby the components of the mould could be centred, and supplied, in the space between the tubes, a reservoir from which liquid sulphur could be drawn as solidification proceeded within the shell.

The casting process. In making a casting it was necessary to ensure that the sulphur cooled slowly and solidified from the bottom upwards. An electric heater was used to control the temperature of the mould, the entire outer surface of which was insulated by a layer of asbestos paper pasted on in the form of strips. The heater consisted of bare nichrome wire wound directly on to the asbestos coating. The frame necessary to keep the heating-wire in position on the spherical portions of the mould surface consisted of four narrow wire helices tied on by means of thread so as to lie vertically on lines of longitude 90° apart. Each helix was of cotton-covered copper wire further insulated, before being mounted, by repeated dipping in shellac varnish. The spacing of consecutive turns on each helix just permitted the insertion of the heating-wire.

The inner mould, receiving heat chiefly by conduction through the bolts at the top, was always coldest at the bottom. To obtain the same condition in the outer mould the heater was wound horizontally to give a gradually increasing concentration of heat-supply from the bottom upwards. The windings on the lower hemisphere were separated by three turns of the helices, while from the equator upwards the spacing passed from two turns to one, the outer tube being covered by practically contiguous turns of heater wire. Finally, a few turns were wound round the projecting portion of the inner tube. No windings were placed round the boss at the base of the mould. The whole system was heavily lagged. The length of heater wire employed was about 15 metres, the total resistance being roughly 20 ohms.

During the process of casting the temperature of the system was indicated approximately by a thermometer resting inside the inner sphere. A current of 1·2 A. was sufficient to keep the temperature steady at about 170° C. The liquid sulphur, obtained from sulphur flowers, was poured while at a temperature of 160° C. into the top of the mould down the annulus between the two tubes, until there were signs of overflow. The adhesion of air bubbles to the brass was impeded by polishing the inner surfaces of the mould before assembly and by tapping the mould when it contained the liquid. After filling, the heating current was reduced by 0·05-ampère stages, being maintained constant at each value for 10 to 12 minutes. Liquid sulphur was continually added to keep the space between the

tubes full. When the current reached the value 0·8 A. solidification was complete. A current of 0·5 A. was passed for half an hour, the system then being allowed to cool in the lagging.

After removal of the heater and asbestos insulation from the mould the bolts and tubes were extracted, the latter by turning at the hexagonal heads. In spite of the initial smearing of the appropriate surfaces with castor oil it was found necessary, in the case of most of the castings, to warm the tubes before they could be turned. The sulphur broke off at the base of the outer tube, which point (see figure 2) was well up in the neck of the outer casing. Completion of the casting to the top of this neck was effected by pouring liquid sulphur into the neck with the inner tube in position.

A very slow rate of cooling during the casting process was essential, not only to secure a uniform specimen but also, since sulphur contracts on freezing, to guard against possible recession from the mould surface. That satisfactory contact was established between this surface and the casting is indicated by the great tenacity with which sulphur was found to adhere to brass even when allowed to solidify quite rapidly.

§ 4. FURTHER EXPERIMENTAL DETAILS

The location of the inner thermocouple. This is shown diagrammatically in figure 3. In a preliminary series of experiments the attempt was made to register the temperature of the inner surface of the sulphur shell by placing the inner thermocouple in a narrow channel *c* cut round the periphery of a vertical diametral section of the inner sphere, the channel being covered in by brass strip. Prevention of leakage of the sulphur past the shoulder at the top of the screw thread on the inner tube proved, however, to be troublesome. The couple junction was therefore placed on the horizontal diametral plane of the inner brass sphere, approximately mid-way between its inner and outer surfaces. To effect this a cylindrical channel *d* was drilled through the upper section of the sphere and inclined at roughly 55° to the horizontal. This channel, 3 mm. wide, was duplicated on the opposite side of the upper section of the sphere in the same vertical diametral plane. Leakage of sulphur into these channels was prevented by treating the thread of the inner tube with a mixture of graphite and seccotine prior to the erection of the mould. As indicated in the figure these thermocouple orifices lay in a plane at right angles to that containing the outer channels originally used. This method of locating the couple had the advantage that the insertion and extraction of the wires were facilitated, owing to the absence of a pronounced bend such as was entailed in the first case.

The lamp and plug system. The 3-watt screw-on-type lamp employed to supply heat to the system was of a length 2·8 cm., so as to be enclosed within the inner brass sphere when standing vertically bulb downwards. The current leads, consisting of flex wire equivalent in section to no. 22 s.w.g., were supported by a thin ebonite rod some 10 cm. long. The metal thread covering the base of the lamp was insulated by a binding of tape and a coating of collodion. The tape binding was continued

round the leads and rod, near the top of which the potential leads were joined to the current wires. Separate potential leads to the lamp itself were not used owing to the additional heat-loss which would thereby be entailed.

The procedure adopted in casting the sulphur plug surrounding the lamp leads was the following. Sufficient mercury was poured into the orifice of the shell to raise the level just above that of the entrances to the outer channel *c*, the uppermost portions of which, being unoccupied by sulphur, were thus filled with the liquid. The system having been brought to a temperature in the vicinity of 0° C. by being left in an ice bath for four hours, mercury was extracted until the surface was just

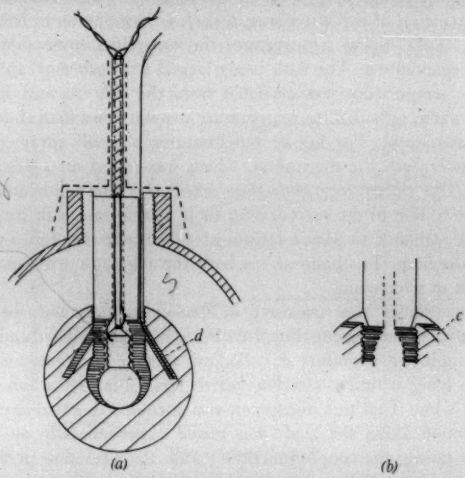

Fig. 3. The lamp and plug system. (*a*) and (*b*) are vertical central sections at right angles to each other.

below the entrances to the outer channels when the lamp was inserted into the orifice and held with the tip of the bulb touching the underside of the inner brass sphere. The lamp-lead system being kept at this depth by a frame which clamped on to the neck of the outer casing, the plug was cast by pouring liquid sulphur down the orifice, which acted as the mould. Contact with the sulphur wall of the orifice and leakage into the outer channels of the inner brass sphere were prevented by covering the surface of the orifice with a thin layer of paper which terminated below the level of the mercury. The paper was treated with glycerin to make it slightly adhesive, and remained attached to the sulphur plug on removal of the lamp system. The base of the plug was cut away a little at the points opposite the outer channels to give free access of mercury to them. A narrow vertical groove was cut along the plug to accommodate the thermocouple leads. Before the plug was

finally inserted for the conductivity measurements, the walls of the orifice were treated with glycerin to give good thermal contact. By casting the plug in this way with the system at 0° it was ensured that the mercury remained in contact with the base of the plug at all temperatures, while the expansion of the mercury at the higher temperatures was taken up within the inner brass sphere.

The sphere system was mounted on a tripod in a 20-gallon copper tank, figure 1, provided with an electro-thermal regulator similar in type to that described by H. T. Barnes [4]. In this method of temperature-control the expansion of a quantity of toluol, contained in a bulb immersed in the water filling the tank, actuates a relay through the medium of a thread of mercury in a glass tube leading from the bulb. The circulation of the water was in such a direction as to leave the surface undisturbed. At the higher temperatures the water was covered with a layer of oil to impede evaporation. The bath being lagged with asbestos, sufficient energy to control the temperature was available from the two 20-watt lamps used as heaters in the water, provided the temperature was not more than about 6° C. above that of the laboratory. For higher temperatures a small spray gas flame was maintained underneath the thermostat, which was raised on a brick support for this purpose. The electric regulator then acted as a fine adjustment. For temperatures close to that of the surrounding air it was necessary in conjunction with the use of the regulator to pass a stream of cold water through a coil of tubing immersed in the bath. To obtain an ice bath the regulator was removed and ice water was kept in circulation.

The thermocouples used consisted of fine constantan and copper elements (36 s.w.g.), the wires passing into the sphere being further insulated and strengthened by repeated dipping in a mixture of collodion, acetone and castor oil. The length of this couple lying within the orifice cut to receive it in the inner sphere was approximately 2 cm. That this immersion was adequate to avoid appreciable error due to conduction along the leads was tested experimentally on a number of occasions. On raising the couple junction 3 mm. the alteration in the differential thermo-electric e.m.f. was less than could be detected with certainty, being lower than 1 in 1000.

In determining the temperature-interval use was made of the relation

$$E_{t_1} = e \,.\, \frac{E_{100}}{s_{100}} + E_{t_2} \frac{b_{100}}{s_{100}} = e\,(1+\lambda) + E_{t_2}\,(1+\beta) \qquad \text{.....(2),}$$

t_1, t_2 in which t_1, t_2 are the temperatures of the hot and cold junctions respectively, the

s_{100}, b_{100}, E e.m.f.'s of which at 100° C. are s_{100} and b_{100}; while E denotes an e.m.f. on the standard

λ curve given by L. H. Adams [5]. The correction λ to the observed differential e.m.f.

e, β e was approximately 0·003, the value of β being generally negligible. The bath temperature was observed by means of standardized mercury thermometers. Owing to the slow change of slope of the standard e.m.f. curve, the correct evaluation of $(t_1 - t_2)$ did not require very great precision in the estimation of the thermostat temperature.

The shape-factor of the casting, $4\pi Rr/(R-r)$, required for its determination

the measurement of the radius of (*a*) the inner surface of the outer brass shell and (*b*) the outer surface of the inner brass sphere. Measurement of the former was effected by screwing together the two sections of the outer shell and weighing it before and after it had been filled with distilled water. In order to ensure that for each determination the sphere was exactly filled with the liquid, a brass stopper provided with a fine central orifice was made to screw down into the neck of the upper hemisphere. When the stopper was in position its lower surface, which was shaped to the appropriate radius, completed the spherical surface under test. The radius of the outer surface of the inner brass sphere was determined from a system of calliper measurements. By marking the sections of the shell and the sphere it was secured that in assembling the mould the sections were screwed into the identical relative positions occupied during the determination of the radii.

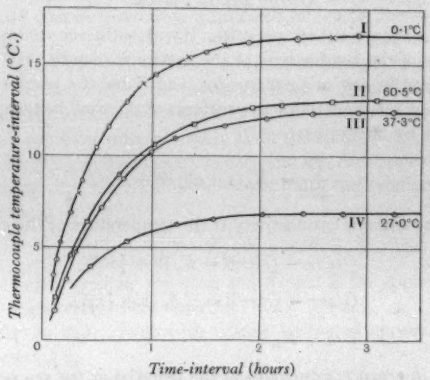

Fig. 4. Mode of attaining thermal equilibrium.

The time taken to attain thermal equilibrium. The manner in which after the heating current was started the temperature of the inner surface of the specimen rose to the value corresponding to thermal equilibrium was determined by observing the time at which the e.m.f. of the differential couple system attained in turn each of a series of suitably spaced values. A selection of the curves so obtained for various power-supplies is given in figure 4, the temperature alongside each curve denoting the thermostat temperature in each case. The extent to which the variation of temperature was of exponential form is indicated in the case of curve I, the ordinates denoted by crosses being calculated from the expression $\theta = 16{\cdot}54\,(T - e^{-0{\cdot}0335t})$ in which the time t is measured in minutes. Curve IV is a plot of observations taken with a double-lead system such as is described later.

During an interval of from 3·0 to 3·5 hours from the commencement the temperature rose by only about two parts in a thousand. The thermal-conductivity

values were determined from the temperature-difference obtaining after approximately four hours had elapsed from the starting of the heater current. During this interval the power-supply was checked from time to time on a four-dial potentiometer of Kelvin-Varley type and was found in general to be constant to one part in a thousand during the last hour. The value during the latter period was taken for the conductivity measurement in each case.

The effective temperature of experiment. Since in the steady state the temperature-gradient along any radial line of the shell was not constant but varied as $1/x^2$ where x is the radius of any elementary shell of thickness dx, it was necessary to define the temperature (called for convenience the effective temperature) to which the deduced conductivity corresponded. The heat-flow Q per second across any elementary shell having a temperature θ is given by

$$Q = -4\pi x^2 K \cdot \partial\theta/\partial x,$$

in which K is a function of the temperature. If with sufficient accuracy the variation with temperature of the conductivity of the specimen may be taken as linear over the range obtaining in any one experiment, and if for the purpose of evaluation the temperatures of the inner and outer surfaces of the shell be taken as θ_1 and zero respectively, then by integration

$$\frac{Q}{4\pi x} = \int K_0 (1 + \alpha\theta)\, d\theta + B,$$

in which K_0 is the thermal conductivity at the temperature of the outer surface.

Hence $$Q/4\pi x - Q/4\pi R = K_0 (\theta + \tfrac{1}{2}\alpha\theta^2),$$

i.e. $$Q/4\pi r - Q/4\pi R = K_0\theta_1 (1 + \tfrac{1}{2}\alpha\theta_1).$$

Thus $$Q/S = K_m\theta_1 \qquad \text{.....(3),}$$

where K_m is the thermal conductivity of the material at the temperature midway between those of the surfaces of the shell. Since the results obtained throughout the range of temperature explored (5° C. to 85° C.) indicated a linear variation with temperature of the conductivity of the specimen, the effective or mean temperature corresponding to each experiment was given by the addition of the thermostat temperature to half the drop across the shell.

A specimen calculation of the apparent thermal conductivity of the casting is as follows.

Data relating to shape-factor: outer radius $R = 5{\cdot}100$ cm.; inner radius $r = 2{\cdot}757$ cm.; $S = 75{\cdot}4$ cm.; bath temperature = 46·7° C.; e.m.f. of differential couple system = 778 μV.; corresponding temperature-interval $\theta_1 = 17{\cdot}99$° C.; mean temperature of experiment = 55·7° C.

Power-supply: current = 0·4790 A.; e.m.f. = 7·006 V.; power = 3·356 W.

Apparent thermal conductivity = $W_1/\mathcal{J}S\theta_1 = 0{\cdot}00059_2$ cal./cm.-sec.-degrees.

Corrections to be applied to the results. Correction for heat-loss. The question of heat-loss was simplified by the absence of a stirring device at the centre of the

system, since uncertain corrections for stirring and for conduction along the stirrer shaft were avoided. Neglecting any possible loss from the surface of the sulphur at the top of the neck of the casing, which loss would in any case be very small, there remained the heat lost by conduction along the current leads and the inner thermocouple elements. The problem of making any theoretical assessment of the value of this loss was complicated by the fact that while the lower portions of the couple wires were at the temperature of the inner brass sphere, the lower parts of the current leads, being attached to the lamp itself and insulated from the mercury, were at a temperature considerably higher by an unknown amount than that of the inner sphere. This made the ratio of the loss along the current wires to that along the couple elements greater to an uncertain extent than the value corresponding to the ratio of the cross-sections. The total loss was eliminated in the following way. An additional determination of the apparent conductivity was made in which the current leads and couple wires were duplicated in exactly the same positions, the other conditions (such as power-supply, mean temperature of experiment, and temperature-drop across the shell) being reproduced as nearly as possible. Although the electric current-densities in the power leads were unequal in the two cases, the dissipation of energy (of the order of 0·001 W.) in these leads was, judging by the results obtained, insufficient to materially affect the conduction of heat along them. The heat-loss obtaining in the primary experiment was thus approximately doubled.

For a primary determination with single leads the relation giving the heat flow per second is given by

$$Q_1 = W_1/\mathcal{J} = K_1 S\theta_1 = KS\theta_1 + H \qquad \text{.....(1),}$$

where H is the heat-loss along the wires and K_1 and K the apparent and corrected conductivities respectively of the material. *H*

Similarly for an experiment with double leads and approximately the same temperature-interval θ_2

$$Q_2 = W_2/\mathcal{J} = K_2 S\theta_2 = KS\theta_2 + 2H \qquad \text{.....(4).}$$

Hence it follows that

$$K = K_1 - (K_2 - K_1)\,\theta_2/(2\theta_1 - \theta_2) \qquad \text{.....(5),}$$

while the fractional heat-losses are given in the respective cases by the expressions

$$(K_1 - K)/K_1 \quad \text{and} \quad (K_2 - K)/K_2 \qquad \text{.....(6), (7).}$$

The heat-loss could thus be corrected for, in the absence of any arbitrary assumption as to its precise nature or as to the relative importance of the losses along the current leads and thermocouple elements. In the case of the secondary experiments with double leads a lamp exactly similar to that used in the primary observations was employed, and a plug was cast round the duplicated lead system as before. An additional copper thermocouple element was laid alongside the inner thermocouple wires and immersed to the same depth. It was unnecessary to duplicate the constantan element since the heat-loss along it was negligible, being only about one-twentieth of that due to the copper element.

Drop of temperature across the mould components. The difference between the temperatures of the thermocouple junctions was very slightly greater than the temperature-drop across the specimen. By calculation the fall of temperature across the 7 mm. of brass intervening between the inner thermojunction and the inner surface of the sulphur shell was 0·02° C. for a power-supply of 2 W. and a corresponding drop of 10° C. across the shell. The temperature-interval across the outer brass casing being of the order of 0·001° C. was negligible. The drop of temperature within the inner sphere, being proportional to the observed temperature interval, involved an increase of only one part in 500 in the value of the thermal conductivity deduced from equation (5).

Since the potential leads used in determining the power supplied to the system were connected to the current wires outside the sphere and not directly to the lamp itself, the measured power was slightly in excess of that expended within the lamp. The resistance of the latter being about 12 ohms and that of the leads within the sphere 0·01 ohm, the correction involved was only 1 part in 1000 and was negligible.

A small uncertainty was involved in estimating the temperature of the outer brass casing by the use of a thermocouple immersed in the thermostat. This position of the couple was essential to preserve adequate insulation of the wires, to avoid error due to heat-conduction along them and to enable easy removal for calibration. Assuming a stagnant layer of water as great as 1 mm. in thickness to cover the surface of the mould, the error produced in the final conductivity result would have been of the order of 0·7 per cent, the presence of the boss reducing the error. Since it is unlikely that with the rapid rate of stirring adopted any stagnant film present was more than a small fraction of 1 mm. thick the probable error involved, being only a few parts in a thousand, was negligible.

§ 5. TEST OBSERVATIONS

Observations carried out with a single-lead system, with various power-supplies at the same effective temperature of experiment but with the inner thermocouple located on opposite sides of the inner sphere, gave the results shown in table 1. Agreement to a few parts in 600 was obtained between the values of apparent conductivity corresponding to a given power-supply.

Table 1.

Couple side *A*				Couple side *B*			
Mean temperature (° C.)	Watts	Temperature interval (° C.)	Apparent conductivity (c.g.s. × 10^{-3})	Mean temperature (° C.)	Watts	Temperature interval (° C.)	Apparent conductivity (c.g.s. × 10^{-3})
30·0	3·378	17·59	$0{\cdot}60_9$	30·1	3·439	18·03	$0{\cdot}60_5$
30·2	2·030	10·58	$0{\cdot}60_9$	29·9	2·042	10·62	$0{\cdot}61_0$
29·7	1·393	7·22	$0{\cdot}61_2$	30·5	1·338	6·92	$0{\cdot}61_4$

A further set of readings was made to test experimentally the method of eliminating the heat-loss at temperatures of experiment up to 80° C. A series of observations was first taken with a single-lead system, the temperature of the thermostat being adjusted to give several groups of readings such that in each group the power-supply varied while the effective temperature of experiment was

Table 2.

Mean temperature of experiment (°C.)	Lead system (single or double)	Ordinal number of experiment	Power-supply (W.)	Observed temperature-interval (°C.)	Apparent conductivity (c.g.s. $\times 10^{-3}$)	Heat-loss along leads (%)	Corrected conductivity (c.g.s. $\times 10^{-3}$)
4·0	S	10	1·543	7·85	0·624	1·1	$0·61_8$
4·0	D	20	1·586	7·98	0·631	2·2	
8·3	S	9	3·224	16·54	0·619	2·3	$0·60_5$
8·3	D	19	3·302	16·53	0·634	4·6	
30·9	S	11	3·184	16·77	0·603	2·2	$0·59_1$
30·6	D	21	2·786	14·26	0·620	4·8	
29·9	S	2	2·042	10·62	0·610	2·3	$0·59_6$
30·2	D	12	2·033	10·31	0·626	4·8	
30·5	S	1	1·338	6·92	0·614	2·6	$0·59_8$
30·5	D	22	1·383	6·97	0·630	5·1	
43·5	S	25	2·373	12·52	0·601	3·2	$0·58_3$
43·3	D	23	2·412	12·33	0·621	6·3	
55·7	S	3	3·356	17·99	0·592	2·9	$0·57_6$
55·4	D	13	3·155	16·35	0·612	6·0	
55·1	S	7	2·294	12·27	0·593	3·2	$0·57_8$
55·4	D	15	2·230	11·50	0·615	6·7	
54·7	S	4	1·445	7·57	0·606	4·1	$0·58_2$
54·9	D	18	1·368	6·84	0·636	8·6	
67·2	S	26	2·454	13·13	0·593	3·9	$0·57_1$
67·6	D	24	2·382	12·18	0·620	8·1	
80·1	S	5	3·280	17·97	0·579	4·0	$0·55_7$
80·1	D	16	3·044	15·90	0·608	8·6	
79·7	S	8	2·255	12·26	0·583	5·3	$0·55_2$
79·4	D	14	2·203	11·27	0·620	11·0	
79·5	S	6	1·451	7·70	0·598	7·0	$0·55_7$
79·4	D	17	1·458	7·16	0·646	13·9	

practically constant. A corresponding series of experiments was then performed with a double-lead system. The method of pairing the observations to eliminate the heat-loss is indicated in the summarized table of data, table 2. The intermediate readings at 43° and 67° were taken with new plug castings. All the measurements were made with the inner thermocouple located in side *B* of the inner brass sphere.

The results show agreement to 1·5 per cent between individual values of the thermal conductivity of the casting obtained for the same mean temperature of

experiment by using power-supplies varying up to nearly 2·5 times the smallest value. This degree of agreement was taken as experimentally substantiating the method of eliminating the heat-loss, in view of the fact that in experiments at the highest mean temperature of observation, for which the heat-loss was greatest, there was no systematic variation of final results with power-supply. The results also justified the neglect of the small surface-loss from the top of the neck of the casting. The increase of heat-loss at the higher temperatures of experiment was probably due to the fact that the temperature of the portions of the leads just outside the plug was largely determined by the atmospheric temperature; this being so, for a given power-supply the temperature-gradient along the leads became greater since the lamp was hotter at the higher temperatures, and an increased heat-loss resulted.

The mean density of the casting was determined by weighing, the masses of the components of the mould being known. The value so obtained was 1·90 gm./cm.^{3} Molten sulphur on solidifying forms monoclinic crystals. As the mass cools below 95° C. these pass into the rhombic form, which is the stable condition below that temperature at atmospheric pressure. During the process of transformation of crystalline structure, the density increases from 1·96, that of the monoclinic crystals, to 2·07, that of the rhombic variety. Hence although the sulphur employed was a specimen of the rhombic form, the maximum value obtainable for the mean density of such a casting was 1·96, the density in the monoclinic state. Thus the casting employed in the test observations possessed a mean density differing by 3 per cent from the maximum possible value. As far as could be ascertained by inspection of the orifice of the shell there appeared to be no corresponding defect in the casting. It was not possible to determine whether the fault was local or whether more or less uniformly distributed throughout the shell.

While therefore the series of test observations considered above were essential to the establishment of the method as applied to a given material, it was necessary in order to obtain absolute values of the thermal conductivity of sulphur to make a further set of experiments with a specimen more satisfactory as regards mean density.

§ 6. DETERMINATION OF THE THERMAL CONDUCTIVITY OF SULPHUR

In making the new casting the same mould system as before was used. As on previous occasions it had been found necessary to remove the last vestiges of sulphur from the spherical surfaces of the mould by means of pumice stone, paraffin oil was employed as a lubricant. In consequence, the shape-factor on redetermination was found to be very slightly reduced, the mould having been employed for several trial castings made intermediately between the two on which systematic observations were performed. The new value of the shape-factor was 75·2 cm., the relevant radii being 5·104 cm. and 2·754 cm. respectively. The casting process finally adopted was carried out as in the case previously described, with the exceptions that the initial working temperatures were lower and the cooling process

was made more protracted. The mould was heated to 135° C. and initially filled with liquid sulphur having a temperature of 127° C. By gradually reducing the current supplied to the electric heater the system was cooled after about 5 hours to 117° C. Solidification was complete after the lapse of a further 40 minutes. The casting so obtained was found to possess a mean density of 1·94 gm./cm^3, which is within about 1 per cent of the maximum possible value.

The lamp and plug systems were assembled in a manner analogous to that adopted for the earlier experiments, and the general method of procedure was the same. A few minor modifications in the apparatus were however introduced. The small cavities employed to take up the expansion of the mercury at the top of the inner brass sphere were filled with glycerin. By keeping the top surface of the plug covered with this liquid the exclusion of air from the system was ensured. With the object of reducing the heat-loss along the lamp leads, thin copper wires (30 s.w.g.) were used, the effective cross-section of the current leads being thereby reduced to about a quarter of the previous value. In addition an attempt was made to regularize this heat-loss by controlling as far as possible the temperature of the portions of the leads immediately outside the sphere. The leads for a distance of roughly 6 cm. from the top of the plug were closely encased in a copper sheath made from pieces of strip the ends of which dipped into the thermostat water. This sheath is indicated by the outer dotted lines in figure 3 (*a*). The outside of the sheath was well lagged. The temperature of the leads just outside the sphere was thus made largely independent of the atmospheric temperature, being chiefly defined by the bath temperature and the current supplied to the system.

New thermocouple wires were used, the temperature-drop across the sulphur shell being roughly 10° C. both in the main experiments, which were carried out as formerly, and in the subsidiary experiments conducted to eliminate the heat-loss along the leads. This loss, being small (of the order of 1 per cent), could be multiplied a number of times without seriously disturbing the thermal distribution of the system. In the auxiliary experiments the loss was increased to six-fold. Each of the leads to the lamp consisted of six strands of 30 s.w.g. copper wire. It was necessary to use both of the thermocouple channels in the inner sphere, one to contain the couple itself and the other to receive the lower ends of six additional copper wires of size 36 s.w.g. The extra copper element was conveniently employed in place of five constantan wires, and provided sufficiently approximate compensation. A narrow vertical groove cut in the sulphur plug on the side opposite to the similar groove containing the couple elements accommodated these additional wires in their passage to the exterior of the apparatus.

With sextuple leads the equation giving the heat flow per second is

$$W_6/\mathcal{J} = K_6 S\theta_6 = KS\theta_6 + 6H \qquad \text{.....(8),}$$

in which $6H$ is the heat-loss and θ_6 and K_6 are the temperature-interval and apparent thermal conductivity respectively. This relation used in conjunction with the single-lead equation (1) gives

θ_6, K_6

$$K = K_1 - (K_6 - K_1)\,\theta_6/(6\theta_1 - \theta_6) \qquad \text{.....(9),}$$

or approximately, since θ_1 and θ_6 are roughly equal,

$$K = K_1 - \tfrac{1}{5}(K_6 - K_1) \qquad \text{.....(10).}$$

Thus, owing to the multiplication of the heat-loss, a given inaccuracy in the value of the apparent conductivity of the specimen was, as regards the effect upon the final result, six times less important in the auxiliary experiments than in the primary ones. The data obtained when three different plug castings were used for the main observations are given in abridged form in table 3.

Table 3.

Single-lead system					Sextuple-lead system				
Effective temperature of experiment (° C.)	Temperature-drop across sulphur shell (° C.)	Apparent conductivity (c.g.s. × 10^{-3})	Ordinal number of experiment	Plug	Effective temperature of experiment (° C.)	Temperature-drop across sulphur shell (° C.)	Apparent conductivity (c.g.s. × 10^{-3})	Ordinal number of experiment	Corrected conductivity (c.g.s. × 10^{-3})
5·1	9·90	0·628	13	II	5·6	9·72	0·684	17	$0{\cdot}62_4$
5·1	9·77	0·633	14	II					
5·1	10·01	0·641	22	III					
25·4	10·21	0·617	3	I	25·0	10·05	0·668	20	$0{\cdot}61_3$
24·0	9·96	0·627	9	I					
30·1	10·33	0·619	23	III	—	—	—	—	$0{\cdot}61_0$
35·0	10·26	0·612	1	I	34·6	10·84	0·659	15	$0{\cdot}60_6$
34·7	10·03	0·618	8	I					
45·9	10·10	0·609	6	I	45·7	10·20	0·648	18	$0{\cdot}60_2$
45·8	10·20	0·608	7	I					
54·3	10·60	0·590	2	I	—	—	—	—	$0{\cdot}59_0$
54·6	10·28	0·604	11	II					
54·9	10·16	0·603	21	III					
64·7	10·57	0·585	4	I	65·1	10·54	0·640	16	$0{\cdot}58_0$
64·5	10·43	0·594	10	I					
85·2	10·82	0·574	5	I	84·8	10·37	0·629	19	$0{\cdot}56_8$
85·1	10·59	0·581	12	II					

A determination with the sextuple-lead system was taken at every temperature of experiment for which primary experiments had been made, with the exception of 30° C. and 55° C. in which cases interpolated values of K_6 were used for the deduction of the corrected conductivity K. This was justifiable since the auxiliary readings showed regular variation and were taken in indiscriminate order. The correction to the power-supply necessitated by the connexion of the potential leads to the current wires outside the sphere and not at the lamp itself amounted to 1 part in 300 in the main experiments. The power-supply was of the order of 2 W. The calculated heat-loss for the single-lead system was sensibly constant, being of the order of 1·5 per cent at each temperature of experiment.

As deduced from the above figures the variation with temperature of the thermal conductivity of the specimen may be represented by a straight line to which the readings in general conform to within 1 per cent. The results are supported by the observations on the test casting to the extent that the test values may be represented by a similar line of practically the same slope but lower by 2 per cent on the conductivity scale, consistently with the small difference in the mean density of the two castings.

The values of the thermal conductivity of sulphur derived from the mean line are the following:

Temperature (° C.)	Conductivity (c.g.s.)
5	$0{\cdot}00062_6$
20	61_5
40	60_2
60	58_8
85	57_0

the temperature-coefficient being $-0{\cdot}12 \times 10^{-2}$ per ° C.

§ 7. COMPARISON OF RESULTS WITH THOSE OF OTHER OBSERVERS

The extent to which the derived values are in harmony with those obtained by other investigators is indicated in figure 5. The dotted curve is a section of that deduced by Kaye and Higgins [6], who conducted an investigation into the thermal conductivity of sulphur at temperatures from 20° C. to 210° C. using a plate method similar to that previously employed by them to determine the conductivity

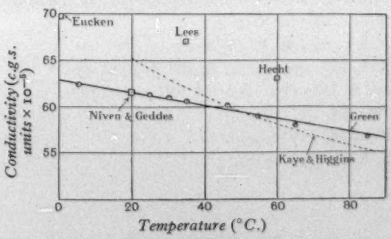

Fig. 5. Thermal conductivity of sulphur.

of liquids. The test layer was only 0·06 cm. thick and the temperature drop across it 4° C., a load being applied to the specimen to secure consistent results. The corrections involved appear to have amounted to some 10 per cent in all. The variation so obtained of the conductivity with temperature was non-linear and more rapid than that indicated by the present experiments, but there is agreement between the values of the conductivity as determined by the two methods at 50° C.

The other values indicated in the figure show rather large divergences among themselves. In addition to the result given, Lees[7] obtained an earlier value 0·00045 for sulphur at 15° C. Eucken[8] worked in the range − 190° C. to 0° C. Hecht[9], having made determinations for a number of poor conductors by an application of Neumann's method, gives the value for the conductivity of sulphur among the less accurate results. The value 0·000615 at a temperature in the vicinity of 20° C. obtained by Niven and Geddes[10] with a central hot vane between sulphur layers agrees well with that derived by the spherical shell method. The observed variation of conductivity with temperature, being linear, is analogous to that exhibited by other poor conductors. The crystals in sulphur castings appear in general to be too small for possible bias in axial orientation to affect the results for the aggregate appreciably, except conceivably in the case of a thin layer.

§ 8. ACKNOWLEDGMENTS

For permission to conduct these experiments at the Imperial College of Science, and for very valuable suggestions regarding them the author is much indebted to the late Prof. H. L. Callendar, C.B.E., F.R.S. Thanks are also due to Mr W. J. Colebrooke for facilitating the manufacture of the apparatus in the Physics Workshop and to Mr C. Hellary for the care taken in the constructional work.

REFERENCES

(1) *Proc. Amer. Acad.* **41**, 454 (1905).
(2) *Proc. Amer. Acad.* **41**, 671 (1906).
(3) *Forsch. ver. d. Ing.* Hefte 63 and 64 (1909); *Dict. of Applied Phys.* 1, 431.
(4) *Phil. Trans.* A, **199**, 208 (1902).
(5) *J. Amer. Chem. Soc.* **36**, 68 (1914).
(6) *Proc. R.S.* A, **117**, 459 (1928); A, **122**, 633 (1929).
(7) *Phil. Trans.* A, **183**, 481 (1892); A, **191**, 399 (1898).
(8) *Ann. d. Phys.* **34**, 185 (1911).
(9) *Ann. d. Phys.* **14**, 1008 (1904).
(10) *Proc. R.S.* A, **87**, 535 (1912).

DISCUSSION

Dr Ezer Griffiths. I should like information on the following points from Mr Green. (1) Is there any possibility of a gas film between the sulphur shell and the metal walls? It is difficult to see how thermal contact is ensured at all temperatures, in view of the fact that the coefficients of thermal expansion differ very considerably. (2) Can it be assumed that a random distribution of sulphur crystals is being studied?

AUTHOR'S reply. Strong evidence against the existence of a gas film between the sulphur and the brass walls of the mould is furnished by the extreme tenacity with which the sulphur adhered to the metal. It was never possible to detach the outer casing without melting out the sulphur. In tests conducted at the conclusion of the conductivity measurements, with the system at laboratory temperature, all attempts to unscrew the upper hemisphere were unsuccessful, even when that hemisphere was subjected to violent knocking.

As far as can be ascertained by inspection, sulphur castings show an absence of directional bias in the orientations of the component rhombic crystals, the very small size of which is indicated by the opacity of the material even in thin layers. Further, such castings do not appear to give any definite cleavage directions on being broken. It is therefore a reasonable assumption that in the case of a specimen over 2 cm. in thickness a random distribution of crystals is being studied.

THE OPTICS OF PHOTOMETRIC MEASUREMENTS

By T. SMITH

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ABSTRACT. The influence of the apertures which limit the light transmitted from the source to the photometer, and of the reflection and transmission coefficients of the specimen and apparatus used, including the source and the photometer themselves, on the quantities determinable from a photometric balance are considered. The way in which the apparatus is set up must take into account the precise information to be obtained. A general theory is given, which leads to the conclusion that in precise photometry the measurement of the light reflected in both directions, in addition to that transmitted, is necessary when the properties of materials are being determined. A suitable experimental arrangement for making the measurements is described. The method of calculating these quantities from the observations is given, and numerical examples are included.

§ 1. STATEMENT OF THE PROBLEM

In the normal work of an optical or photometric laboratory measurements are frequently required of the transmission factors of some materials and the reflecting factors of others. The measurements obtained are sometimes interpreted in a way which would be correct if no light were reflected in passing from the source to the observer's eye. The purpose of this paper is to indicate the significance of the photometric balance when reflected light is taken into account.

§ 2. OBSTACLES AND APERTURES

It is necessary in the first place to define every quantity we are concerned with in experimental work with some care, for its significance is slightly different from that of the similarly named quantity introduced in theoretical discussions on the properties of materials. In a typical measurement we are concerned with light which leaves a source of finite size and ultimately reaches the entrance window of a photometer. On the way it encounters various obstacles, including the specimen under examination and any lenses or mirrors used as experimental aids. To fix ideas we will suppose that all these materials are transparent and that the light we are interested in travels along paths grouped about a mean straight path from the source to the photometer. In confining our attention to this special case we are not limiting the general applicability of our conclusions.

It is important that all the light which reaches one side of the photometer should be derived from the appropriate source and should have traversed all the obstacles. To ensure this each obstacle is surrounded by a large opaque screen capable of absorbing all the light falling upon it. The obstacles and screens thus mark off a series of isolated spaces, and light can only pass from one space to the

next through the obstacle which serves as a window between the two. If then we are considering the transmission factor of a particular obstacle, this factor, in the absence of other limitations, will be defined for light which would pass through this obstacle and through the apertures in the two neighbouring screens left by the removal of the preceding and succeeding obstacles. Any light which impinges on any screen is lost, and we must recognize that the quantities we employ are to be interpreted as though such light had no existence. In many measurements the transmission factor relates to parts only of the obstacles. To find the effective portions we form in the space under consideration the optical images of all the apertures (i.e. the transparent obstacles) and of the source and the entrance pupil of the photometer, and determine the intersection with the obstacle of the largest cone which lies within all these images. This procedure is familiar in the theory of optical instruments. If the apertures effective in the determination of the cone change with small movements of the source or photometer the experimental arrangements are faulty and the interpretation of the measurements somewhat uncertain. As will be seen below, by the size of the source we may have to understand something larger than the incandescent body from which the radiant energy flows.

§ 3. REFLECTION FACTORS

We have not only to deal with directly transmitted light. Whenever light is refracted some is also reflected. To determine the meaning of reflection factor we should strictly construct the images of the different apertures by both reflection and refraction and thus determine the exact significance to be given to the term under the existing experimental conditions. In some cases this procedure might be unavoidable, but more frequently simple considerations show that we can only make trivial errors by assuming the effective apertures for the reflection factors to coincide with those for refraction. The chief point to bear in mind is that after a number of reflections the light has ultimately to be refracted through later apertures to reach the entrance pupil of the photometer, and thus after reflection it must normally lie within the cone for effective refracted rays.

With the determination of the limiting apertures, or alternatively of the limiting cone, the terms transmission factor and reflection factor assume definite significance. We have to note that for their meanings to be useful in other work the experimental conditions must be arranged to ensure that the illumination is reasonably uniform over the several apertures. The only other vital point to bear in mind is that a given obstacle has one transmission factor and two reflection factors*. In general all these are changed if the obstacle is reversed so that the directly incident light traverses it in the reverse direction. Only when the limiting cones for the spaces on the two sides of an obstacle are symmetrical can the factors remain unchanged on reversal.

* See "On the light transmitted and reflected by a pile of plates" and "Note on the immutability of transmissive factors with reversal of light," *Trans. Opt. Soc.* **27**, 31 (1925–6).

§ 4. ILLUSTRATION

The distinction between the two reflection factors, quite apart from the effect of the apertures, persists unless there is complete symmetry. As this is often not realized it may be helpful to give a simple example—in this case involving a broken path.

Let the obstacle consist of a refracting prism, on one side of which the light is incident normally while on the other the incidence is oblique. If the transmitted light travels in the direction $PABQ$, a certain fraction r_A of the incident light is reflected directly back to P from A and another fraction t_A enters the glass. When the light reaches B part is reflected in the direction BR and is to be regarded as lost, and another part, say the fraction t_B, is refracted to Q. Thus the total reflected light is measured by r_A and the total transmitted by $t_A a t_B$ where a differs from unity on account of the absorption of the material of the prism.

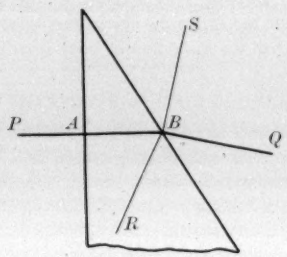

Fig. 1.

Now consider the corresponding factors when the light travels in the reverse direction from Q to P. When B is reached some of the light is reflected in the direction BS and lost, and the fraction t_B is refracted towards A. When A is reached the fraction t_A is transmitted and the fraction r_A reflected back towards B. On arrival at B the part reflected along BR is lost as before and the fraction t_B is refracted towards Q. Thus the total transmitted from Q to P is $t_B a t_A$, which agrees with the fraction transmitted from P to Q, and the fraction reflected back to Q is $t_B a r_A a t_B = r_A a^2 t_B^2$, which is less than r_A, the fraction reflected back to P; the two reflection coefficients are therefore in this case necessarily unequal. If the two surfaces of the prism were in a similar condition we should clearly have equality in the reflection coefficients (usually zero values, as the reflected light would probably strike a screen) we are at the moment considering if the prism were placed in the position of minimum deviation.

§ 5. MATHEMATICAL THEORY

We have now to consider how the factors so defined enter into an expression for the light entering the photometer. Let three successive obstacles, in the order in which they are encountered by directly transmitted light, be identified by the numbers $n-1$, n, $n+1$. Let I_n denote the total light falling on the effective aperture of obstacle n in the forward direction, and J_n that falling on it in the reverse direction. Let t_n be the transmission factor, and r_n and r_n' the reflection factors for light incident in the forward and reverse directions respectively. From these definitions we have the general equations

n, I_n

J_n

t_n, r_n, r_n'

$$I_{n+1} = I_n t_n + J_n r'_n,$$
$$J_{n-1} = J_n t_n + I_n r_n.$$

We may combine these results in the form

$$(J_n \quad -I_{n+1}) = (J_{n-1} \quad -I_n)\,\mu_n,$$

μ_n

where the matrix μ_n is given by

$$\mu_n = \begin{pmatrix} \dfrac{1}{t_n} & -\dfrac{r_n'}{t_n} \\ \dfrac{r_n}{t_n} & t_n - \dfrac{r_n r'_n}{t_n} \end{pmatrix},$$

so that $|\,\mu_n\,| = 1$. When there are l obstacles, numbered from 1 to l, we have

l

$$(J_l \quad -I_{l+1}) = (J_0 \quad -I_1)\begin{pmatrix} a & -b \\ c & d \end{pmatrix} = (J_0 \quad -I_1)\,\mu_1\mu_2 \ldots \mu_l,$$

where $bc + ad = 1$*. Here we must interpret I_{l+1} as the radiation falling on the

*. If
$$\mu_1 = \begin{pmatrix} a_1 & -b_1 \\ c_1 & d_1 \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} a_2 & -b_2 \\ c_2 & d_2 \end{pmatrix},$$
the product $\mu_1\,\mu_2$ is given by
$$\mu_1\,\mu_2 = \begin{pmatrix} a & -b \\ c & d \end{pmatrix},$$
where
$$\begin{aligned} a &= a_1a_2 - b_1c_2, \\ b &= a_1b_2 + b_1d_2, \\ c &= c_1a_2 + d_1c_2, \\ d &= -c_1b_2 + d_1d_2. \end{aligned}$$

When more than two matrices are to be multiplied together we may proceed by finding the product of two factors, and reapplying the formula with this product as one of the factors. Note that $\mu_1\mu_2$ is not equal to $\mu_2\mu_1$, so that the order of the factors is important. From the above equations it is easy to show that

$$(bc + ad) = (b_1c_1 + a_1d_1)\,(b_2c_2 + a_2d_2),$$

and the left side is equal to unity if both factors on the right have this value.
The equation

$$(P \quad Q) = (R \quad S)\begin{pmatrix} a & -b \\ c & d \end{pmatrix}$$

may be regarded as a method of writing the two equations

$$P = aR + cS, \qquad Q = -bR + dS,$$

and the solution of these equations when $bc + ad = 1$ is

$$R = dP - cQ, \qquad S = bP + aQ,$$

or
$$(R \quad S) = (P \quad Q)\begin{pmatrix} d & b \\ -c & a \end{pmatrix}.$$

photometer, and J_0 as the reflected radiation falling on the source or that part of the surrounding apparatus which lies within the effective cone of rays. In general both the source and the photometer reflect light. Let their reflection factors
s, p (corresponding to r_0' and r_{t+1}) be s and p respectively. Then the light incident on the first obstacle consists of that directly emitted by the source, which we will call i, and reflected light, i.e.

$$I_1 = i + J_0 s.$$

Also we clearly have $$J_t = I_{t+1} p.$$

The ratio ordinarily utilized in photometric measurements is the ostensible
T transmission factor $T = I_{t+1}/i$. From the above equations

$$\frac{I_{t+1}}{i} = \frac{J_0 b + I_1 d}{I_1 - J_0 s}$$

and $$p = \frac{J_t}{I_{t+1}} = \frac{J_0 a - I_1 c}{J_0 b + I_1 d}.$$

The latter equation gives

$$\frac{J_0}{I_1} = \frac{c + dp}{a - bp},$$

so that $$T = \frac{I_{t+1}}{i} = \frac{b(c + dp) + d(a - bp)}{a - bp - (c + dp)s} = \frac{1}{a - bp - cs - dps}.$$

This expression shows that the usual interpretation of a photometric balance may be seriously in error. In general we must know both p and s. If both are known to be small and the light reflected by the whole system of obstacles is also small we may neglect the terms involving b, c, d, since they are then second order quantities. Even when these conditions are satisfied we have only determined a, and this is insufficient to give the transmission factor of a particular obstacle unless no other obstacles are present. In the more general case we can only find the properties of a particular obstacle when we know the properties of each auxiliary obstacle and can make observations from which the a, b, c, d of the complete series of obstacles can all be determined.

We note that if the photometer and source are interchanged, and if the effective portions of the obstacles are thereby unchanged, the quantity measured by the photometric balance is

$$T' = \frac{J_0}{j} = \frac{1}{a - bs' - cp' - dp's'},$$

s', p' where s' and p' are the reflection factors of the source and the photometer in their new positions. If, as will often be the case, s' and p' are substantially equal to p and s respectively, no new information can be derived from the second measurement.

§ 6. REFLECTION MEASUREMENTS

Clearly there is only one way of obtaining the additional information required. We must arrange to have the source and the photometer on the same side of the
R series of obstacles and make measurements of the apparent reflection factors R

and R'. In taking these measurements the apparatus must be so contrived that the effective areas of the obstacles are unaltered, that the source and photometer do not interfere with one another, and that all the light falling on the photometer is received from the system under measurement. Let both photometer and source be situated before the first obstacle, and let q be their joint reflection factor. Also let u be the reflection factor of any objects at the other end of the system under examination. The ratio to be measured is then J_0/i' under the conditions

R'

u

$$I_1 = i' + J_0 q, \quad J_1 = I_{1+1} u.$$

This is readily found to give

$$R = \frac{J_0}{i'} = \frac{c + du}{a - bu - cq - dqu}.$$

Similarly let q' be the joint reflection factors of the source and photometer when placed behind the last obstacle, and u' the reflection factor of objects before the first obstacle. The conditions are $J_1 = j' + I_{1+1} q'$, $I_1 = J_0 u'$, giving

$$R' = \frac{I_{1+1}}{j'} = \frac{b + du'}{a - bq' - cu' - dq'u'}.$$

We now have five equations connecting a, b, c, d, viz. the expressions for T, T', R, R' and the identity $bc + ad = 1$. From a suitable set of four we can determine a, b, c, d, and thence determine the properties of a single obstacle under the conditions of the experiment.

It is not difficult to arrange the apparatus so that

$$p = s' = q' = u, \text{ and } p' = s = q = u'.$$

The equations then give $T = T'$,

$$a = (1 + uR')(1 + u'R)/T - uu'T,$$
$$b = (1 + u'R)R'/T - u'T,$$
$$c = (1 + uR')R/T - uT,$$
$$d = T - RR'/T.$$

These results are best expressed in the form

$$\begin{pmatrix} a & -b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & u' \\ . & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{T} & -\frac{R'}{T} \\ \frac{R}{T} & T - \frac{RR'}{T} \end{pmatrix} \begin{pmatrix} 1 & . \\ -u & 1 \end{pmatrix},$$

where the matrix involving the experimental measurements corresponds in form to the matrix expressing the properties of an individual obstacle. If the purpose of the measurements is to determine the properties of obstacle n, we obtain

$$\mu_n = M_{1,n-1} \begin{pmatrix} 1 & u' \\ . & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{T} & -\frac{R'}{T} \\ \frac{R}{T} & T - \frac{RR'}{T} \end{pmatrix} \begin{pmatrix} 1 & . \\ -u & 1 \end{pmatrix} M_{n+1,1},$$

where $M = \begin{pmatrix} d & b \\ -c & a \end{pmatrix}$, and the suffixes refer to the extreme obstacles in the products of the μ's for which a, b, c, d are to be computed.

§ 7. EXPERIMENTAL ARRANGEMENTS

The theoretical considerations we have been investigating help us to realize what kind of experimental arrangement is desirable in making tests. The arrangement shown in figure 2 is suitable when the photometer occupies only a small portion of the illuminated field. The obstacles are arranged in their correct positions

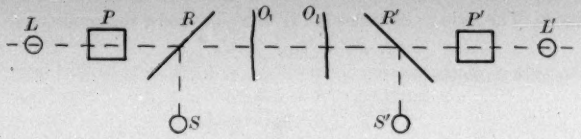

Fig. 2.

on a photometric bench and at each end is placed a photometer P, P'. The comparison lamps by which a photometric balance is obtained are denoted by L, L'. It is important that no other parts of the apparatus but L and L' should be moved, for then the effective apertures will be the same for all the observations. The systems are viewed by the photometers through windows cut out of reflectors R, R', which are used to reflect light from sources S, S' into the system. Readings with the photometer P and comparison lamp L are made

(1) when S is on and S' and L' off,

(2) when S' is on and S and L' off;

and similar readings are made with photometer P' and comparison lamp L'

(3) when S' is on and S and L off,

(4) when S is on and S' and L off.

§ 8. OPAQUE MATERIALS

The modifications required in the apparatus when reflecting rather than refracting obstacles are present are sufficiently obvious. The theory is simpler, inasmuch as we then have a single reflection coefficient and no transmission coefficient for such obstacles. A word should, however, be said about measurements on matt surfaces. In some respects the considerations resemble those for reflecting surfaces, but the size of the specimen is of much importance. Particular care should be taken to see that the extent of the specimen illuminated and that visible from the photometer correspond to the measurements significant in the application of the results obtained.

§ 9. NUMERICAL EXAMPLES

The following examples are intended to illustrate the numerical calculations required according to the foregoing theory and to indicate for typical experimental measurements the magnitude of the discrepancy between results computed according to this theory and to the current approximate theory.

(1) We readily see that in the direct photometry of lamps the reflection factors of the lamp and of the photometer may safely be neglected. For if we assume that

the minimum separation between the lamp and photometer is 50 cm., the area of the hemisphere of this radius is about 15,000 cm². If the photometer has a diffusing surface, the light falling upon it will be scattered in all directions. If the intensity of this light scattered in the direction of the lamp were twice that of the mean for equal areas on the hemisphere the light falling on a lamp of 100 cm. projected area would only be 1·3 per cent of that falling on the photometer screen. If the lamp scattered the light falling on it in the same way as the photometer screen, and the area of the latter were as great as that assumed for the lamp, the doubly reflected light falling on the photometer screen would be less than 0·02 per cent of that directly incident. With the smaller screen size of real photometers this fraction would be reduced. The effects are therefore entirely negligible.

As another example consider a photometer which reflects 10 per cent of the incident light specularly. The reflected light is equivalent to a lamp rated at 10 per cent of the real source placed at the same distance as the latter from the photometer, but on the opposite side. The light reflected back to the lamp is thus $2\frac{1}{2}$ per cent of that incident on the photometer. In the most unfavourable case this will be scattered as in the previous example, and the effects are again entirely negligible.

(2) In many measurements the reflected light is by no means negligible. An example is afforded by the derivation of reflection factors from measurements of light transmitted. Unless special arrangements are made the direct measurement of reflection factors is not easy. These difficulties are often increased by the small amount of light reflected. The transmission measurement is simpler, but the uncertainty in the measurement is considerable because the reflection factor is derived from the difference between two slightly different measurements. Thus if the reflection factor of the polished surface of glass is to be measured, a plate of glass will be used, and if the required factor is of the order of 4 per cent the transmission for the plate will be about 92 per cent. An error of 1 per cent in the measurement of the latter corresponds to an error of more than 12 per cent in the reflection factor. To increase the accuracy several similar plates may be arranged in series, and the usual theory will then lead to an erroneous result. By way of illustration let us suppose that no light is lost, so that the sum of the reflection and transmission coefficients is unity. Putting $r/t = r'/t = x$, we find

$$\begin{pmatrix} \frac{1}{t} & -\frac{r'}{t} \\ \frac{r}{t} & t - \frac{rr'}{t} \end{pmatrix} = \begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix}.$$

Moreover, by the multiplication rule,

$$\begin{pmatrix} 1+x_1 & -x_1 \\ x_1 & 1-x_1 \end{pmatrix}\begin{pmatrix} 1+x_2 & -x_2 \\ x_2 & 1-x_2 \end{pmatrix} = \begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix},$$

where $x = x_1 + x_2$. Thus when there is no loss of light the quantities r/t are simply additive, and the order of the obstacles is insignificant unless a rearrangement modifies the assumed conditions.

Now suppose that in the absence of any loss of light n similar obstacles are placed in series. The measurable transmission factor T is connected with the individual reflection factor r by the equation

$$\frac{1}{T} = 1 + nx = 1 + \frac{nr}{1-r}.$$

According to the approximate theory sometimes adopted the relation is

$$T_1 = (1 - r_1)^n.$$

It will not escape notice that the exact expression is the simpler of the two for numerical computation. For $n = 1$ the two expressions agree, but they differ for other values of n. For example, if $r = r_1 = 4$ per cent we obtain the following values:

n	2	4	8	12	24
T	0·923	0·857	0·750	0·667	0·500
T_1	0·922	0·849	0·721	0·613	0·375

Clearly with as few as two plates ($n = 4$) the error due to the incorrect theory is of the same order as the photometric uncertainty, and increases rapidly as more plates are used. We may exhibit the theoretical error in another form, by finding the value of r_1 that would be deduced from exact observations when the real value of r is 4 per cent. They are as follows:

n	2	4	8	12	24
$100r_1$	3·92	3·78	3·53	3·32	2·85

These results indicate that even in very simple systems it is important to employ the exact theory.

(3) The comparison given above is unduly favourable, for in measurements of this type the light reflected by the photometer should be taken into account. Let us suppose that the reflection factor of the photometer is 10 per cent, and that of the source negligible. With the same plates as before the value of T will be given by

$$\frac{1}{T} = a - bu = 1 + x - x/10 = 1 + 0{\cdot}9x.$$

The numerical values of T, together with the values of r_1 derived from them according to the approximate theory, are given in the following table. The extent of the departures from the correct value of 4·00 per cent for r_1 show that it is essential to take the reflection factor of the photometer into account.

n	2	4	8	12	24
T	0·9302	0·8696	0·7692	0·6896	0·5263
$100r_1$	3·55	3·43	3·23	3·05	2·64

It may be observed that the percentage inaccuracy increases with r, and that the ordinary theory is now incorrect even for $n = 1$.

This example shows that the introduction of even a single thin sheet of glass may vitiate the simplest deductions derived on the usual theory from photometric

measurements. Still more erroneous conclusions may be drawn when two or three optical parts such as lenses and filters are traversed by the light.

(4) A simple measurement not infrequently required is the determination of the transmissivity of a given material. The value is derived from the transmittance of a suitably prepared specimen of known dimensions. As a rule the refractive index of the material will be known, and if the surfaces are freshly polished the surface effects can be computed from Fresnel's formula. The experimental conditions are usually arranged so that light passes normally through the surfaces, and the losses that take place may be attributed entirely to absorption. It is easy to show that if τ is the transmittance, and the ratio r/t for each surface is equal to x, the matrix for the specimen is τ

$$\begin{pmatrix} a & -b \\ c & d \end{pmatrix} = \begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix} \begin{pmatrix} \tau^{-1} & 0 \\ 0 & \tau \end{pmatrix} \begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix}.$$

Let us suppose the refractive index is 1·5. Then

$$x = \frac{(1{\cdot}5-1)^2}{4\times 1{\cdot}5} = \frac{1}{24},$$

$$\begin{pmatrix} a & -b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{25}{24} & -\frac{1}{24} \\ \frac{1}{24} & \frac{23}{24} \end{pmatrix} \begin{pmatrix} \tau^{-1} & 0 \\ 0 & \tau \end{pmatrix} \begin{pmatrix} \frac{25}{24} & -\frac{1}{24} \\ \frac{1}{24} & \frac{23}{24} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{25}{24\tau} & -\frac{\tau}{24} \\ \frac{1}{24\tau} & \frac{23\tau}{24} \end{pmatrix} \begin{pmatrix} \frac{25}{24} & -\frac{1}{24} \\ \frac{1}{24} & \frac{23}{24} \end{pmatrix},$$

or

$$a = \frac{(625\tau^{-1}-\tau)}{576},$$

$$b = c = \frac{(25\tau^{-1}+23\tau)}{576},$$

$$d = \frac{(-\tau^{-1}+529\tau)}{576}.$$

If the simplest equipment is used for these measurements we shall have to deal with such values of u and u' as 0·1 and zero. Assuming these, the relation for finding τ becomes

$$T = \frac{576}{\{625\tau^{-1}-\tau-0{\cdot}1\,(25\tau^{-1}+23\tau)\}}$$

$$= \frac{576}{(622{\cdot}5\tau^{-1}-3{\cdot}3\tau)}.$$

Taking in turn (i) 0·80 and (ii) 0·20 as measured values of T, we compute corresponding values of τ, using the first term in the denominator to obtain first approximations:

(i) $\tau \doteq 0{\cdot}80 \times 622{\cdot}5/576 = 0{\cdot}8646$,

(ii) $\tau \doteq 0{\cdot}20 \times 622{\cdot}5/576 = 0{\cdot}2161$.

Inserting these in the second denominator term we obtain the corrected values

$$\text{(i)}\ \tau = 0{\cdot}8612, \qquad \text{(ii)}\ \tau = 0{\cdot}2161,$$

showing that the second term is only significant for the more transparent material. We may compare these values with those given by the common formula

$$T = \tau\,(1 - 0{\cdot}04)^2;$$

these are $\text{(i)}\ \tau = 0{\cdot}8681, \qquad \text{(ii)}\ \tau = 0{\cdot}2170.$

With the more transparent material the difference would not usually be considered negligible.

It would be easy to multiply examples and thus show that in more elaborate measurements, such as those made with some spectrophotometers, the application of an inexact theory may lead to much greater errors than those already illustrated. The examples given have been confined intentionally to measurements where it has been possible to avoid the direct measurement of reflection factors. The less simple determinations are probably not of general interest, and will tend to be regarded as tasks for specialists, who should themselves be capable of a critical examination of the procedure followed. It will therefore suffice to add a single caution. The exact significance of the quantities entering into the equations must be studied. Thus the "ostensible transmission factors" are not necessarily the ratios determined from measurements with and without a specimen. For example, this will not be the case if neither the reflection factor for the source nor that for the photometer is negligible. For assuming we are testing a specimen in which there is no loss of light, employing the variable x to denote its properties, we have

$$T = \frac{1}{1 + x - (u + u')\,x - uu'\,(1 - x)}$$

$$= \frac{1}{1 - uu' + x\,(1 - u)\,(1 - u')}.$$

A measurement made when the specimen is removed is represented by putting $x = 0$, and the corresponding value of T is not unity but T_0, where $T_0 = (1 - uu')^{-1}$. The quantity determinable from observation is usually T/T_0.

SOME PRINCIPLES GOVERNING THE DESIGN OF KERR CELLS

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Communicated by Dr W. H. Eccles, F.R.S., December 22, 1931.
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ABSTRACT. The paper describes some principles that were employed in designing a Kerr cell for use in a television system. The volts/intensity characteristic of the cell is developed and the theory is extended to apply to the case when an optical bias is used. The chromatic effects obtained with high potentials are discussed and finally the treatment of the cell as an integral part of an optical system is given. It is shown that from this aspect the most efficient form of cell is one using shaped plates. The frequency-response is briefly discussed.

§ 1. INTRODUCTION

In recent years a great deal of attention has been directed towards the development of methods of modulating a light-source, more particularly in connexion with recording sound on film, transmission of pictures by wire or wireless and television. Generally speaking, the two main requirements are (*a*) a good over-all frequency-response and (*b*) the power to handle as large a flux of light as possible. The frequency range varies according to the use to which the modulating device is to be applied, and the amount of light required also is dependent on the application in view; for instance, a photographic record may be required or the result may be viewed directly by the eye. Again, the choice of a suitable device depends on the circumstances under which it is to be used: if a picture is to be projected in a hall, the use of an arc lamp can be contemplated; for other purposes it may be essential to keep the light-intensity constant, in which case an arc lamp would be distinctly undesirable. Whatever the purpose, however, the demand is usually made for higher frequency-response and more light, and the development of a Kerr cell for use in projecting a television picture has followed these lines. The principles on which this development has been based at the laboratories of The Gramophone Company are described in this paper.

§ 2. THEORY OF A SIMPLE KERR CELL

The effect which is used in the Kerr cell was first observed by Dr Kerr in 1875*. He found that certain substances became doubly refracting when subjected to an electric stress, so that a plane-polarized beam of light, on passing through the

* J. Kerr, *Phil. Mag.* **1**, 337–446 (1875).

strained medium, became elliptically polarized. A simple form of apparatus is shown diagrammatically in figure 1. Light from an arc is focussed by L_1 on to the Kerr cell K after the light has been plane-polarized by the nicol N_1. On the far side of K is the analysing nicol N_2. The cell itself consists of a bath of nitrobenzene in which two parallel metal plates are immersed at a small separation. The nicols are set in the crossed position, with the polarizing planes at 45° to the direction of the electric stress that exists in the nitrobenzene when a potential is applied between the two immersed plates.

The light incident on the cell is plane-polarized by N_1, but owing to the double refraction occurring in the Kerr cell, a phase retardation is introduced between the components parallel and at right angles to the electric stress. The light on emerging from the cell is therefore elliptically polarized and has a component that can be transmitted by the second nicol N_2. The intensity of the transmitted light can be readily calculated* and the results are briefly summarized below.

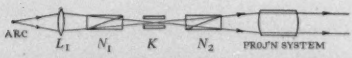


Fig. 1. Optical system for use with Kerr cell.

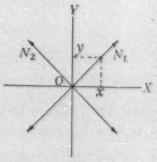


Fig. 2.

In figure 2 the vector ON_1 represents the light incident on the Kerr cell; OX, OY represent the stress directions in the nitrobenzene and ON_2 the plane of the light transmitted by N_2. ON_1 has components Ox, Oy in the X, Y planes and these components will travel with slightly different velocities owing to the double refraction. On emerging from the stressed medium there will be a phase retardation ϕ given by

ϕ

$$\phi = Kl\,(E/d)^2 \qquad \text{.....(1)},$$

l, E d, K where l is the length of the light path in the strained medium, E the potential applied across the Kerr cell plates, d the separation between the plates and K a constant depending on the medium subjected to the stress and on the wave-length of light under consideration.

I Straightforward calculation then gives the value of the intensity I of the light transmitted by the second nicol thus:

$$I = a^2 \sin^2 \tfrac{1}{2}\phi \qquad \text{.....(2)}$$

$$= a^2 \sin^2 [\tfrac{1}{2}Kl\,(E/d)^2] \qquad \text{.....(3)},$$

a where a corresponds to the amplitude of vibration when I is a maximum. For a

* See "Kerr cell method of recording sound," *Trans. Soc. Motion Picture Engineers*, **12**, 748 (1928).

cell of given dimensions in which l and d are fixed, the relation between light intensity and applied potential becomes

$$I = a^2 \sin^2 K_1 E^2 \qquad \text{.....(4),}$$

K_1 being another constant. K_1

From figure 3, which represents this relation diagrammatically, various conclusions may be drawn. For sound recording the relation between intensity and voltage must be linear or very nearly so: to attain this result the portion AB of the curve is used. This necessitates a constant biasing potential corresponding to the point C and modulation between the limits set by A and B. In this way a practically undistorted sound-record can be made.

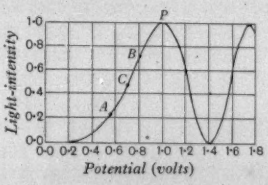


Fig. 3. Kerr-cell characteristic curve.

For television the no-distortion criterion must be very largely neglected. At B the intensity is little more than three times that at A, so that the normal contrasts between light and shade of a picture, which may be 100 or 1000 to 1, would fail altogether to be reproduced were this portion of the curve alone utilized. It is therefore necessary, in television reproduction, to modulate over a rather longer portion of the curve, say between potentials of 0·2 and 1·0 in figure 3, with a biasing voltage of 0·6. This will give a greatly increased contrast to the picture, but owing to the pronounced curvature of the characteristic the light and shade will not be accurately reproduced. The error, fortunately, is one to which the eye is not highly sensitive.

§ 3. USE OF AN OPTICAL BIAS

The difficulty referred to above is due in particular to the small change in intensity for a given voltage-change at the low-intensity end of the curve. If we suppose that by some suitable means the Kerr effect corresponding to the point P can be neutralized, then P would be the zero-intensity point and O the point of maximum intensity. In practice all that is required to neutralize the Kerr effect at P is the introduction of an optical retardation of a half wave-length between the nicols N_1 and N_2, figure 1, in the orientation necessary to oppose the retardation in the Kerr cell. That this is correct is evident from equation (2) above, for the first intensity maximum P will occur when $\phi = \pi$ which, of course, is a half-wave-length retardation. The equation giving the intensity relation for the combined Kerr cell and optical bias is thus:

$$\begin{aligned} I &= a^2 \sin^2 \tfrac{1}{2}(\phi - \pi) \\ &= a^2 \sin^2 (K_1 E^2 - \tfrac{1}{2}\pi) \end{aligned} \qquad \text{.....(5).}$$

This curve is plotted in figure 4 and it will be realized that it is of the same shape as that in figure 3 except that it is inverted. The advantage of this new characteristic over the old is the steeper slope at the low-intensity portion of the curve, so that a given potential-change on the Kerr-cell plates will produce a much greater proportional intensity-change than before. For example, with a biasing potential of 0·8 a very satisfactory picture could be produced by modulating between 0·6 and 1·0. This is only half the modulation previously required. Actually the modulation can be made still smaller as the visually significant quantity is not the intensity I itself but log I. This means that the upper half of the characteristic curve is of very much less importance than the lower half.

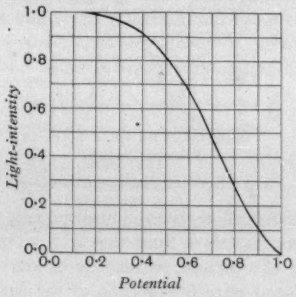


Fig. 4. Characteristic curve for Kerr cell with optical bias of $\frac{1}{4}\lambda$.

Apart from the advantage of a smaller modulating voltage, a useful point in itself, a gain in definition of the televised picture should also be obtained, inasmuch as the use of a final step-up transformer with its attendant distortion, due to phase-shift and frequency-attenuation, may be avoided. A still greater improvement should result from the more nearly linear relation between voltage and intensity that is obtained with an optical bias.

The method of applying this result is apparently quite simple, although what might appear as the simplest method does not, in practice, prove to be so. All that is needed is a piece of mica or other doubly refracting substance of just the thickness necessary to produce a half-wave-length retardation. This is not easy to obtain, and when it is available some strain in the glass walls of the vessel containing the Kerr cell is liable to upset all calculations. The more satisfactory method is to introduce a piece of clear celluloid held in a device with which it can be subjected to a strain sufficient to make it doubly refracting and thus produce the required retardation. With this arrangement it is, of course, very easy to allow for any strains already existing in intermediate glass surfaces.

§4. USE OF HIGH BIASING POTENTIAL

An alternative method of obtaining a straighter characteristic is to use a high biasing potential of, say, 1·2 (see figure 3) and modulate between 1·0 and 1·4. At first sight this would appear to give satisfactory results as the curve deviates from a straight line by only small amounts and the necessary modulating potential is greatly reduced. Unfortunately chromatic dispersion becomes of increasing importance as the biasing potential is raised. The formula (1) for ϕ contains a constant K which varies inversely as the wave-length of light considered, so that the shorter the wave-length the greater the phase-difference introduced. This is shown in figure 5 where the volts/intensity curves for three wave-lengths are given. This shows very clearly that where, for instance, the red is a minimum, at a potential of 1·3, the green and blue are nearly at a maximum with a blue-green as the resultant colour for that potential. Again, at the green minimum, both the red and blue have finite values with the resultant colour a purple. At the blue-minimum the colour is a yellow or reddish brown. An accurate analysis of the colour effects would require several more curves taken for wave-lengths at closer intervals throughout the spectrum, in addition to the absorption curve of the nitrobenzene. Figure 5, however, is sufficient to indicate the nature of the phenomenon. Practical confirmation of the above was obtained on a particular Kerr cell with the results given in the table.

Potential on Kerr-cell plates (volts)	Colour observed	Estimated intensity (arbitrary units)
1200	Greenish white	100
1300	Yellow	50
1400	Reddish yellow	25
1450	Orange	15
1500	Orange pink	10
1550	Pink	15
1600	Pinkish white	20

The estimated intensity cannot be regarded in the light of an accurate measurement, but merely as a rough indication. The evidence is sufficient, however, to show that for ordinary purposes a Kerr cell cannot be operated successfully on the steeper slopes of its characteristic, although there might be a very definite and useful application for television in colours.

The chromatic dispersion, of course, exists at even low potentials, but as the effect is additive no result is noticeable at low voltages, as can be seen from figure 5. The first upward slopes of all three curves are very nearly the same, the blue-maximum being reached slightly before the red.

As a theoretical problem it is interesting to consider the possibility of correcting for the chromatic dispersion in some way analogous to the achromatizing of a lens. The problem is very definitely academic and of no practical importance, but it may be pointed out that the only possibility would appear to be to use two Kerr

cells containing different liquids and to make use of their different refractive chromatic dispersions. This dispersion, which is connected with refractive index, must be distinguished from the Kerr chromatic dispersions which would always remain proportionately the same whatever liquids were used. The use of right- or left-handed quartz is of no value, since the net result would be that for most wave-lengths the Kerr cell would no longer be between crossed nicols.

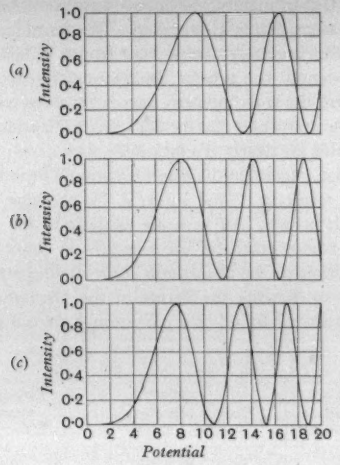


Fig. 5. Kerr-cell characteristic curves for three wave-lengths. (*a*) $\lambda = \cdot 70\mu$ (red); (*b*) $\lambda = \cdot 53\mu$ (green); (*c*) $\lambda = \cdot 46\mu$ (blue).

§ 5. KERR CELLS WITH SHAPED PLATES

In designing a Kerr cell it is necessary to consider it as part of the optical system by which the television picture is being projected. In this system, a given small area or spot of light is required to radiate within a given cone, the actual dimensions of spot and cone being determined by such optical constants of the system as aperture and focal length.

Suppose that, for the system with which we are dealing, the area and cone of light required are as shown in figure 6 (*a*). Then to satisfy these requirements with a parallel-plate Kerr cell, the plates would have to be as shown in (*b*) although their length and separation could be varied somewhat. Obviously, however, the arrangement shown in (*c*), in which the plates are shaped so as to fit the cone, is the more efficient form since the separation between the plates is at each point the narrowest that will still allow the required cone of light to be transmitted. At each point, therefore, the Kerr effect is a maximum for a given potential and the total retardation is also a maximum.

The expression (1) for ϕ shows that l must be large and d small if ϕ is to be of a useful magnitude and E is not to be excessive. With l long and d small, in a parallel-plate Kerr cell, the cone of light transmitted must necessarily be very limited. With the shaped plates l can be increased indefinitely without cutting down the cone of light, although the gain is eventually counterbalanced by the increased absorption in the nitrobenzene.

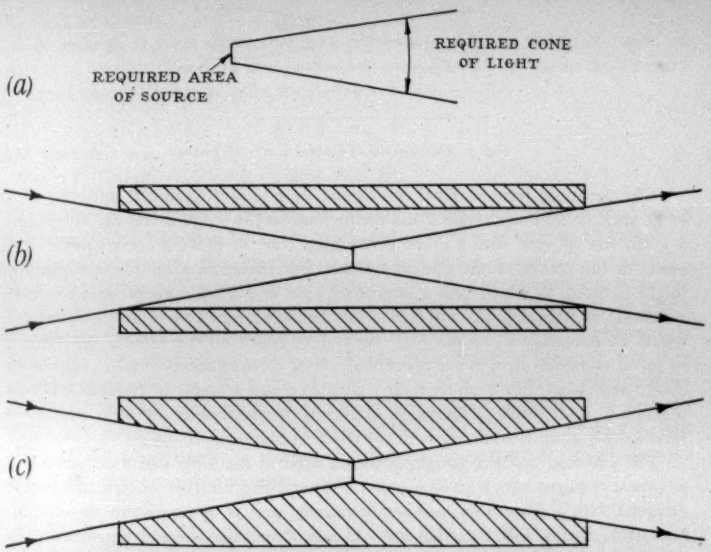


Fig. 6. (*a*) Explanatory diagram. (*b*) Parallel-plate Kerr cell. (*c*) Shaped-plate Kerr cell.

The total retardation, ϕ_n, of the new type of cell is readily calculated by means of a simple integration. With reference to figure 7, the retardation produced across the short distance dx is given by ϕ_n x

$$d\phi_n = \frac{KE^2 dx}{(d + 2Dx/l)^2},$$

and the total retardation is given by

$$\phi_n = \int_{-\frac{1}{2}l}^{+\frac{1}{2}l} \frac{l^2 KE^2}{(dl + 2Dx)^2}\, dx,$$

which becomes

$$\phi_n = (KlE^2)/dD \qquad \text{.....(6),}$$

where d is the separation at the mid-point of the plates and D the separation at each end. If this is compared with the formula for the retardation for parallel plates d, D

$$\phi = (KlE^2)/D^2$$

at the separation D that would be necessary to provide the same cone of light and area of source, the gain is immediately apparent. In the cells used at The Gramophone Company's Laboratories, $D/d = 4$ approximately, so that only one-half of the biasing and modulating voltages were necessary compared to those required if parallel plates had been used.

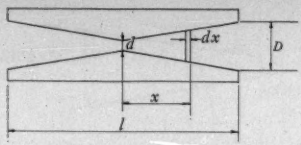


Fig. 7. Dimensions of shaped Kerr-cell plates.

The optimum form of Kerr cell cannot be finally determined until the effect of varying the mid-separation d and the inclination $(D - d)/l$ is known. To produce a given size of spot with a given converging cone of light, a considerable range exists in the choice of the optical system. For instance, a microscope objective might be used, in which case a very small spot and a big cone of light would be required; or were a projection lens used, a much larger source and smaller cone would be necessary to produce the equivalent result on the screen. In practice, so far as television images are concerned, other considerations enter to restrict the choice somewhat, but even so there exists a certain amount of freedom. It then remains to find which combination of Kerr cell and lens is the most efficient in producing a given retardation with the smallest biasing potential.

The practical problem usually presents itself in the form that a certain size of picture is required with a given number of lines using a mirror wheel that has been designed with a reasonable number of mirrors and is of reasonable dimensions; reasonable, that is, with regard to cost and ease of manufacture and adjustment. The size of picture and number of lines determines the size of spot required, the size of picture and number of mirrors determines the distance of the mirror wheel from the screen and the size of mirror wheel determines the aperture of the projection lens and thence the maximum cone of light that could be employed. The remaining unknown is the focal length of the projection lens in order to give the most efficient Kerr-cell operation.

A well-known property of any optical system is the fact that the product of the image height, the sine of the convergent angle of the cone of light and the refractive index is a constant for all the conjugate foci in the system. As the size of spot on the screen and the final convergent cone are fixed, their product will be constant and the same rule will apply at other focussed points in the system, in particular at the Kerr cell which is in a conjugate plane to the screen. Accordingly, if full use is to be made of the projection lens and mirror wheel, the cell must be designed so that

$$d\,(D - d)\,N/l = Q \qquad (7),$$

where Q is the invariant referred to above, d of course corresponds to the image height, $(D-d)/l$ to the cone of light, and N is the refractive index of the nitrobenzene. Then, from (6) and (7)

$$\phi_n = K_2 \frac{D-d}{D} E^2$$

$$= K_2 (1 - d/D) E^2,$$

K_2 being a constant.

To make ϕ_n as great as possible for a given value of E, the ratio d/D should be as small as possible. The difference between ϕ_n when $d/D = \frac{1}{4}$, and in the theoretically best case when $d/D = 0$, is quite small. When $d/D = \frac{1}{4}$,

$$\phi_n = \tfrac{3}{4} \cdot K_2 \cdot E^2;$$

and when $d/D = 0$,

$$\phi_n = K_2 \cdot E^2.$$

The difference amounts, in practice, to only 15 per cent. reduction in the biasing potential. It can therefore be safely concluded that there is little to be gained by reducing d/D below $\frac{1}{4}$. For determining the actual values of d and D, equation (7) shows that as d and D are diminished l also is reduced. The shorter l can be made, the smaller is the loss of light due to absorption in the nitrobenzene. It might possibly have been advantageous to have reduced d below the value of 0·02 in. that was used. The entire absence of sparking and other troubles, however, was very adequate compensation. With the relative values of d, D and l determined on theoretical grounds and the absolute values on practical grounds, it is then an elementary problem in optics to determine the focal length of projection lens required, use being made, of course, of the quantities—aperture, image-distance and so on—that are already known.

§ 6. FREQUENCY RESPONSE

In the introduction it was pointed out that the two main requirements for a light-modulating source were a good frequency-response and high light-efficiency. The design of a Kerr cell for maximum light-efficiency has been dealt with in considerable detail, but nothing has been said with reference to the frequency-response. The reason for this is that little can be done to improve it or make it worse. So far as is known, the Kerr effect in itself will respond to very high frequencies indeed. The limiting factor occurs in applying the high frequencies to a given Kerr cell owing to the capacity of the cell. This consideration will therefore affect the design of the cell to the extent of making it desirable to keep the area of the plates as small as possible and to increase the separation if necessary. Actually, with pure nitrobenzene, little attenuation need be anticipated until frequencies above, roughly, 15,000 ~ have to be dealt with. It may then become necessary to compromise between light-efficiency and frequency-response. In any case the

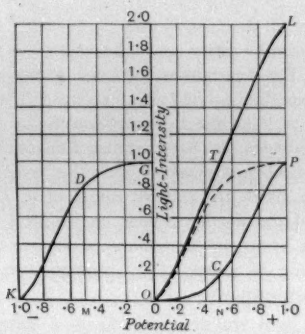
mechanical design of the cell should be such that the areas of the plates are the minimum required for theoretical reasons; the mounting of the plates, for instance, should not involve any increase in their area.

§ 7. ACKNOWLEDGMENT

In conclusion I would like to express to The Gramophone Co. Ltd. my thanks for permission to publish these results, which were obtained in their laboratories.

DISCUSSION

Mr A. J. MADDOCK. As the potential applied to a Kerr cell is increased, the successive maxima of intensity occur at correspondingly decreased separation. It



OCP, characteristic curve for cell *A*.
KDG, „ „ „ *B* (with optical bias of $\frac{1}{4}\lambda$).
OTL, resultant curve for combined effect of two cells.

seems possible, therefore, that at sufficiently high potentials only a narrow spectral region of, say, the blue will be at a minimum with a resultant white colour—an effect somewhat analogous to a thin film of oil on water appearing white when of sufficient thickness. In this way it would appear that for white light at high potentials on the cell the Kerr effect has apparently disappeared though still existing for the various spectral regions. I should like to ask the author if he has experienced this.

As a method of obtaining a linear relation between potential and emergent light-intensity and maximum variation of this latter the following scheme has occurred to me. The light after polarization by the first nicol is divided into two separate beams each passing through a separate Kerr cell and being recombined before traversing the analysing nicol. Corresponding plates of the two cells are connected

in parallel and to the source of modulating potential. Means are arranged for biasing each cell to half the maximum potential and in such a sense that corresponding plates of the two cells are of opposite polarity; thus if the top and bottom plates are correspondingly connected together and to the modulating source, then in cell *A* the top plate will be made positive with respect to the bottom plate and in cell *B* the top plate will be negative with respect to the bottom plate. One cell, *B*, has an optical bias of $\frac{1}{4}\lambda$ as described by the author. Cell *A* will thus have the characteristic curve *OCP* in the accompanying figure biased to the point *C*, whilst cell *B* will have the curve *KDG* biased to point *D*; the intensity of the final emergent beam being given by the sum of the ordinates *DM* and *CN*, i.e. point *T*. As the modulating voltage swings on the positive half-cycle, curves *CP* and *DG* are traced out for the two cells, whilst during the negative half-cycle we obtain *CO* and *DK*. The intensity of the final beam is the sum of the ordinates of these two curves and is shown at *OTL* (the dotted curve *OP* being the curve *KDG* translated for easy addition of the ordinates). It will be seen that this is sensibly linear over the whole of the range of modulating potential and that the light intensity varies from zero to the full intensity of the initial plane-polarized beam, except for losses in the optical system.

AUTHOR'S reply. Dr Maddock's anticipation that a point can be reached at which no change in intensity occurs is theoretically quite correct. But it would be difficult to observe such an effect in practice, and personally I have never done so, owing to the very high potential required to produce the phenomenon. An exactly analogous result, however, is produced in photo-elastic experiments and I have discussed this particular point recently*.

The scheme suggested to obtain a more linear relation between potential and intensity is ingenious, and provided it could be successfully put into practice should achieve the desired result. The diagram, however, is somewhat misleading as the apparent advantage of the curve *OTL* over *KDG* is to some extent due to the fact that it is drawn on twice the scale. It would be better shown lying between *OCP* and the dotted curve and drawn as an average rather than as the sum of these two. It is evident that the new curve in that case would be less steep at the lower end than *KDG*, although over the whole range from *O* to *P* the result would certainly be a closer approximation to a straight line.

* R. V. Band and W. D. Wright, "The Analysis of the Colors observed in Photo-elastic Experiments," *J. Opt. Soc. Am.* **20**, 381 (1930).

ON THE FORCES ACTING ON DROPS IN AN ELECTRIC FIELD

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ABSTRACT. If whilst a drop of electrolyte is falling in distilled water, a horizontal electric field be established, the drop rapidly spreads out into a filament parallel to the lines of force. An explanation of this phenomenon is given on the basis of the charge at the boundary between conductors that necessarily accompanies the transport of electricity. Experiments with dielectrics and with drops that contract instead of expand are described, and the paper is illustrated with photographs.

§ 1. INTRODUCTION

If two metal plates are immersed in distilled water and a small drop of some coloured electrolyte, such as a solution of potassium permanganate, is allowed to fall between them, then the moment an electric field is established across the plates, the drop begins to spread out laterally into a long filament parallel to the lines of force. With 200 volts across the plates, and with a slowly falling drop, the experiment is very striking and the rate of lateral spreading may easily exceed that of vertical fall under gravity. Experiments can be extended without difficulty to colourless electrolytes by making use of the differences of their refractive indices from that of water. A point source of light casts an easily observable shadow of such a drop, and when the plates are connected to the supply mains the drop is seen to spread out in a similar way.

The classical experiments of Quincke on liquid dielectrics* and the recent experiments of Macky† on the breaking of water globules in strong electric fields deal with related subjects, but the author has so far been unable to find any direct references to the present phenomenon, and as it is possibly new the following investigation has been undertaken.

§ 2. EXPERIMENTAL ARRANGEMENTS

As has already been mentioned the phenomenon may be observed with the very simplest apparatus, but as all is over in a second or two it is desirable to make permanent records which may be examined at leisure. With this in view recourse was had to the shadow cast on a photographic plate by the condensed spark discharge from the secondary of an induction coil. The primary was actuated by contacts carried on the back of the plate carrier, and the plate itself and the carrier were pushed upwards by hand behind a hole of such a size that four ex-

* *Phil. Mag.* **16**, 1 (1883). See also Addenbrooke, *ibid.* **3**, 1116 (1927).
† *Proc. R.S.* **133**, 565 (1931).

Potassium permanganate

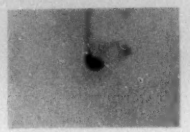

Fig. 1. Time, 0 sec.

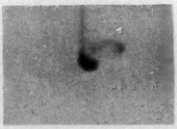

Fig. 2. Time, 0·6 sec.

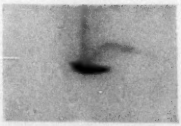

Fig. 3. Time, 1·3 sec.

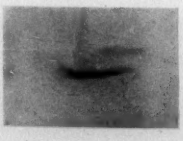

Fig. 4. Time, 1·9 sec.

Sulphuric acid

Fig. 5. Time, 0 sec.

Fig. 6. Time, 0·4 sec.

Fig. 7. Time, 1·0 sec.

Fig. 8. Time, 1·3 sec.

Sulphuric acid

Fig. 9. Time, 0 sec.

Fig. 10. Time, 0·6 sec.

Fig. 11. Time, 1·2 sec.

Fig. 12. Time, 1·6 sec.

[*To face p.* 336

posures were possible on one plate. The first two exposures were made without any field, whilst the last two were made with the field switched on. From the first two photographs it was possible to examine any changes taking place in the shape of the drop due to causes other than the electric field, and, from the last two, any changes directly due to this field.

A lens which formed the bob of a short pendulum focussed light from a point source on the side of the plate, and thus provided a time scale of reference. Generally the photographs were taken at intervals of about 0·6 sec.

§ 3. RESULTS

These are embodied in the photographs, figures 1 to 24. Figures 1–12 exhibit drops of initially simple shapes. More often the drops are of complicated shapes, as in figures 13–16. Here the same effect can be traced although it is not at once so obvious. Figures 17–20 represent an effect not yet discussed. Here instead of a drop of electrolyte being allowed to fall into distilled water, a drop of distilled water, whose density had been raised by the admixture of a small quantity of glycerine, was allowed to fall into weak sulphuric acid. The significance of the result will be considered later. In all these cases the difference of potential between the electrodes, which were of platinum, was about 220 V., the positive pole was on the left, and the potential gradient was about 49 V./cm.

Figures 21–24 deal with the case of so-called non-conducting liquids. Here a drop made up of a mixture of nitrobenzene and alcohol was allowed to fall into xylol. The potential difference was much higher and was derived from a small transformer with a step-up ratio of twenty. The r.m.s. voltage on the primary was 65 and the secondary voltage was therefore assumed to be 1300, giving a gradient of 290 V./cm. The significance of this result will likewise be discussed later.

The distilled water used was of the ordinary laboratory type, no special care having been taken with its preparation. Its specific resistance τ varied from τ $1 - 2 \times 10^5$ ohm-cm. and was thus much below the standard of conductivity water.

Data for some solutions used

	Composition of drop	Liquid in which drop falls
Figs. 1–4	0·265 per cent solution potassium permanganate in water $\tau = 635$ ohm-cm.	Water $\tau = 1{\cdot}3 \times 10^5$ ohm-cm.
Figs. 5–12	0·371 per cent solution sulphuric acid in water $\tau = 64$ ohm-cm.	Water $\tau = 2{\cdot}1 \times 10^5$ ohm-cm.
Figs. 13–16	0·37 per cent solution eosin in water $\tau = 1520$ ohm-cm.	Water $\tau = 1{\cdot}0 \times 10^5$ ohm-cm.
Figs. 17–20	0·37 per cent solution glycerine in water $\tau = 1{\cdot}6 \times 10^5$ ohm-cm.	Very weak sulphuric acid $\tau = 1{\cdot}48 \times 10^4$ ohm-cm.
Figs. 21–24	Alcohol 5 parts (Dielectric constant = 27); nitrobenzene (about 1 part) (Dielectric constant = 34)	Xylol (Dielectric constant = 2·4)

§ 4. DISCUSSION OF RESULTS

Although the phenomenon under observation was first noticed with drops of an electrolyte in distilled water, it is theoretically simpler to start with the case of the so-called non-conducting liquids exhibited in figures 21–24. Here a liquid of specific inductive capacity about 28 is dropped into one of specific inductive capacity 2·4. As a result, the lines of force crowd into the globule in the well-known fashion*, and the concentration at the poles of a spherical globule rises to $3K_2/(K_2 + 2K_1)$ times the undisturbed value, where K_2 is the dielectric constant of the globule and K_1 that of the medium into which it is dropped. In the present case, therefore, the concentration is 2·6 times the normal field, whilst for a drop of infinite dielectric constant the concentration would be 3·0 times.

K_2
K_1

This concentration is a very obvious feature, but, if we wish to explain the sideways spreading of the drop, we must look to the change in tension that occurs when a tube of force passes from one medium to another, even when there is no such concentration. The surfaces of a slab of dielectric, for instance, placed between the flat plates of a condenser, experience forces tending to pull them across to the plates. The tension per cm.^{2} of dielectric surface is easily calculated, for the tension per cm.^{2} outside the slab is $K_1F_1^2/8\pi$ whilst that within the slab is $K_2F_2^2/8\pi$, where F_2 and F_1 represent the respective potential gradients within and without the slab. The resultant tension per cm.^{2} towards either plate is therefore

F_2, F_1

$$K_1F_1^2/8\pi - K_2F_2^2/8\pi \qquad \text{.....(1)},$$

or, using the normal boundary condition, we may write it

$$(K_1F_1^2/8\pi)\,(1 - K_1/K_2) \qquad \text{.....(2)}.$$

For a dielectric of infinite specific inductive capacity this would be $K_1F_1^2/8\pi$, whilst for the dielectrics in this experiment it would be 0·91 of this value.

In the region of the poles of a spherical droplet, therefore, where the tubes of force pass almost normally into the surface, there is a tension not far short of that which would be experienced by a metallic globule of the same size, and it is to this tension that we must attribute the lateral spreading shown in figures 21–24.

The case of an electrolyte is a little more complicated. First of all the drop is in general not quite at the same potential as the solution into which it falls, and thus has a resultant charge. This resultant charge is small, however, and in any case will only cause a drift of the drop to one or other electrode. It will thus in no wise explain the lateral spreading.

As the voltage gradients employed in this work are comparatively large, we shall, as a simplification, use Ohm's Law and deal with our electrolytes as with metallic conductors, writing

$$(1/\tau_1)\,(dv_1/dn) = (1/\tau_2)\,(dv_2/dn) \qquad \text{.....(3)},$$

where τ_1 and τ_2 are the specific resistances of the two media and where dv_1/dn and dv_2/dn represent the respective normal potential gradients.

τ_1, τ_2, v_1
v_2, n

* J. J. Thomson, *Electricity and Magnetism*, chap. 5.

Since the equation is formally similar to the normal condition at a dielectric boundary

$$K_1 dv_1/dn = K_2 dv_2/dn \qquad \text{.....(4)},$$

and since moreover the tangential conditions are also the same, the concentration of flow-lines round a highly conducting sphere will be exactly similar to that of the tubes of force in the electrostatic problem.

On the other hand, we must do more than concern ourselves with formal similarities. In the actual problem electrostatic and electrodynamic conditions must be simultaneously satisfied. Yet as they stand (3) and (4) either imply that K_1/K_2 is equal to τ_1/τ_2 or else are mutually exclusive. The difficulty disappears however if there is a free charge σ per cm.^{2} of boundary*, for we may then write σ

$$K_1 dv_1/dn - K_2 dv_2/dn = 4\pi\sigma \qquad \text{.....(5)},$$

and it is now possible for (3) and (4) to be simultaneously true. Thus when we consider a droplet of comparatively high conductivity placed in a medium of low conductivity, we have not only to consider the bound charges at the surface resulting from the polarized ends of the molecules, but also the superposed free charges. Although therefore equation (1) is still true, (2) is not, and we must now substitute from (3) instead of from (4). Thus we have

$$\text{Resultant tension per cm.}^2 = \left(\frac{dv_1}{dn}\right)^2 \left(K_1 - \frac{K_2\tau_2^2}{\tau_1^2}\right) \Big/ 8\pi \qquad \text{.....(6)}.$$

It might at first be thought that this expression should tend towards (2) as the specific resistances became very great. It will be observed, however, that it does not. The truth is that the electrostatic condition (4) and the electrodynamic condition (3) represent respectively the far removed initial and the steady conditions. In the case of a very poor conductor, the attainment of the latter would take a considerable time and the free surface charge would only slowly develop, whereas, in the case of a comparatively good conductor such as an electrolyte, the steady condition would be attained in the merest fraction of a second. Actually, in the case of the nitrobenzene-and-alcohol droplet in xylol, it is probable† that quite a considerable amount of free surface charge would be developed even in 0·01 sec. Strictly, therefore, in this case the phenomenon should be regarded as intermediate between the two extremes. Keeping all this in mind, let us now proceed to make a comparison between the rates of vertical fall and lateral spreading of a drop. Let us first consider a central vertical column of radius r that runs through a drop of radius R. The resultant gravitational pull downwards is given by r R

$$\pi r^2 . 2R(\rho_2 - \rho_1) g,$$

where $(\rho_2 - \rho_1)$ is the difference of the densities of the drop and the water, and where g is the acceleration of gravity. Hence the resultant pressure at the bottom ρ_2, ρ_1 g

* J. J. Thomson, *Electricity and Magnetism*, chap. 9; or J. H. Jeans, *Electricity and Magnetism*, chap. 10.

† The time relaxation is given by $\exp(-4\pi t/\kappa\tau)$; see J. H. Jeans, *Electricity and Magnetism*, chap. 10.

of the drop is $2R(\rho_2 - \rho_1)g$. In the case of the drop of potassium-permanganate solution, $(\rho_2 - \rho_1) = 0{\cdot}0017$ gm./cm. and $R = 0{\cdot}15$ cm. roughly. Hence the resultant pressure at the lowest point is approximately 0·5 dyne/cm.²

To obtain the lateral pull, we must revert to (1) and estimate the values of the potential gradients within and without the drop in the region of a pole. We know that within the drop*

$$\frac{dv_2}{dn} = \left(\frac{3dv}{dn}\right) \Big/ \left(\frac{\tau_1}{\tau_2} + 2\right),$$

where dv/dn is the potential gradient in a region remote from the drop. Now in the case of the drop under consideration $\tau_1/\tau_2 = 1{\cdot}3 \times 10^5/635$ and hence

$$dv_2/dn = 0{\cdot}014\ dv/dn.$$

Outside the drop dv_1/dn has τ_1/τ_2 times this value or nearly $3dv/dn$. Hence the resultant tension per cm.² may be written

$$\left\{K_1\left(\frac{3dv}{dn}\right)^2 - K_2\left(\frac{0{\cdot}014dv}{dn}\right)^2\right\} \Big/ 8\pi.$$

Now K_1 the dielectric constant of water is 80, and that of an electrolyte is of the same order†, hence the tension per cm.² may be written $28{\cdot}6\ (dv/dn)^2$. The potential gradient used, namely 49 V./cm.‡, corresponds to 0·16 e.s.u./cm. and hence the tension per cm.² works out to 0·7 dyne. It thus appears that, in the case of the permanganate droplet, the lateral and vertical forces per cm.² are of the same order, being respectively 0·7 and 0·5 dyne. Unless therefore the droplet offers any appreciable resistance to deformation of shape§, we should expect the initial lateral movement not to be very different from the rate of fall.

An inspection of the photographs shows that the lateral spreading is rather faster than has been indicated, but there is no difficulty in accounting for this. We have considered only a spherical drop. The concentration of lines of force at the poles of a spheroidal drop, with its long axis parallel to the field, is very much greater, and there is thus a correspondingly increased tension on the ends.

There is one further matter that needs consideration. The elongated droplet referred to, carries a much denser current than the liquid around. Now the cross section of the drop can be split into a number of current-carrying filaments, and each of these filaments attracts its neighbours, so that there is thus a tendency for the drop to reduce its diameter. An estimate of the order of the pressure tending to cause this reduction can be obtained from the expression for the lateral pressure of the magnetic tubes of force, but a more exact calculation is not difficult, and, for a long cylinder, the pressure works out to be the square of the current divided by the cross-section. As the current is a mere fraction of an ampère the pressure is quite negligible, and we need not concern ourselves any more with this factor.

* J. J. Thomson, *Electricity and Magnetism*, chap. 5.

† R. T. Lattey and W. G. Davies, *Phil. Mag.* **12**, 1111 (1931).

‡ It will be noted that, owing to the high dielectric constant of water, this value is much less than in the case of xylol.

§ It would be necessary to take account of this resistance if there were an appreciable interfacial surface tension, or if the internal and external lateral pressures of the tubes of force around an equatorial band were seriously different.

So far then our explanations appear to be satisfactory, but one obvious confirmatory experiment remains to be performed, namely, the production of a lateral contraction instead of a lateral spreading. For this to happen, it is clear that we must make the tension inside the drop greater than that outside. In other words, the drop must be of low conductivity and the surrounding liquid of comparatively high conductivity. With this in view, therefore, a mixture of glycerine and water of specific resistance $1{\cdot}6 \times 10^5$ ohm-cm. was dropped into weak sulphuric acid of specific resistance $1{\cdot}5 \times 10^4$ ohm-cm. Here dv_2/dn works out to be $1{\cdot}4\, dv/dn$, whilst dv_1/dn is approximately one-tenth of this value. Hence since K_1 and K_2 are not very different, the resultant contracting tension may be written

$$80\left\{\left(\frac{0{\cdot}14\,dv}{dn}\right)^2-\left(\frac{1{\cdot}4\,dv}{dn}\right)^2\right\}\Big/8\pi,$$

which $= -0{\cdot}16$ dyne/cm^2. This is approximately one-quarter the previous expanding tension, but an inspection of figures 17–20 leaves no doubt about its existence, and the theory is thus confirmed.

It appears possible, therefore, to give a satisfactory explanation of the lateral spreading of drops of one electrolyte in another on the basis of the change of tension in the tubes of force as they pass a surface of separation. We can further regard this change of tension as arising, in dielectrics, from a polarization charge at the surface, and, in conductors, from a combination of free and polarization charges.

DISCUSSION

Mr R. H. Humphry (communicated). The experiments which Dr West has carried out have an important bearing on the observation of cataphoresis in colloidal solutions. In the usual method of making such observations the motion of a surface of separation in an electric field is observed, and if there is a difference between the conductivities or between the dielectric constants of the solution and the dispersion medium, as is generally the case, effects such as those mentioned in this paper are likely to vitiate the results of such measurements. Experiments not unlike those of Dr West were carried out some time ago in the course of an investigation of cataphoresis in non-aqueous media*. In subsequent experiments by Dr R. S. Jane and myself†, in which we used the Toepler *schlieren* method to render visible a stream of a solution of rubber in benzene flowing in benzene between electrodes, we were careful to arrange that the dielectric constant of the solution was practically the same as that of the dispersion medium, namely 2·2 for rubber and 2·29 for benzene. It was found that careful drying of the substances used completely changed the effect, no spreading of the stream being observed in the case of the dry solution; apparently water, with its high dielectric constant, may, even in very small quantities, play an important part in these "non-aqueous"

* R. H. Humphry, *Koll. Zeit.* **38**, 306 (1926).

† R. H. Humphry and R. S. Jane, *Trans. Faraday Soc.* **22**, 420 (1926); *Koll. Zeit.* **41**, 293 (1927).

solutions. These experiments have been discussed critically by Büchner and van Royen* who also considered the conditions for the spreading (or otherwise) of drops in electric fields.

It would be of interest to know whether any experiments have been carried out with alternating fields. Since the pull is proportional to the square of the electric field, the spreading of the drop should occur equally in an alternating field. On the other hand, if electric charges are present the force on these would be proportional to the first power of the field and an alternating field would have no resultant effect on them. In the experiments referred to on rubber and benzene we found that, while spreading occurred in a direct field, no such effect was observed when an alternating field of the same intensity was used.

AUTHOR'S reply. Mr Humphry's experiment, in which a cylindrical stream of rubber solution is drawn out into a fan by an electric field, certainly resembles the present experiments with electrolytes, whilst those he mentions of Büchner and van Royen resemble them still more closely. The methods used differ, of course, but, despite this, there is good agreement between the conclusions reached by these workers and by the author.

It is suggested that in Mr Humphry's undried solutions a certain amount of conduction takes place. If, further, the rubber solution were the better conductor, the stream would spread for much the same reason as do the drops of electrolyte in the present work. Careful drying would greatly reduce the effect by reducing the conductivity and thus increasing the time the charges take to form—i.e. the "time of relaxation" referred to in the paper.

In reply to Mr Humphry's last query: experiments made with 50 ~ alternating currents showed that the effect still took place. Any charges developed at the ends of a drop change sign with the field in much the same way as would the charges at the ends of a metallic conductor in a similar alternating field. Owing mainly to the higher resistance of the drop, however, the time taken for the charges to reach their full values would be greater, and, with sufficiently high frequencies, the tendency to spread would thus become progressively smaller. Apparently the time the charges take to form in Mr Humphry's high-resistance solutions is considerably greater than the period of the alternating field. In the absence of such charges, therefore, and with equal dielectric constants, there is no reason why the stream should spread. The present work would thus appear to throw some light on Mr Humphry's results.

* E. H. Büchner and A. H. H. van Royen, *Koll. Zeit.* **49**, 249 (1929).

THE FIRST SPARK SPECTRUM OF ARSENIC (As II)

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ABSTRACT. The spectrum of arsenic has been examined under varying conditions of excitation, and lines belonging to the singly-ionized atom have been identified. Combinations between the deep terms of the singlet and triplet system of As II have been discovered; about 70 lines have entered into the classification. A provisional term scheme, based on two members of the singlet series $5s\ ^1P - mp\ ^1D$, is set up. The largest term $4p\ ^3P_0$ (ν 162788) leads to an ionization-potential of about 20·1 V.

§ 1. INTRODUCTION

In continuation of the previous work by the writer*, on the second spark spectrum of arsenic, the following paper gives a preliminary account of the regularities that have been obtained in the first spark spectrum of arsenic. It is based on an experimental investigation of the spectrum under different conditions of excitation. The work has been made possible in the first instance by the data, kindly supplied by Prof. R. A. Sawyer, of the spark spectrum of arsenic in the vacuum-grating region down to λ 370 Å. Of the published wave-lengths in this region, those of Queney† also were made use of. When the author, on the basis of the above results, had worked out the preliminary analysis of the spectrum of As II, discussed below, he had an opportunity of looking into the unpublished results obtained independently by K. R. Rao, with which there is found to be a very good agreement in the main features of the scheme. It is therefore considered desirable to publish this preliminary account pending a more detailed study of the spectrum, which is in progress.

§ 2. EXPERIMENTAL

The spark spectrum of arsenic was studied in the visible and the quartz regions by a 10-ft. concave grating in an Eagle mounting and a Hilger E_2 quartz spectrograph. The spark was produced between electrodes of pure arsenic in air and in hydrogen at varying pressures, by a ½-kilowatt 20,000-volt transformer. To distinguish lines due to different stages of ionization, the spectrum was photographed under varying degrees of discharge; to this end a variable self-inductance and capacity were included in the secondary circuit. An auxiliary spark-gap in air was also placed in series with the experimental spark-gap. Discharge through the vapour

* *Ind. J. Physics*, **3**, 437 (1929).
† *J. de Phys. et la Rad.* **10**, 448 (1929).

of pure metallic arsenic and arsenic trichloride was also studied, the length of the spark-gap being varied. The lines of the tube discharge were very sharp. The chlorine lines visible on the plates were first identified and eliminated. For measuring the wave-lengths in these regions the lines of the iron arc were used as standards. The already-published analyses of As I, As III, and As IV also have helped in the identification of lines belonging to the spectrum of singly-ionized arsenic.

§ 3. RESULTS

The terms which are to be expected in the spectrum of As II according to Hund's theory are given below in table 1.

Table 1. Predicted terms of As II.

1_1	2_1 2_2	3_1 3_2 3_3	4_1 4_2 4_3 4_4	5_1 5_2 5_3	Term prefix	Terms
2	2 6	2 6 10	2 2 . .	. . .	$4p$	$^3P\ ^1D\ ^1S$
2	2 6	2 6 10	2 1 . .	1 . .	$5s$	$^3P\ ^1P$
2	2 6	2 6 10	2 1 1 .	. . .	$4d$	$^3F\ ^3D\ ^3P\ ^1F\ ^1D\ ^1P$
2	2 6	2 6 10	2 1 . .	. 1 .	$5p$	$^3D\ ^3P\ ^3S\ ^1D\ ^1P\ ^1S$
2	2 6	2 6 10	2 1 . 1	. . .	$4f$	$^3G\ ^3F\ ^3D\ ^1G\ ^1F\ ^1D$

It is evident from the table of predicted terms of As II that the strongest lines in the spectrum should correspond to the transitions $4p - 5s$ and $4d$. The location of the triplet $4p\ ^3P - 5s\ ^3P$ formed the starting point of the classification. The position of this triplet in the analogous spectra of other elements is represented below.

	$mp\ ^3P_2 - (m+1)s\ ^3P_2$		
C I	60350	N II	148943
Si I	39732	P II	86656
Ge I	37708	As II	(78967)
Sn I	35201	Sb II	—

It is expected from such a comparison that in As II this group may occur in the region ν 80000, which is found to be also a region in which some of the strongest unclassified lines of the arsenic spark spectrum occur. Further, the elimination from the lines in this region of a few belonging to As IV and As III* at once led to the identification of the combination $4p\ ^3P - 5s\ ^3P$, represented in table 2. It is gratifying to note that in one of the plates taken by K. R. Rao, at Upsala, of the vacuum spark spectrum with a certain amount of inductance in series, this group stands out clearly isolated in the appropriate region. About 5000 cm.$^{-1}$ from this group and towards the region of larger wave-length, where the strong singlet $4p\ ^1D_2 - 5s\ ^1P_1$ might be expected, there are two intense lines λ1375 and 1369. Of these the former has proved to be the required line, supported by the presence of all the intercombination lines. The level $4p\ ^1S_0$ has been suggested to the writer by K. R. Rao.

* Rao and Badami, *Proc. R.S.* A, **131**, 154 (1931); K. R. Rao, *Proc. Phys. Soc.* **43**, 68 (1931).

The intervals between the terms of the deepest $4p$ levels being now known, the extension of the regularities further into the violet was considerably facilitated. The relative intensities among the lines constituting any level have led to the probable assignment of the level, which is indicated in table 2.

Table 2. Combinations in As II.

Terms		$4p\ ^3P_0$ 162788		$4p\ ^3P_1$ 161725		$4p\ ^3P_2$ 160248		$4p\ ^1D_2$ 152693		$4p\ ^1S_0$ 140189	
			1063		*1477*		*7555*		*12504*		
$5s\ ^3P_0$	84059	—		77666	(9)	—		—		—	
	397										
$^3P_1$	83662	79128	(10)	78063	(8)	76586	(10)	69030	(3)	56527	(2)
	2381										
$^3P_2$	81281	—		80445	(8)	78967	(10)	71412	(4)	—	
$5s\ ^1P_1$	79972	82816	(5)	81753	(2)	80278	(1)	72724	(10)	60219	(8)
$4d\ ^3D_1$	63723	99065	(8)	98002	(6)	—		88964	(2)	76470	(4)
	483										
$^3D_2$	63240	—		98485	(10)	97008	(2)	—		—	
	842										
$^3D_3$	62398	—		—		97851	(10)	90294	(6)	—	
$4d\ ^1D_2$	60305	—		—		99944	(3)	92387	(10)	—	
$4d\ ^1P_1$	61302	101485	(4)	100423	(3)	98945	(2)	91391	(6)	78889	(10)
$4d\ ^1F_3$	52916	—		—		107335	(1)	99774	(8)	—	
α	79688	—		—		80561	(3)	73004	(5)	—	
β	78156	84634	(3)	83570	(1)	82092	(1)	74537	(1)	62030	(1)
γ	73962	—		87765	(2)	86285	(2)	—		—	
δ	73240	—		—		87007	(4)	79453	(2)	—	
ϵ	60193	—		101531	(5)	100054	(3)	92501	(4)	79996	(1)

Terms		$5p\ ^3S_1$ 61702	$5p\ ^1D_2$ 60399	$5p\ ^1P_1$ 63610	$5p\ ^1S_0$ 58735
$5s\ ^3P_0$	84059	22358·1 (5)	—	20449·4 (8)	—
	397				
$^3P_1$	83662	21960·4 (7)	23263·1 (6)	20052·1 (9)	24926·5 (5)
	2381				
$^3P_2$	81281	19580·5 (8)	20882·9 (6)	17671·6 (8)	—
$5s\ ^1P_1$	79972	18270·0 (6)	19572·4 (8)	16360·3 (10)	21235·5 (7)

The group $4d\ ^3D$ has been tentatively suggested. The alternative group shown in table 3 has been identified by K. R. Rao.

Table 3.

	$4p\ ^3P_0$	$^3P_1$	$^3P_2$
$4d\ ^3D_1$	99065 (8)	98002 (6)	—
− 149			
$^3D_2$	—	97851 (10)	96373 (4)
2111			
$^3D_3$	—	—	98485 (10)

The partial inversion of the term $4d\,^3D$ is, of course, then in keeping with the corresponding feature in Ge I. But the line ν 96373 is not found in the list used by the writer; the first group is therefore retained. Until the analysis has been extended so as to identify the combinations of the $4d$ term with the $5p$, or more probably with the $4f$ terms, the allocation of the various other energy levels in table 2 is difficult; so only arbitrary symbols have been used to designate these levels. Besides those which are given in this table many more could be found by searching through the list of recurring frequency-differences. They have, however, been omitted.

The spark spectrum of arsenic is exceedingly rich in lines, in the visible and the quartz regions, which have to be assigned to the singly-ionized atom of the element. Our knowledge of the intervals of the $5s\,^3P$ term affords a clue to the extension of the analysis of the spectrum into this region. A search for lines giving the differences 2381 cm.$^{-1}$ and 397 cm.$^{-1}$ (equal to $5s\,^3P_1 - 5s\,^3P_2$ and $5s\,^3P_0 - 5s\,^3P_1$) did not lead to the location of any satisfactory group which could be ascribed to the combination $5s\,^3P - 5p\,^3P$. It is believed that this group, as also the group $5s\,^3P - 5p\,^3D$, lies partially in the red region, which has not yet been quite satisfactorily investigated. Experiments with a view to identifying these are in progress. Some of the singlets, supported by intercombinations, have been discovered and have led to a preliminary evaluation of the characteristic terms. It is evident from an examination of these that the strong and diffuse groups of lines in the visible region referred to above are probably due to the transition of the series electron $4d \rightarrow 4f$, which gives rise to several multiplets of the type $^3D\,^3F$, $^3D\,^3D$, $^3P\,^3D$, etc. Further attempts at analysis are being made on the lines of this suggestion.

There are three strong pairs with wave-number difference 720 cm.$^{-1}$, which within limits of experimental error is equal to the difference between the levels $(\gamma - \delta)$. Hence the classification of these pairs shown in table 4 is adopted. The terms a, b, c may belong to the configuration $4f$. A few other combinations between these and other known levels are also given below.

Table 4. (Cf. table 2.)

Terms		a 41268	b 43887	c 41318
γ	73962	32690·6 (6)	35309·6 (6)	32739·9 (6)
δ	73240	31970·4 (6)	34589·1 (3)	32019·7 (4)
$4d\,^3D_1$	63723	—	25072·9 (1)	—
	483			
$^3D_2$	63240	21975·7 (7)	—	—
	842			
$^3D_3$	62398	21131·7 (8)	—	—
$^1D_2$	60305	19033·7 (5)	21651·8 (6)	19083·5 (4)

The differences 397 and 2381 occur also in the pairs, given in table 5, which might form part of the transition $5s \rightarrow 5p$.

Table 5.

λ	Int.	ν	δν
5620·86	10	17786·0	
5498·01	7	18183·4	397·4
6405·95	4	15606·2	
5558·34	8	17986·0	2379·8
5331·47	7	18751·4	
4730·90	8	21131·7	2380·3

§4. TERM VALUES

A provisional estimate of the term values is made possible by the detection of two members of the singlet series $5s\ ^1P_1 - mp\ ^1D_2$.

$$5s\ ^1P_1 - 4p\ ^1D_2 = -\ 72724$$
$$5s\ ^1P_1 - 5p\ ^1D_2 = \quad 19572$$
$$\text{Limit}\quad 5s\ ^1P_1 = \quad 79972.$$

This value is adopted for $5s\ ^1P_1$ in the following list of characteristic term values in the spectrum of As II.

Table 6. Term values of As II.

$4p\ ^3P_0$	162788	$^3P_2$	81281	$5p\ ^1P_1$	63610	ϵ	60193
$^3P_1$	161725	$^1P_1$	79972	$4d\ ^3D_2$	63240	$5p\ ^1S_0$	58735
$^3P_2$	160248	α	79688	$4d\ ^3D_3$	62398	$4d\ ^1F_3$	52916
$^1D_2$	152693	β	78156	$5p\ ^3S_1$	61702	b	43887
$^1S_0$	140189	γ	73962	$4d\ ^1P_1$	61302	c	41318
$5s\ ^3P_0$	84059	δ	73240	$5p\ ^1D_2$	60399	a	41268
$5s\ ^3P_1$	83662	$4d\ ^3D_1$	63723	$4d\ ^1D_2$	60305		

The comparison in table 7 between the fundamental multiplets in the spectra of Ge I, As II and Se III* is interesting. The wave-numbers given in this table have been reduced for purposes of comparison, according to the method of Millikan and Bowen†.

Table 7.

		$4p\ ^3P_0$	δν	$^3P_1$	δν	$^3P_2$	δν	$^1D_2$	δν
$5s\ ^3P_0$	Ge I			34426					
	As II			67794	33368				
	Se III			102320	34526				
$^3P_1$	Ge I	35234		34677		33824			
	As II	69256	34022	68191	33514	66714	32890		
	Se III	104568	35312	102829	34638	100632	33918		
$^3P_2$	Ge I			36093		35240			
	As II			70573	34480	69095	33855		
	Se III			106437	35864	104242	35147		
$^1P_1$	Ge I							30427	
	As II							62852	32425
	Se III							96411	33559

* Badami and Rao, *Nature*, **128**, 3229 (1931).

† *Phys. Rev.* **26** (1925).

Finally, a list of all wave-lengths of As II that have been classified in this investigation is given below in table 8.

Table 8. List of classified lines of As II.

λ (intensity)	ν	Classification	λ (intensity)	ν	Classification
6110·66 (10)	16360·3	$5s\ ^1P_1 - 5p\ ^1P_1$	1287·57 (9)	77666	$4p\ ^3P_1 - 5s\ ^3P_0$
5657·23 (8)	17671·6	$5s\ ^3P_2 - 5p\ ^1P_1$	1281·01 (8)	78063	$4p\ ^3P_1 - 5s\ ^3P_1$
5471·95 (6)	18270·0	$5s\ ^1P_1 - 5p\ ^3S_1$	1267·61 (10)	78889	$4p\ ^1S_0 - 4d\ ^1P_1$
5252·37 (5)	19033·7	$4d\ ^1D_2 - a$	1266·36 (10)	78967	$4p\ ^3P_2 - 5s\ ^3P_2$
5238·69 (4)	19083·5	$4d\ ^1D_2 - c$	1263·78 (10)	79128	$4p\ ^3P_0 - 5s\ ^3P_1$
5107·82 (8)	19572·4	$5s\ ^1P_1 - 5p\ ^1D_2$	1258·61 (2)	79453	$4p\ ^1D_2 - \delta$
5105·71 (8)	19580·5	$5s\ ^3P_2 - 5p\ ^3S_1$	1250·07 (1)	79996	$4p\ ^1S_0 - \epsilon$
4985·62 (9)	20052·1	$5s\ ^3P_1 - 5p\ ^1P_1$	1245·67 (1)	80278	$4p\ ^3P_2 - 5s\ ^1P_1$
4888·76 (8)	20449·4	$5s\ ^3P_0 - 5p\ ^1P_1$	1243·09 (8)	80445	$4p\ ^3P_1 - 5s\ ^3P_2$
4787·27 (6)	20882·9	$5s\ ^3P_2 - 5p\ ^1D_2$	1241·29 (3)	80561	$4p\ ^3P_2 - \alpha$
4730·90 (8)	21131·7	$5s\ ^3D_3 - a$	1223·19 (2)	81753	$4p\ ^3P_1 - 5s\ ^1P_1$
4707·79 (7)	21235·5	$5s\ ^1P_1 - 5p\ ^1S_0$	1218·14 (1)	82092	$4p\ ^3P_2 - \beta$
4617·27 (6)	21651·8	$4d\ ^1D_2 - b$	1207·50 (5)	82816	$4p\ ^3P_0 - 5s\ ^1P_1$
4552·37 (7)	21960·4	$5s\ ^3P_1 - 5p\ ^3S_1$	1196·60 (1)	83570	$4p\ ^3P_1 - \beta$
4549·22 (7)	21975·7	$4d\ ^3D_2 - a$	1181·55 (3)	84634	$4p\ ^3P_0 - \beta$
4471·41 (5)	22358·1	$5s\ ^3P_0 - 5p\ ^3S_1$	1158·95 (2)	86285	$4p\ ^3P_2 - \gamma$
4297·45 (6)	23263·1	$5s\ ^3P_1 - 5p\ ^1D_2$	1149·33 (4)	87007	$4p\ ^3P_2 - \delta$
4010·66 (5)	24926·5	$5s\ ^3P_1 - 5p\ ^1S_0$	1139·40 (2)	87765	$4p\ ^3P_1 - \gamma$
3987·24 (1)	25072·9	$4d\ ^3D_1 - b$	1124·05 (2)	88964	$4p\ ^1D_2 - 4d\ ^3D_1$
3126·99 (6)	31970·4	$\delta - a$	1107·49 (6)	90294	$4p\ ^1D_2 - 4d\ ^3D_2$
3122·18 (4)	32019·7	$\delta - c$	1094·20 (6)	91391	$4p\ ^1D_2 - 4d\ ^1P_1$
3058·10 (6)	32690·6	$\gamma - a$	1082·40 (10)	92387	$4p\ ^1D_2 - 4d\ ^1D_2$
3053·49 (6)	32739·9	$\gamma - c$	1081·07 (4)	92501	$4p\ ^1D_2 - \epsilon$
2890·24 (3)	34589·1	$\delta - b$	1030·84 (2)	97008	$4p\ ^3P_2 - 4d\ ^3D_2$
2831·26 (6)	35309·6	$\gamma - b$	1021·96 (10)	97851	$4p\ ^3P_2 - 4d\ ^3D_3$
λ vac.			1020·39 (6)	98002	$4p\ ^3P_1 - 4d\ ^3D_1$
1769·06 (2)	56527	$4p\ ^1S_0 - 5s\ ^3P_1$	1015·38 (10)	98485	$4p\ ^3P_1 - 4d\ ^3D_2$
1660·60 (8)	60219	$4p\ ^1S_0 - 5s\ ^1P_1$	1010·66 (2)	98945	$4p\ ^3P_2 - 4d\ ^1P_1$
1612·11 (3)	62030	$4p\ ^1S_0 - \beta$	1009·44 (8)	99065	$4p\ ^3P_0 - 4d\ ^3D_1$
1448·64 (1)	69030	$4p\ ^1D_2 - 5s\ ^3P_1$	1002·27 (8)	99774	$4p\ ^1D_2 - 4d\ ^1F_3$
1400·32 (4)	71412	$4p\ ^1D_2 - 5s\ ^3P_2$	1000·56 (3)	99944	$4p\ ^3P_2 - 4d\ ^1D_2$
1375·07 (10)	72724	$4p\ ^1D_2 - 5s\ ^1P_1$	999·46 (3)	100054	$4p\ ^3P_2 - \epsilon$
1369·78 (5)	73004	$4p\ ^1D_2 - \alpha$	995·79 (3)	100423	$4p\ ^3P_1 - 4d\ ^1P_1$
1341·53 (1)	74537	$4p\ ^1D_2 - \beta$	985·37 (4)	101485	$4p\ ^3P_0 - 4d\ ^1P_1$
1307·76 (3)	76470	$4p\ ^1S_0 - 4d\ ^3D_1$	984·92 (5)	101531	$4p\ ^3P_1 - \epsilon$
1305·72 (10)	76586	$4p\ ^3P_2 - 5s\ ^3P_1$	931·66 (1)	107335	$4p\ ^3P_2 - 4d\ ^1F_3$

§ 5. ACKNOWLEDGMENTS

In conclusion I wish to express my grateful thanks to Prof. A. Fowler and Dr T. Royds for their interest in the work, and to Dr K. R. Rao for his very helpful suggestions.

THE PHOTOGRAPHIC MEASUREMENT OF THE ABSORPTION COEFFICIENTS OF GAMMA-RAYS FROM RADIUM (B + C)

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ABSTRACT. In continuation of earlier work in which the photographic action of γ-rays was examined, the absorption of the γ-rays from radium (B + C) filtered by 1·6 cm. of lead has been measured by a photographic method. With this lead filter the absorption is exponential. The absorption μ_e per electron for the lighter elements is found to be constant, in accordance with the Klein-Nishina formula; and the value, $1{\cdot}56 \times 10^{-25}$, corresponds to a wave-length of 7·0 X.U. The increase of μ_e for the heavier elements in excess of this is found to vary as Z^3, so that the additional absorption is assumed to be due to the photo-electric effect. It has been found that, by assuming the photoelectric term to vary as $\lambda^{2\cdot2}$, an approximate agreement is obtained with the absorption of short X-rays. The intensity of the main γ-rays in the radium (B + C) spectrum after they have passed through 1·6 cm. of lead is calculated by assuming the $\lambda^{2\cdot2}$ law, and it is shown that the intensities are not in contradiction to an effective wave-length of 7·0 X.U. It is shown that the use of limiting channels between the source and absorbers causes scattered, softened γ-rays to be superposed on the main beam.

§ 1. INTRODUCTION

In a previous paper* the photographic action of γ-rays was investigated and the variations of the photographic density produced by γ-rays on photographic film with (i) the time of exposure, and (ii) the intensity of γ-rays, were examined. It was also shown that the photographic method could be used to obtain values of the absorption of γ-rays in lead. If the γ-rays from a constant source, passing through thicknesses d_1 and d_2 of an absorber, took times t_1 and t_2 respectively to produce a certain photographic density, the absorption coefficient μ was shown to be given by

d_1, d_2, t_1, t_2

μ

$$\mu = (\log t_1 - \log t_2)/(d_1 - d_2).$$

In practice the exact times t_1 and t_2 to produce this particular density need not be known, but if times of exposure were chosen so that densities which did not exceed 0·8 were obtained, the times to produce this given density could be calculated, since, under these conditions, the density was directly proportional to the time of exposure. In any experiment, the object was to produce densities of the same order, as the linear relation between density and time had then to be used only over a small range. The densities used were in the vicinity of 0·6, as the reproducibility of the densities, under the same conditions of exposure, is greatest when the densities are between 0·5 and 0·7.

* J. S. Rogers, *Proc. Phys. Soc.* **43**, 59 (1931).

The relation above has been used to find the absorption coefficients of twenty-two elements. For any element, a series of films were exposed to γ-rays which had passed through different thicknesses of the element. The values of μ were obtained by plotting values of log t against d and measuring the slope of the straight line so produced.

§ 2. PHOTOGRAPHIC TECHNIQUE

Agfa duplitized film was used and was exposed in aluminium film-holders of the type previously described. From a sheet of film 7 in. × 5 in. it was possible to cut 22 pieces of film, and for any one determination the pieces cut from one film were used. After exposure these films were all developed and fixed together, a frame being made so that the pieces could all be clipped in a vertical position, and during development and fixing these were continually agitated in the solutions. A hydroquinone-metol developer was employed, development being for 5 minutes at 18° C. The accuracy of the method depends on the uniformity of the photographic emulsion and generally this was found to be reasonably good. Occasionally a discordant reading in an otherwise concordant set could be accounted for only by an irregularity in the emulsion. In the early stages of the work, the pieces of film for any one determination were not necessarily all selected from the one sheet, but when it was found that pieces from different sheets of film, when exposed and developed together, did not produce densities as uniform as those produced by pieces from the same sheet, all subsequent determinations were made from pieces cut from one sheet. The worst discrepancy from pieces cut from two different films was as follows. The times of exposure to produce a given density from two pieces cut from one sheet were 56 and 55 min. and from four pieces cut from another sheet 61, 63, 63 and 62 min., although the six pieces were all developed together. This variation appears to be exceptional, but it was considered advisable to make the remaining determinations with pieces cut from one film. The films were photometered with a Moll microphotometer, the galvanometer readings being obtained visually.

In all the determinations except some of those with lead and mercury, which were made with the film pressed between aluminium on one side and velvet on the other, lead intensifying-screens were used to shorten the times of exposure. Thin lead foil of mass 0·103 gm./cm^2 was pressed against each side of the film. Before the foil was used its intensifying effect was examined. Lead foil was bent and wrapped so that half of the film, back and front, was covered with the lead foil. Figure 1 shows a reproduction of the film together with a graph of the photometered readings. It will be seen that the density-variation from where there was no lead to where the lead was present is quite sharp, indicating that the lateral scattering was small. An apparent gradation in density is due to the heights of the lead foil differing slightly at the back and front of the film. The density produced under the lead foil was approximately twice that where there was no lead, the actual densities

being 0·771 and 0·384. The variation of density with time (constant source), with lead intensifying-screens was found to be a linear one provided the densities did not exceed 0·8.

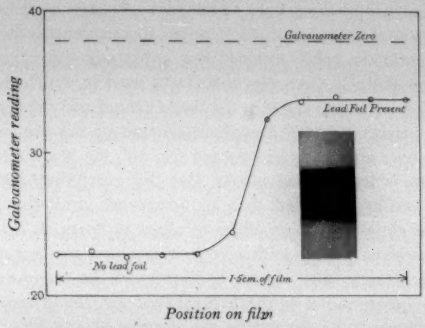

Fig. 1. Effect of lead foil in intensifying the photographic action of γ-rays. Half of the duplitized film was covered on both sides with lead foil of surface-density 0·10 gm./cm.^{2}

§ 3. EXPERIMENTAL PROCEDURE

A sketch of the apparatus is shown in figure 2. The film is held with its plane vertical in its holder. The rays pass from the source *S* on to the absorbers *A* and, after passing through a narrow beam is selected by means of the lead blocks *B*, 15·3 cm. long, 2·3 cm. wide and 7·5 cm. high so that the rays fall almost normally on the film. An important point to note in the method is that a narrow beam of γ-rays is selected by limiting these *after* they have passed through the absorbers, instead of by the method customarily used in absorption experiments in which the

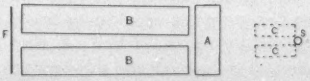

Fig. 2. Sketch of apparatus showing positions of film *F*, lead blocks *B* limiting γ-rays falling on film, absorbing screens *A*, source *S*, and channel *C*, *C* (when used).

beam is limited by a lead channel *before* it reaches the absorbers. The beam selected had a different angular opening when viewed from above from that which it had when viewed from the side. The angular opening from above was 1·2°, but when the beam was viewed from the side, γ-rays at greater angle could affect the film. The distance from the source to film was at least 21·6 cm., but was usually 25 cm. or more. The height of the source when a single tube was used was 4·8 cm. and the height of the film was 0·8 cm. When the distance between these was least, i.e. 21·6 cm., the ray at greatest angle to the horizontal which could pass from source to film was less than 8°, and as cos 8° = 0·990, the departure of the beam from being truly parallel could be neglected. A more likely source of error in the experimental arrangement is that rays could be scattered at greater angles than this

by the absorbers and affect the film. The amount of scattering so produced was small on account of the lateral limitation of the beam and it did not introduce any serious error, as it was found that the values of the absorption coefficients did not depend on the distance of the source to the film. A more conclusive argument is that a determination of the value of μ for copper was made with the solid lead block described below, and the value obtained was 0·391 cm^{-1}. (For this determination, a single glass tube of $RaCl_2$ (100 mg. of Ra) was used in which the volume of the salt was small. The centres of the salt, the lead channel and the film were carefully aligned and the blackening of the exposed portion of the film was found to be uniform.) The γ-rays after they had passed through the absorbers were scattered at the walls of the selecting lead blocks, but this scattering does not affect the accuracy of the results, provided that all beams are similarly treated and that the density on the exposed portion of the film was uniform, as it was found to be.

In measurements of γ-rays with electroscopes and ionization chambers, the natural leak of the vessels must always be found. In the photographic method as used here, what corresponded to the natural leak was being measured while the film was being exposed. To determine the density of any film, the photometer readings through the exposed and unexposed portions were taken. These two readings were obtained on a piece of film 8 mm. wide and 0·24 mm. thick, and the distance between them was 2 cm. or less. The method makes the assumption that the γ-rays scattered by the walls of the room, etc., were of uniform intensity over this portion of film. The natural leak is due to these extraneously scattered rays and in the method the effect of these was being continually recorded on the unexposed portion of the film. In this lies an important advantage of the photographic method, for no ionization vessel could be constructed of as small a volume as that of the film used to determine the density, nor does it seem possible to construct an ionization vessel which will record the main ionization current and the natural leak simultaneously. The experiments were carried out in the strong room of the Commonwealth Radium Laboratory in which there was 1 gm. of radium solution. This was, however, screened by 15 cm. of lead. A glass tube from this solution passed unscreened up one wall of the room and through it to the purifying-apparatus in another room. Several times per week radon was withdrawn from the solution so that there must have been considerable stray γ-radiation in the room. Varying amounts of this was received at different times on the films, as the varying blackening on the unexposed portions of different films showed, but as the same amount was received on the exposed and unexposed portions of any one film, the density due to the γ-rays from the source could still be determined.

The sources used were in glass tubes, each containing approximately 100 mg. of radium, which was in some in the form of the carbonate and in others in the form of the chloride. When no lead filtering was used, a single tube was mounted vertically in a V-shaped slot in the front of a soft wood block. For most of the determinations four tubes contained in brass holders were employed. The holders, 0·8 cm. in diameter, were mounted vertically in holes in a wooden block so that their axes were in line with the central line of the lead channel.

§ 4. MATERIALS USED

When the elements could be obtained in sufficient quantity, a series of rectangular blocks were made, each being carefully machined so as to be of uniform thickness. Their height and width were chosen so that they just covered the ends of the blocks of the lead channel, and hence the addition of a block cut down proportionally the radiation falling on both the exposed and the unexposed portions of the film. For those elements of which only small amounts were obtainable, a tapering rectangular channel was made in a solid lead block 15 cm. long, 7·5 cm. high and 5·5 cm. wide. The dimensions of this channel were 2 cm. × 0·7 cm. at the source end and 2 cm. × 1 cm. at the film end. A single piece of the absorber of known thickness was placed in front of the channel and the absorption of the material was measured relative to lead, a thickness of lead being selected to produce practically the same absorption. The accuracy obtainable with this arrangement, as judged by the reproducibility of the results, was not so great as when a series of blocks could be provided, for with the latter the value of μ was obtained from a straight line drawn through the points from a number of readings. Further, when the channel in the solid block was used, marked variations in the density of the exposed portion of the film were observed, whereas in the other arrangement the density was uniform. This lack of uniformity in the density was due to incorrect alignment of the radium salt within the brass holders with the axis of the lead channel and it makes the results less reliable. The liquids, mercury and bromine, were contained in rectangular glass vessels. Absorption by the glass walls of these vessels was allowed for by obtaining some exposures when the vessels were empty and others when they contained the liquids. A similar glass vessel was used for the uranium, which was in the form of a metal powder rammed tightly into the vessel.

The elements were obtained as pure as possible. The most impure was molybdenum (Kahlbaum) which appeared to be a metal powder compressed with a binding agent. The sample contained a certain amount of air, and when an attempt was made to determine its density by weighing in water, air escaped. The density was found, from a rectangular block, to be 5·20 gm./cm^3, while the true density of molybdenum is 9·0 gm./cm^3. The density of the uranium powder was found by measuring the volume of the powder rammed into a rectangular shape in its glass vessel. Most of the other densities were found by weighing the substances in air and in water, and from the densities the elements appear to be very pure.

The carbon was graphite sheet, which from an analysis by Mr G. Ampt of the Chemistry Department, was 99·75 per cent carbon. The aluminium (British Aluminium Company) was 99·47 per cent pure. The sulphur was cast into rectangular blocks from Merck's "purissimum" crystals, special care being taken that there should be no holes in the blocks. The chromium was prepared by Mr Z. A. Merfield in an arc; the iron was Armco iron; the nickel was 99·53 per cent pure. The copper was cut from a piece of commercial bus bar; a very pure sample of electrolytic copper was discarded on account of tiny holes. The zinc, 99·98 per cent pure, and the cadmium, 99·98 per cent pure, were products of the Electrolytic

Zinc Company of Australia. The bromine and tin were from British Drug Houses, Ltd. The silver, platinum, mercury and bismuth were commercial products. The antimony was "star" antimony, the tungsten was a portion of the anticathode of a Coolidge X-ray tube, the gold was a pure sample which was pressed into uniform thickness. The lead, 99·9915 per cent pure, was from the Broken Hill Associated Smelters, and the uranium powder was from Schering-Kahlbaum*.

§ 5. EXPERIMENTAL RESULTS

A re-examination of the variation of the photographic density with time and with intensity for the γ-rays from radium (B + C) unfiltered except by the glass wall of the containing tube and 3·8 mm. of aluminium film-holder. In the previous paper it was found that with the exception of the γ-rays which were unfiltered, the variations

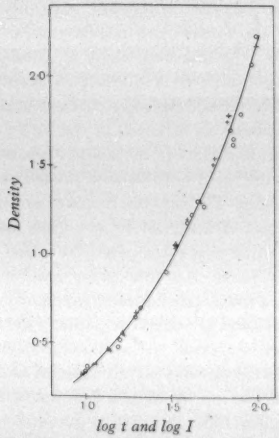

Fig. 3. Filtering of γ-rays by 3·8 mm. of Al + thickness of glass holding radium salt.
○ Points obtained with source of constant intensity, time of exposure being varied.
\+ " " " " " " constant.

of density both with log (time), for a source of constant intensity, and also with log (intensity), for a constant time of exposure, were represented by exactly similar curves. The curves for no-lead-filtering did not coincide, but the readings for these curves were more scattered than for the other curves. As the pieces of film from which the curves were obtained were not selected from the same sheet, it was decided that the readings should be repeated. A larger number of readings than

* The thanks of the writer are due to Mr Ampt for the magnesium, to Mr Merfield for the chromium, to the Defence Department of the Commonwealth of Australia for the nickel, to Prof. Greenwood for the molybdenum and gold, to Prof. Hartung for the bromine and platinum and to the Electrolytic Zinc Company for the zinc and cadmium.

previously were taken for this series, and as far as possible two pieces of film were exposed for each point. Further, the two sets of films, one for the Hurter-Driffield curve (constant intensity) and the other for the characteristic C curve (constant time of exposure) were developed in the same sample of developer, the development of the second set following as quickly as possible after the first. The C curve was obtained from readings spread over a fortnight with a source of originally 100 mc. of radon. The result is shown in figure 3, in which it will be seen that the one curve represents the two sets of readings. The non-agreement of the curves previously published was due, apparently, to experimental errors, and it is to be noted that the earlier curves differed most when the densities exceeded 1·0, in which region the accuracy with the photometer used was least.

Method of obtaining absorption coefficients. In order to illustrate the method and to show the reduction of the results, the readings for a determination of the absorption coefficient of sulphur are given in table 1.

Table 1. Typical set of results.

Element, sulphur. Filtering, 1·6 cm. of lead.

Thickness d of absorber (*cm.*)	Time of exposure (*min.*)	Density D	Time t for $D = 0{\cdot}6$ (*min.*)	
6·89	136	0·618	132	129·5
	112	0·529	127	
4·95	105	0·562	113	110·5
	111	0·617	108	
2·61	80	0·547	88	84·5
	71	0·617	81	
1·27	66	0·508	78	76·0
	62	0·501	74	
0	53	0·491	65	65·5
	73	0·669	66	

($\mu = 0{\cdot}097_6$ cm.$^{-1}$ from the straight line obtained when d is plotted against log t.)

The straight line which fits these results is plotted in figure 4, in which the observations for aluminium, cadmium and antimony also are given. It will be seen that the experimental results lie very well on straight lines, so that the value of μ can be obtained from the slope of the straight line drawn through the points. When the readings did not lie so close to the straight line, the method of least squares was employed to calculate μ.

The values of the absorption coefficients of the γ-rays filtered through 1·6 cm. of lead. In the paper already referred to it was shown that the γ-rays from Ra (B + C) were exponentially absorbed in lead when the thickness was not much greater than 1 cm. Throughout these determinations a filter of 1·42 cm. of lead was used, but as the wall of the film-holder was of aluminium 0·38 cm. thick, and as the γ-rays had to pass through the wall of the glass container, the total filtering was equivalent to 1·6 cm. of lead. When the elements could be obtained in block form, sufficient

thicknesses were used to show that d plotted against log t gave a straight line. The results of the determinations are set out in table 2, in the third column of which are the values of μ in cm.$^{-1}$ At least three values were obtained for each element,

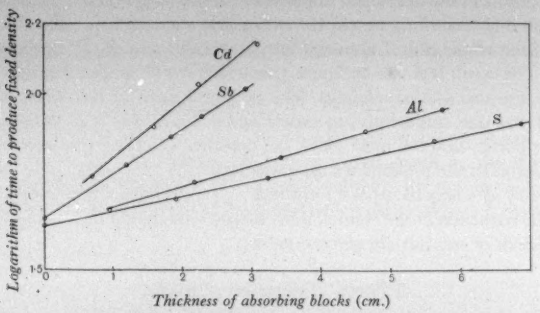

Fig. 4. Absorption curves. Filtering, 1·6 cm. of lead.

Table 2. The absorption coefficients of γ-rays filtered by 1·6 cm. of lead.

Element	Z	μ (cm.$^{-1}$)		ρ (gm./cm^3)	$\mu/\rho \times 10^2$	$\mu_e \times 10^{25}$	
		Rogers	Kohlrausch			Obs.	Calc.
C	6	0·080	0·087	1·67	4·79	1·58	1·56
Mg	12	0·081	0·083	1·75	4·63	1·55	1·56
Al	13	0·124	0·126	2·70	4·59	1·57	1·56
S	16	0·094	0·091	1·94	4·84	1·60	1·56
Cr	24	0·293	—	6·96	4·21	1·53	1·57
Fe	26	0·345	0·356	7·87	4·38	1·55	1·57
Ni	27	0·400	0·408	8·99	4·45	1·54	1·57
Cu	29	0·381	0·395	8·91	4·27	1·55	1·57
Zn	30	0·315	0·322	7·14	4·40	1·58	1·58
Br	35	0·126	0·118	3·10	4·42	1·54	1·59
Mo	42	0·230	0·373	5·20	4·42	1·67	1·61
Ag	47	0·462	0·451	10·33	4·46	1·68	1·63
Cd	48	0·372	0·350	8·65	4·46	1·65	1·64
Sn	50	0·293	0·299	7·26	4·04	1·58	1·65
Sb	51	0·291	0·272	6·73	4·32	1·68	1·65
W	74	0·815	0·850	18·81	4·33	1·78	1·84
Pt	78	0·95	—	21·5	4·41	1·83	1·90
Au	79	0·88	0·901	19·26	4·57	1·88	1·91
Hg	80	0·628	0·621	13·6	4·64	1·92	1·92
Pb	82	0·532	0·533	11·37	4·68	1·95	1·95
Bi	83	0·481	0·383	9·81	4·99	2·02	1·96
U	92	0·173	—	3·50	4·94	2·12	2·11

and for those tested in the form of blocks the accuracy, as judged from the mean departure from the mean, was 2 per cent. The accuracy for the other elements was less, about 5 per cent. In the fourth column are given, for comparison, the results of Kohlrausch, taken from page 105 of his book *Probleme der γ-Strahlung*, but the filtering used by him was 4 cm. of lead. It will be seen that except for bismuth,

the agreement between the two sets of results is satisfactory, as Kohlrausch estimated his experimental error to be 5 per cent*. In the fifth column are given the experimentally determined densities ρ, and in the sixth the values of μ/ρ. The mean value of this for the elements investigated is $0{\cdot}045 \pm 0{\cdot}002$, both the light and the heavy elements producing greater values than those in the range $Z = 24$ to 51. In the seventh and eighth columns are given the values of μ_e, the absorption per electron, which is equal to $\mu A/\rho LZ$ where A is the atomic weight, Z the atomic number and L Avogadro's number. In the seventh column are the experimentally determined values of μ_e while the values in the eighth column will be discussed later. The experimental values of μ_e are plotted against Z in figure 5.

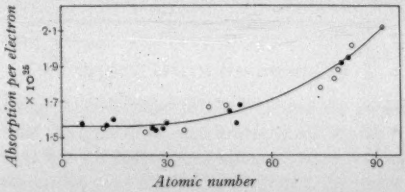

Fig. 5. The variation of the absorption per electron with atomic number. The points shown with filled-in circles are obtained from readings with a series of blocks of the elements. The full curve is for the equation $\mu_e = 1{\cdot}56 \times 10^{-25} + 7{\cdot}1 \times 10^{-32} Z^3$.

Lead filtering necessary to produce γ-rays from Ra $(B + C)$ which are exponentially absorbed. From table 2 it will be seen that the numerical values of the absorption coefficients agree with those of Kohlrausch, but whereas he found it necessary to filter the rays with 3·5 cm. of lead before they were exponentially absorbed, in these experiments much less lead filtering was required. It would appear that the differences arise from the difference between the experimental arrangements. In the method of Kohlrausch† the source was placed near the centre of an iron sphere 30 cm. in diameter, which was filled with mercury, and a narrow beam of γ-rays passed out through an iron channel before they encountered the absorbers. In the experiments here described, the γ-rays, after being filtered, pass through the absorbing sheets, and then a narrow beam selected by the lead channel can affect the photographic film. It was decided that the conditions of Kohlrausch should be reproduced at least approximately; i.e. the beam was limited before it fell on the absorbers. With the apparatus remaining as before, the source was placed within a lead block and the γ-rays passed out through a narrow channel 6 mm. wide, 3·5 cm. high and 2·5 cm. long. The position of the sides of this channel is shown in figure 2 by C, C. After emergence from this limiting channel the rays fell on the absorbers. The source was in the glass tube alone and the absorption in lead was

* The experimental value of $0{\cdot}230$ cm.$^{-1}$ for molybdenum corresponds to a value of $0{\cdot}400$ cm.$^{-1}$ for the pure metal.

† K. W. F. Kohlrausch, *Wien. Ber.* **126**, 441, 683 and 887 (1917).

measured. The result is shown in figure 6, in which it will be seen that the γ-rays are not exponentially absorbed (with μ equal to 0·523) until they have passed through more than 3 cm. of lead. On the same figure is shown for comparison the absorption of lead when no limiting channel is used. It would thus appear that this limiting channel had a marked effect on the radiation which passed out from it, as a softer radiation was produced. The apparent explanation of this fact is that some of the γ-rays strike the walls of the channel and are scattered into the beam with diminished wave-length (Compton scattering). Thus the relative proportion of the softer radiation present in the beam is increased and a greater thickness of lead is required to filter the beam before it becomes absorbed exponentially. To compare these results further with those of Kohlrausch, the curve shown was

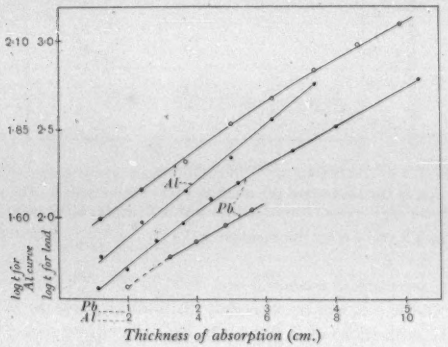

Fig. 6. Graphs of log t/thickness for Al and Pb. The filled-in circles are for readings obtained with lead channel between source and absorbers; the open circles for readings with no lead channel.

analyzed and the absorption coefficient of the softer radiation was calculated. The value found was 1·46 cm.$^{-1}$, while Kohlrausch obtained 1·49; the ratio of the harder to the softer radiation was 4 : 3, again agreeing with his value. The source was then taken out of the lead block and two copper blocks 7·5 cm. × 4·1 cm. were placed in the positions C, C of figure 2. The value of the absorption coefficient of the softer component was then 1·40 cm.$^{-1}$ and the ratio of the harder to the softer radiation was found to be 3 : 2. In the absence of the channel limiting the rays before they encountered the absorbing sheets, there was no evidence of this softer radiation, but a softer radiation with μ equal to 3·0 cm.$^{-1}$ was found. That found by Kohlrausch was 4·6 cm.$^{-1}$

The influence of using the limiting lead channel was also investigated by absorbing the γ-rays in aluminium. The source was placed at the same distance from the film (a) within the lead block and (b) when no such limiting device was used. The results of these two experiments are both shown in figure 6. In the first arrangement the absorption between 1 and 7 cm. was nearly exponential,

with μ equal to 0·178 cm.$^{-1}$, while Kohlrausch found with no lead filter that for the first 6 cm. the value of μ for aluminium was 0·176 cm.$^{-1}$ In the second arrangement, the slope up to 6 cm. was less, and then the absorption was exponential with μ equal to 0·126 cm.$^{-1}$ The absorption coefficient of the softer radiation here present was 0·6 cm.$^{-1}$ as compared with Kohlrausch's 0·57, but no evidence was found of the component with μ equal to 0·229, reported by Kohlrausch. The curves show further that the presence of the lead limiting channel increased the intensity of the γ-rays which fell on the film, and indicated therefore that additional rays were scattered into the beam by it. Thus, when a thickness of 6 mm. of aluminium absorbing sheets was used, in arrangement (*a*) the time to produce a density of 0·6 was 34 min. while with (*b*) the time was 40 min.

§ 6. DISCUSSION OF RESULTS

The relation between the absorption per electron and the atomic number. When a number of different thicknesses of an element were available, the value of μ, from which that of μ_e was calculated, was obtained from a straight line μ which showed that the absorption was exponential. Hence the value of μ had more significance than when a single thickness of absorber was used. The general agreement of the values with those obtained by Kohlrausch has already been mentioned. Although he did not obtain his values of μ by a series of absorbing layers, except for lead, aluminium and tin, he did what was substantially the same thing: he measured the absorption of a single thickness of the absorbing material when the γ-rays were filtered by different thicknesses of lead. He found, from the absorption measurements in lead, that the γ-rays could be analyzed into two components and the relative intensity of the two could be determined. The values of the absorption coefficients of the other elements could then be found. It is the values of μ for the harder component of the γ-rays used by Kohlrausch that agree with those found here. But, as was previously mentioned, the writer has found no evidence of the softer component having μ equal to 1·49 cm.$^{-1}$ in lead in the γ-rays which arise from the source. It was, however, the presence of this softer component in the radiation used by Kohlrausch, that made it necessary to use 3·5 cm. of lead filter before the rays were exponentially absorbed.

When blocks were used, the reading with no absorber present, i.e. for the lead filter alone, was usually omitted, the first reading for the curve being obtained when the lead filter plus a thin thickness of the absorber was present. This arrangement was adopted after it had been found that the reading for the filter alone fell slightly below the straight line representing the points for different thicknesses of the absorber. It was just on this account that the absorption of an element, of which a single thickness only was obtainable, was compared with that of a single thickness of lead which produced nearly the same absorption. In this discussion more weight will be given to the values obtained from straight lines than to the others.

The theory of Klein and Nishina* has been found recently by a number of authors† to account successfully, at least in part, for the absorption of γ-rays. According to this theory, if the absorption is due to scattering alone, the absorption per electron should be independent of the atomic number and be given by

$$\mu_e = \frac{2\pi e^4}{m^2c^4}\left[\frac{1+\alpha}{\alpha^2}\left\{\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\log(1+2\alpha)\right\} + \frac{1}{2\alpha}\log(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2}\right],$$

where $\alpha = h\nu/mc^2$. It will be seen, figure 5, that for the lighter elements up to $Z = 35$ μ_e is approximately constant and equal to $1{\cdot}56 \times 10^{-25}$, so that the formula appears valid for the γ-rays from Ra (B + C) filtered by 1·6 cm. of lead for these elements. For the elements of greater atomic number, however, μ_e is greater and increases with the atomic number. A similar increase of μ_e with Z was also found by Ahmad‡, although his values were all higher than those found here. Meitner and Hupfeld§, who measured the absorption in carbon, aluminium and lead of the γ-rays from Ra (B + C) filtered by 4 cm. of lead, found that $\mu_e = 1{\cdot}53 \times 10^{-25}$ for carbon and aluminium and $1{\cdot}996 \times 10^{-25}$ for lead, for which the value here is $1{\cdot}95 \times 10^{-25}$. Tarrant‖, using 3·2 cm. of lead filter, obtained higher values, viz. $1{\cdot}727$ and $1{\cdot}691 \times 10^{-25}$ for magnesium and aluminium, and $2{\cdot}279$ and $2{\cdot}250 \times 10^{-25}$ for lead and bismuth. In all these determinations the same general result appears, viz. that, while the absorption per electron for the lighter elements is constant, it increases with the atomic number for the heavier ones. The conclusion is therefore, that, while scattering of the type considered by Klein and Nishina accounts for the absorption of the rays from Ra (B + C) in the lighter elements, there must be an additional process for those of greater atomic number. The results here obtained can be accounted for by considering that this additional process is the photoelectric one. The difference between $1{\cdot}56 \times 10^{-25}$ and the values of μ_e for the heavier elements was found to vary as Z^3, and the values of μ_e for all elements can be expressed by the formula:

$$\mu_e = 1{\cdot}56 \times 10^{-25} + 7{\cdot}1 \times 10^{-32} Z^3\text{¶}.$$

The curve drawn in figure 5 represents this relation, and the eighth column in table 2 is calculated from it. Among elements yielding the more accurate readings, tin shows the greatest departure from the curve, the other elements falling on it within the experimental error. In the X-ray region the atomic absorption coefficient, due to the photoelectric effect, varies approximately as Z^4, and therefore the absorption per electron varies as Z^3. Hence, as the additional absorption for the heavier elements varies as Z^3, it can be assumed that this extra absorption is due to the same process that produces the Z^3 law in the X-ray region—i.e. to the photoelectric effect.

* O. Klein and Nishina, *Z. f. Phys.* **52**, 853 (1928).

† E.g. E. C. Stoner, *Phil. Mag.* **7**, 841 (1929); D. Skobelzyn, *Z. f. Phys.* **65**, 773 (1930); C. Y. Chao, *Phys. Rev.* **36**, 1519 (1930); G. T. P. Tarrant, *Proc. R.S.* **128**, 345 (1930); Lise Meitner and H. H. Hupfeld, *Z. f. Phys.* **67**, 147 (1931).

‡ N. Ahmad, *Proc. R.S.* **104**, 507 (1924). § *Loc. cit.* ‖ *Loc. cit.*

¶ Cf. the equation $\mu_e = 1{\cdot}60 \times 10^{-25} + 5{\cdot}5 \times 10^{-32} Z^3$ given by Kohlrausch on p. 115 of *Probleme der γ-Strahlung*.

If the value of μ_e is taken as $1{\cdot}56 \times 10^{-25}$, the effective wave-length of the γ-rays from Ra (B + C) filtered by 1·6 cm. of lead can be calculated from the Klein and Nishina formula, and the value found is 7·0 X.U. This wave-length agrees with the conclusion of Gray and Cave* that the γ-rays from Ra (B + C) filtered by 1·61 cm. of lead must have an effective wave-length which is less than 8 X.U. and must be nearer 7 than 8 X.U. Meitner and Hupfeld, from their results in carbon and aluminium, calculate that the effective wave-length of the rays, filtered by 4 cm. of lead, is 6·7 X.U.

Meitner and Hupfeld found that the absorption per electron for γ-rays of wave-length 4·7 X.U.† was not constant even for the lighter elements but increased continually with Z. To account for their results, they found it necessary to take into consideration a nuclear scattering term. For the γ-rays here employed, of wave-length 7·0 X.U., this process does not appear to be significant. On the other hand, Meitner and Hupfeld consider that when the effective wave-length is 6·7 X.U.‡ the photoelectric term is not more than 10 per cent of the total absorption.

The lead filtering necessary for the γ-rays from Ra (B + C) to be exponentially absorbed. The absorption per electron found for lead is $1{\cdot}95 \times 10^{-25}$. As mentioned above, this can be split up into two terms, one due to scattering in terms of the Klein and Nishina formula and the other due to photoelectric absorption. For the first the value found is $1{\cdot}56 \times 10^{-25}$, the corresponding wave-length is 7·0 X.U. and the photoelectric term is $0{\cdot}39 \times 10^{-25}$. To find the absorption for other wave-lengths in lead the variation of both these terms with wave-length has to be considered. The Klein and Nishina formula itself gives the necessary variation for the first term, but an assumption must be made as to the variation of the photoelectric term with wave-length. In the X-ray region the λ^3 law holds, at least approximately, but in the γ-ray region no definite exponent of λ has yet been established. Ellis and Aston§ consider that the exponent is less than 3. Gray and Cave‖ consider that the exponent is of the order 2·05. If a value of 2·2 is assumed, the photoelectric term for the absorption in lead corresponding to any wave-length λ expressed in X.U. will be $0{\cdot}39\,(\lambda/7{\cdot}0)^{2{\cdot}2}$. The validity of this assumption can be tested by calculating the values of μ for the shortest X-rays and comparing it with the values experimentally found. Allen¶ found for λ 81 X.U. in lead that $\mu = 28{\cdot}8$ cm.$^{-1}$ and for λ 126 X.U., $\mu = 59{\cdot}3$ cm.$^{-1}$ For these wave-lengths the Klein and Nishina term is only 5 per cent and 2 per cent respectively of the photoelectric terms, and the values of μ for the sum of the two terms are 24·2 cm.$^{-1}$ and 62·8 cm.$^{-1}$ respectively. While the agreement is not as close as could be wished, it is satisfactory since the calculated value depends mainly on the photoelectric term, which is obtained from the difference between two experimentally determined quantities,

* J. A. Gray and H. M. Cave, *Proc. R.S. Canada*, **21**, 163 (1927).
† Source Th (B + C + C″) filtered by 4 cm. of lead.
‡ The filtered rays from Ra (B + C).
§ C. D. Ellis and G. H. Aston, *Proc. R.S.* **129**, 180 (1930).
‖ J. A. Gray and H. M. Cave, *loc. cit.*
¶ S. J. Allen, *Phys. Rev.* **27**, 266 (1926).

viz. the value of μ_e for lead and that for the lighter elements. On the other hand, if the assumption is correct and there is no other method of absorption other than the two mentioned, it should account for the absorption in lead found by Meitner and Hupfeld* for wave-length 4·7 X.U. For this wave-length the scattering term is $1{\cdot}235 \times 10^{-25}$, and if the above variation with wave-length be used, the photo-electric term is $0{\cdot}162 \times 10^{-25}$. The total absorption per electron would thus be $1{\cdot}40 \times 10^{-25}$, whereas the experimentally found value was $1{\cdot}74 \times 10^{-25}$. The validity of the formula, with regard to the variation with wave-length, therefore, rests on the approximate agreement obtained with the absorption for the short-wave-length X-rays.

If this law of absorption is valid, the variations in intensity of the different wave-lengths of the γ-rays from Ra (B + C), when they pass through 1·6 cm. of lead, can be calculated. The formula can be expressed in the form

$$\mu_e = \sigma_{KN} + 0{\cdot}39\,(\lambda/7{\cdot}0)^{2\cdot2},$$

σ_{KN}

where σ_{KN} represents the Klein and Nishina term. From this the value of μ for 24·2 X.U. is 2·6 $cm.^{-1}$ and therefore γ-rays of this wave-length would be reduced to 1·5 per cent of their intensity in passing through 1·6 cm. of lead. γ-rays of longer wave-length would be reduced to negligible intensity in passing through this thickness. Thus the intense γ-rays in the Ra (B) spectrum of wave-lengths 50·8, 41·6 and 34·9 X.U.† would all be absorbed. The most intense lines in the Ra (C) spectrum according to Ellis and Aston are set out in table 3. The table also gives the intensities of the lines, the values of μ_e calculated with the above formula, the corresponding values of μ, and the calculated intensities after the γ-rays have passed through 1·5 cm. of lead.

Table 3. Intensity of the γ-rays from Ra (C) filtered by 1·6 cm. of lead.

Wave-length (X.U.)	Intensity (Ellis and Aston)	$\mu_e \times 10^{25}$	μ ($cm.^{-1}$)	Emergent intensity
20·2	0·658	6·66	1·82	0·0360
16·0	0·065	4·76	1·30	0·0087
13·1	0·067	3·67	1·00	0·0146
10·9	0·206	2·99	0·817	0·0558
9·89	0·063	2·71	0·742	0·0193
8·89	0·064	2·43	0·665	0·0221
6·94	0·258	1·94	0·530	0·1183
5·57	0·074	1·64	0·448	0·0352

It will be seen from this table that, whereas initially the γ-rays of wave-length 20·2 X.U. are nearly three times as intense as the next strongest line of 6·94 X.U., after passing through 1·6 cm. of lead the strongest line is 6·94 X.U., the next strongest is 10·9 X.U. which has 47 per cent of its intensity, while the γ-ray 20·2 X.U. is only 30 per cent of the intensity of the line 6·94 X.U., the line 5·37 being also of this

* Lise Meitner and H. H. Hupfeld, *loc. cit.*

† C. D. Ellis and G. H. Aston, *loc. cit.*

intensity*. It has now to be decided whether or not the intensities of the γ-rays, as shown in the last column of table 3, are consistent with the effective wave-length of 7·0 X.U.; i.e. whether the average absorption coefficient of the γ-rays, calculated from the sums of the incident and emergent intensities for the individual rays, is consistent with that actually measured. Unfortunately, as the variation of photographic action with wave-length is not known, the calculation cannot be done with accuracy. Further, γ-rays of shorter wave-length than those given in the table have been reported: e.g. Steadman† measured eight γ-rays of considerable intensity and of wave-length shorter than those given in table 3. It is not possible, however, to find an accurate effective wave-length of γ-rays merely by considering the initial intensity of the lines and the absorption of the individual lines in passing through different thicknesses of absorber. If it were possible, the absorption coefficient in lead would steadily decrease as greater thicknesses of lead were employed, for with additional thicknesses the longer wave-lengths would be gradually reduced to negligible intensity, the effective wave-length would decrease and so, in consequence, would the absorption. In contrast to this, it has been previously found‡ that the value of the absorption coefficient is constant for thicknesses of lead between 1·4 and 8·0 cm. There must, in consequence, be a softening process of the harder rays, as they pass through the absorber, so that exponential absorption represents an equilibrium between the reduction in intensity of the longer-wave-length γ-rays and an increasing degradation of the wave-length of the shorter ones.

The conclusion that filtering by 1·4 cm. of lead, with the apparatus used, is sufficient to produce exponential absorption has been supported by measuring the absorption of the γ-rays unfiltered by lead in carbon, aluminium, copper and tin. It was found that the absorption was exponential after a thickness for carbon of 7·5 cm., for aluminium 6 cm., for copper 2·0 cm. and for tin 2·5 cm. When the densities of these substances are considered together with the density of lead, it is found that they are consistent among themselves, since $d \times \rho$ is nearly constant, where d is the thickness required to produce exponential absorption and ρ is the density. d, ρ

Brommer§ examined the absorption in mercury of the γ-rays from Ra (B + C) by placing the source along the axes and near the centres of brass or glass cylinders containing mercury. The γ-rays were measured in an ionization vessel faced with 3 mm. of lead. He found that the absorption was exponential throughout, but as the smallest thickness of mercury was 3·6 mm., the filtering used amounted to the equivalent of 7·4 mm. of lead, i.e. a smaller thickness than was found necessary

* It should be noted that the values have been calculated by assuming that the lead filter was 1·6 cm. thick, whereas the actual filter-thickness was 1·42 cm., the other 0·18 cm. being an allowance for the thickness of the aluminium film-holder and the thickness of the wall of the glass tube containing the radium salt. The laws governing the absorption of the last two substances are not exactly the same as for lead, but the error introduced by the assumption made does not alter the general trend of the argument, nor is the error introduced very great.

† Luville T. Steadman, *Phys. Rev.* **36**, 460 (1930).

‡ J. S. Rogers, *loc. cit.*

§ A. Brommer, *Wien. Ber.* **121**, 1563 (1912); *Phys. Z.* **13**, 1037 (1912).

in these experiments. The value of μ for mercury found by him was 0·641 cm.$^{-1}$ as compared with the value found here of 0·628 cm.$^{-1}$

Effect of using a limiting channel before the absorbers. The effect of introducing a limiting channel between the source and the absorbers has been described above. As a result of this it would appear that the radiation measured by Kohlrausch, which had μ equal to 1·49 cm.$^{-1}$, does not arise from the source directly but is formed by the γ-rays which encounter the walls of the limiting channel being re-scattered into the main beam. If the variation with wave-length of μ_s given above is valid, an absorption coefficient of 1·49 cm.$^{-1}$ in lead corresponds to a wave-length of 17 X.U. It is therefore necessary to account for a radiation of effective wave-length 7 X.U. being changed in part to one of effective wave-length 17 X.U.

θ To do this the Compton formula for a γ-ray scattered at angle θ gives the change $\delta\lambda$ of wave-length thus:

$$\delta\lambda = 24{\cdot}2\,(1 - \cos\theta).$$

The intensity of the radiation scattered at angle θ is given by Klein and Nishina.

F According to them, the fraction F of the incident energy scattered per unit solid angle per electron in a direction θ is given by

$$F = \frac{e^4}{2m^2c^4}\,\frac{1+\cos^2\theta}{\{1+\alpha\,(1-\cos\theta)\}^3}\left[1+\frac{\alpha^2\,(1-\cos\theta)^2}{(1+\cos^2\theta)\,\{1+\alpha\,(1-\cos\theta)\}}\right].$$

In table 4 are set out for different angles the wave-length of the γ-rays of 7 X.U. scattered at various angles (Compton relation) and the relative intensity of the radiation at these angles (Klein and Nishina relation).

Table 4. Wave-length and intensity of γ-rays of 7 X.U. scattered at various angles.

Angle (degrees)	Wave-length (X.U.)	Relative intensity
0	7·0	1
5	7·09	0·957
30	10·23	0·298
60	19·1	0·052
90	31·2	0·018

It will be seen from the table that the scattering must be at large angles before the necessary change of wave-length can be produced, as small-angle scattering would not be adequate to account for it. The intensity of the radiation scattered at large angles is however small: at 60° it is some 5 per cent of that scattered at 5°. When a limiting channel has been used, generally the source has been pushed up very close to one end, so that the intensity of the γ-rays from the source falling on this near end of the channel is very much greater than that at the far end. Further, it is just at this near end that the γ-rays can be scattered at large angles and sent into the beam proceeding to the absorbers. Thus although the fractional intensity of the rays scattered at large angles is small, in view of the much greater intensity of the γ-rays falling so that they could be scattered at large angles, it

would appear that, qualitatively, account can be given of the softening effect produced by the scattering of γ-rays by the channel walls.

Various experimental determinations of absorption coefficients will now be examined to see to what extent they support this argument. In the first place the same softening was observed above whether a lead channel 2·5 cm. long was used or a copper one 4·1 cm. long, and approximately the same softening was observed by Kohlrausch with a channel 12 cm. long; but although the lengths of the channels were so different, in all three the conditions would be approximately the same at the source end of the channel, where the greatest change in wave-length would be expected. Next, Meitner and Hupfeld used a channel 36 cm. long and 1·2 × 1·0 cm. in cross-section, but their source was placed back from the near end of the channel so that there was no opportunity for the very intense radiation to produce large-angle scattering into the main beam. They found that the same values of the absorption coefficients were obtained whether the absorbers were placed at the near end or at the far end of the channel; this showed that the experimental conditions were such that there was no softening of the radiation in its passage down the channel. Further they found that, for Th (B + C + C″), 1 cm. of lead filtering produced the same absorption coefficients as 4 cm., while Chao* with a filter of 1·36 cm. of lead found that $\mu = 0{\cdot}565$ cm.$^{-1}$, and with 6·8 cm. that $\mu = 0{\cdot}477$. In Chao's apparatus, however, the conditions were such that large-angle scattering could occur close to the source, and hence the apparent discrepancies between the two sets of results are accounted for.

Ahmad†, in order to use small sources, employed a comparatively wide-angle beam and made allowance for scattering, except the scattering produced at the walls of the channel. The amount of scattered radiation that would be sent into the main beam with such a wide channel would be considerable, and hence the values of μ due to the greater proportion of softer radiation were greater than those found by Kohlrausch or by the author. Thus with filtering of 3·41 cm. of mercury, he found μ_e for lead to be $2{\cdot}540 \times 10^{-25}$. Owen, Fleming and Fage‡ placed their source in a small cylindrical hole so that it was possible for high-angle scattering radiation from the walls of the hole to enter the main beam. The amount of this radiation was apparently considerable, as is shown by their high value of μ, which was 0·720 cm.$^{-1}$ for the γ-rays from Ra (B + C) filtered with 2·3 cm. of lead. Gray and Cave§ used two arrangements, in both of which there were γ-rays scattered at large angles near the source, but in one of these they used a limiting channel through which the rays passed before they encountered the absorbers. The values of μ/ρ obtained with this arrangement were always higher than those in the absence of this limiting channel, as table 5 shows.

Although the differences obtained were not great, being at most 4 per cent, they were always in such a direction as to indicate the softening of the radiation

* C. Y. Chao, *Proc. Nat. Acad. Sci.* **16**, 431 (1930).
† N. Ahmad, *Proc. R.S.* **105**, 507 (1924); **109**, 206 (1925).
‡ E. A. Owen, N. Fleming and Winifred E. Fage, *Proc. Phys. Soc.* **36**, 355 (1924).
§ J. A. Gray and H. M. Cave, *loc. cit.*

at the walls of the channel. The amount of softening to be expected in these experiments would not be great, as the channel was not placed next to the source and hence the scattering would be confined to smaller angles. Attention has already been drawn by L. H. Gray* to the softening of γ-rays by scattering at the walls of a channel.

Table 5. Values of μ/ρ obtained by Gray and Cave.

Filter thickness (cm. of lead)	μ/ρ			
	Lead		Copper	
	without channel	with channel	without channel	with channel
2·2	0·0530	0·0552	0·0470	0·0479
3·25	0·0502	0·0513	0·0455	0·0460
4·30	0·0478	0·0487	0·0448	0·0452

§ 7. CONCLUSIONS

(*a*) The absorption coefficients of 22 elements for the γ-rays from Ra (B + C) filtered by 1·6 cm. of lead have been measured by a photographic method.

(*b*) For elements up to $Z = 30$ the absorption per electron is constant, in agreement with the Klein-Nishina formula, and has the value $1{\cdot}56 \times 10^{-25}$. This value corresponds to a wave-length 7·0 X.U.

(*c*) For values of Z greater than 30, μ_e increases continually with Z so that $\mu_e = 1{\cdot}56 \times 10^{-25} + 7{\cdot}1 \times 10^{-32} Z^3$.

(*d*) On account of the Z^3 variation it is assumed that the extra absorption for the heavier elements is due to a photoelectric term.

(*e*) Both the Klein-Nishina term and the photoelectric term must be separately considered in calculating the variation of μ_e with wave-length. It has been assumed that the second of these varies as $\lambda^{2\cdot2}$.

(*f*) The absorption per electron can be expressed thus:

$$\mu_e = \sigma_{KN} + 7{\cdot}1 \times 10^{-32} \times 0{\cdot}39 \left(\frac{\lambda}{7{\cdot}0}\right)^{2\cdot2} Z^3$$

$$= \sigma_{KN} + 3{\cdot}83 \times 10^{-34} \lambda^{2\cdot2} Z^3.$$

(*g*) The limiting channel generally used in absorption experiments to limit the γ-rays before they fall on the absorbers has been shown to cause the main beam to receive a certain amount of scattered and softened γ-radiation.

§ 8. ACKNOWLEDGMENTS

These experiments were carried out in the Commonwealth Radium Laboratory, University of Melbourne. The author wishes to express his thanks to Mr A. H. Turner, M.Sc., Physicist of that laboratory for making available its facilities. He also wishes to acknowledge his indebtedness to Prof. T. H. Laby, F.R.S. for assistance in preparing the paper.

* L. H. Gray, *Proc. R.S.* **128**, 361 (1930).

DISCUSSION

Dr G. H. Aston. The author finds that a $\lambda^{2\cdot2}$ law for the photoelectric absorption of γ-rays enables him to deduce approximately correct values for the absorption in the X-ray region. It is probable, however, that if the photoelectric absorption were to be measured for a different effective γ-ray wave-length, a different power of λ would be required to give the right value in the X-ray region. It is unlikely that a simple power law can be used to represent the photoelectric absorption in the γ-ray region. An empirical formula for this absorption should agree with the X-ray measurements not only in magnitude but also in slope. L. H. Gray* has deduced such a formula, using data from absorption and intensity-measurements of γ-rays.

* *Proc. Camb. Phil. Soc.* **27**, 103 (1931).

THE DERIVATION OF MAXWELL'S EQUATIONS FROM THE EQUATIONS OF THE QUANTUM THEORY

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ABSTRACT. In a theory developed by Flint and Fisher the fundamental equations of the quantum theory are expressed in terms of a five-dimensional geometry based upon the use of two inter-related tensors A and B. In the present paper Maxwell's equations are derived from the five-dimensional equations previously proposed, and the tensors A and B are exhibited as geometrical quantities related to the electromagnetic potential.

In their attempt to develop a unitary physical theory to include the fundamental equations of the quantum theory, Flint and Fisher*, by making certain modifications of the five-dimensional geometrical scheme of Kaluza and Klein, have been led to an equation of the form

$$\text{div } A = 0 \qquad \ldots\ldots(1),$$

A where A is a contravariant tensor of the second rank.

They express the opinion that this tensor has some geometrical or metrical significance, but the actual meaning remains at present undiscovered. The equation (1) is interpreted in accordance with the metrical scheme proposed and leads directly to Dirac's equations without any artificial introduction of operators.

In a later paper† it has been shown that the occurrence of positive and negative charges and the asymmetry in the masses of the proton and electron can be accounted for in terms of geometry and metrics.

The work of Kaluza and Klein and later of de Broglie has shown that in the five-dimensional system the tracks of all particles in gravitational and electromagnetic fields are geodesics. The work of Flint and Fisher shows that the geodesics are null geodesics and that the generalized mass of all particles is zero. Thus the analogy with the track of a ray of light in the space-time of Einstein is complete and, as has been stated before, this may well form the basis of a wave-theory of matter. The occurrence of mass in physics is due, according to this theory, to our description of phenomena in space and time.

The theory points to the fundamental character of the tensor A of equation (1) and it is important to follow up any line of study which will indicate its meaning. The author has previously studied this question‡ and has shown a connection between the components of electronic magnetic and electric moments. The procedure adopted led to results analogous to those obtained by Dirac.

* *Proc. R.S.* A, **126**, 644 (1930).
† *Proc. R.S.* A, **131**, 170 (1931).
‡ *Proc. Phys. Soc.* **43**, 124 (1931).

In the original paper on this subject† two tensors A and B were introduced but there exist a number of relations between the components of the two tensors, so that actually one fundamental tensor alone is introduced. These conditions were adopted by analogy with the usual procedure in the relativistic treatment of Maxwell's equations, and also in order to show that the work was able to account for previous suggestions made by other writers. *B*

The present purpose is twofold. Firstly we show that Maxwell's equations can be derived by simple combinations of the fundamental equations proposed, and secondly by developing the relations between the electric and magnetic intensities and the tensors A and B we connect these tensors with geometrical quantities, for the electromagnetic potential is in this scheme a geometrical quantity and the intensities are derived from it.

The equations proposed‡ are

$$\frac{\partial A^{mn}}{\partial x^n} = -\frac{\partial \psi_0}{\partial x^m} + \frac{2\pi ie}{h}\phi_n A^{mn} + \frac{2\pi ie}{h}\phi_m \psi_0 - \frac{2\pi imc}{h} A^{m5} \quad \text{.....(2),} \qquad \theta, \phi, \psi$$

and

$$\frac{\partial B^{mn}}{\partial x^n} = -\frac{\partial \theta_0}{\partial x^m} + \frac{2\pi ie}{h}\phi_n B^{mn} + \frac{2\pi ie}{h}\phi_m \theta_0 - \frac{2\pi imc}{h} B^{m5} \quad \text{.....(3).}$$

The conjugate equation of (1) is

$$\frac{\partial A^{*mn}}{\partial x^n} = -\frac{\partial \psi_0^*}{\partial x^m} - \frac{2\pi ie}{h}\phi_n A^{*mn} - \frac{2\pi ie}{h}\phi_m \psi_0^* + \frac{2\pi imc}{h} A^{*m5} \quad \text{.....(2*).}$$

To obtain the set of equations corresponding to

$$\frac{1}{c}\frac{\partial H}{\partial t} + \operatorname{curl} E = 0 \quad \text{.....(4),} \qquad H, E$$

and

$$\operatorname{div} H = 0 \quad \text{.....(5),}$$

we multiply the equation (2) by A^{*mk} and the equation (2*) by A^{mk} and subtract.

We obtain

$$\left(A^{*mk}\frac{\partial A^{mn}}{\partial x^n} - A^{mk}\frac{\partial A^{*mn}}{\partial x^n}\right)$$

$$= -\left(A^{*mk}\frac{\partial \psi_0}{\partial x^m} - A^{mk}\frac{\partial \psi_0^*}{\partial x^m}\right) + \frac{2\pi ie}{h}\phi_n\left(A^{*mk}A^{mn} + A^{mk}A^{*mn}\right)$$

$$+ \frac{2\pi ie}{h}\phi_m\left(A^{*mk}\psi_0 + A^{mk}\psi_0^*\right) - \frac{2\pi imc}{h}\left(A^{*mk}A^{m5} + A^{mk}A^{*m5}\right),$$

or

$$\frac{\partial}{\partial x^n}\left(A^{*mk}A^{mn} - A^{mk}A^{*mn}\right)$$

$$= A^{mn}\frac{\partial A^{*mk}}{\partial x^n} - A^{*mn}\frac{\partial A^{mk}}{\partial x^n} - A^{*mk}\frac{\partial \psi_0}{\partial x^m} + A^{mk}\frac{\partial \psi_0^*}{\partial x^m}$$

$$+ \frac{2\pi ie}{h}\phi_n\left(A^{*mk}A^{mn} + A^{mk}A^{*mn} + A^{*nk}\psi_0 + A^{nk}\psi_0^*\right)$$

$$- \frac{2\pi imc}{h}\left(A^{*mk}A^{m5} + A^{mk}A^{*m5}\right) \quad \text{.....(6).}$$

† *Proc. R.S.* A, 126, 644 (1930).

‡ *Ibid.*, p. 650.

Putting k equal to 4 in equation (6), we obtain the following equation:

$$\frac{1}{c}\frac{\partial}{\partial t}i(A^{*m4}A^{m1}-A^{m4}A^{*m1})+\frac{\partial}{\partial x^2}(A^{*m1}A^{m2}-A^{m1}A^{*m2})-\frac{\partial}{\partial x^3}(A^{*m3}A^{m1}-A^{m3}A^{*m1})$$

$$=\left(A^{mn}\frac{\partial A^{*m1}}{\partial x^n}-A^{*mn}\frac{\partial A^{m1}}{\partial x^n}-A^{*m1}\frac{\partial \psi_0}{\partial x^m}+A^{m1}\frac{\partial \psi_0^*}{\partial x^m}\right)$$

$$+\frac{2\pi ie}{h}\phi_n(A^{*m1}A^{mn}+A^{m1}A^{*mn}+A^{*n1}\psi_0+A^{n1}\psi_0^*)$$

X_1

$$-\frac{2\pi imc}{h}(A^{*m1}A^{m5}+A^{m1}A^{*m5})=X_1\,(\text{say}) \qquad \ldots\ldots(6{\cdot}1).$$

Equation (6·1) suggests a comparison with

$$\frac{1}{c}\frac{\partial H_1}{\partial t}+\frac{\partial E_3}{\partial x^2}-\frac{\partial E_2}{\partial x^3}=0,$$

and thus we may write

$$E_3=ai(A^{*m3}A^{m1}-A^{m3}A^{*m1}),\quad E_2=a(A^{*m1}A^{m2}-A^{*m2}A^{m1}),$$

$$H_1=a(A^{*m1}A^{m4}-A^{*m4}A^{m1}),$$

where a is a constant factor.

Similarly

$$E_1=ai(A^{*m2}A^{m3}-A^{m2}A^{*m3}),\quad H_2=a(A^{*m2}A^{m4}-A^{*m4}A^{m2}),$$

$$H_3=a(A^{*m3}A^{m4}-A^{*m4}A^{m3}).$$

The quantity on the right of equation (6·1) may be regarded as representing a

σ_1 component of magnetic current and if we denote this current component by σ_1 we have

$$\sigma_1=aci\,X_1.$$

Writing $k=4$, we obtain

$$\frac{\partial}{\partial x^n}(A^{*m4}A^{mn}-A^{m4}A^{*mn})=X_4,$$

X_4 where X_4 denotes the right-hand side of (6) with k equal to 4.

If this is compared with (5), we can write

$$H_n=a(A^{*mn}A^{m4}-A^{*m4}A^{mn}).$$

The condition, $X_4=0$, is suggested but recent developments indicate that free magnetic charges are not excluded by the quantum theory. In order to take into consideration the possibility that (5) should be extended to

$$\text{div}\,H=\sigma,$$

we write

$$aX_4=-\sigma.$$

The other group of Maxwell's equations is

$$-\frac{1}{c}\frac{\partial E}{\partial t}+\text{curl}\,H=\frac{j}{c} \qquad \ldots\ldots(7),$$

and

$$\text{div}\,E=\rho \qquad \ldots\ldots(8),$$

j, ρ where j is the electric current vector and ρ the density of charge.

To obtain the corresponding equations from the quantum equations, we multiply (3) by A^{*mk} and (2*) by B^{*mk} and subtract.

We obtain

$$\frac{\partial}{\partial x^n}(A^{*mk}B^{mn} - B^{*mk}A^{mn}) = \left(B^{mn}\frac{\partial A^{*mk}}{\partial x^n} - A^{mn}\frac{\partial B^{*mk}}{\partial x^n} - A^{*mk}\frac{\partial \theta_0}{\partial x^m} + B^{*mk}\frac{\partial \psi_0}{\partial x^m}\right)$$
$$+\frac{2\pi ie}{h}\phi_n(A^{*mk}B^{mn} - B^{*mk}A^{mn} + A^{*nk}\theta_0 - B^{*nk}\psi_0)$$
$$-\frac{2\pi imc}{h}(A^{*mk}B^{m5} - B^{*mk}A^{m5}) \qquad \text{.....(9).}$$

By inserting the value $k = 1$, we obtain on the left-hand side

$$\frac{\partial}{\partial x^2}(A^{*m1}B^{m2} - B^{*m1}A^{m2}) + \frac{\partial}{\partial x^3}(A^{*m1}B^{m3} - B^{*m1}A^{m3})$$
$$+\frac{1}{c}\frac{\partial}{\partial t}\cdot\frac{A^{*m1}B^{m4} - B^{*m1}A^{m4}}{i}.$$

This should be compared with the left-hand side of the equation

$$\frac{\partial H_3}{\partial x^2} - \frac{\partial H_2}{\partial x^3} - \frac{1}{c}\frac{\partial E_1}{\partial t} = \frac{j_1}{c},$$

and if we regard the equations as identical we can write

$$E_1 = a(A^{*m1}B^{m4} - B^{*m1}A^{m4}), \quad H_2 = ai(A^{*m1}B^{m3} - B^{*m1}A^{m3}),$$
$$H_3 = ai(B^{*m1}A^{m2} - A^{*m1}B^{m2}).$$

Similarly we obtain

$$E_2 = a(A^{*m2}B^{m4} - B^{*m2}A^{m4}), \quad H_1 = ai(B^{*m2}A^{m3} - A^{*m2}B^{m3}),$$

and

$$E_3 = a(A^{*m3}B^{m4} - B^{*m3}A^{m4}).$$

If we denote the right-hand side of equation (9) by Y_k, we have Y_k

$$j_k = -aicY_k.$$

Writing $k = 4$ we obtain

$$\frac{\partial}{\partial x^n}(A^{*n4}B^{mn} - B^{*m4}A^{mn}) = Y_4,$$

where Y_4 denotes the right-hand side in this case. Y_4

We may then compare this equation with (8) and write

$$E_n = a(B^{*m4}A^{mn} - A^{*m4}B^{mn}),$$

and

$$\rho = -aY_4.$$

We have thus obtained three different forms for the intensities and it remains to show that they are equal to one another. We proceed to show that this is the case for E_1 and the rest follows by symmetry. The demonstration of the identity of these various forms for the intensities rests upon certain relations adopted in the paper already referred to†. These relations are:

$$A^{23} = iB^{41}, \quad A^{31} = iB^{42}, \quad A^{12} = iB^{43},$$
$$A^{41} = iB^{23}, \quad A^{42} = iB^{31}, \quad A^{43} = iB^{12},$$

† Flint and Fisher, *loc. cit.* p. 651.

and they were adopted by analogy with Maxwell's equations as mentioned above. The relations lead in a remarkable way from the general equation proposed in that paper to the first-order equations of the quantum theory.

We have obtained the following expressions for E_1:

$$a\,(A^{*m1}B^{m4} - B^{*m1}A^{m4}), \quad a\,(B^{*m4}A^{m1} - A^{*m4}B^{m1}),$$
$$ai\,(A^{*m2}A^{m3} - A^{m2}A^{*m3}).$$

By expansion of the first of these (m having the values 2 and 3 only since the components with both affixes the same vanish), and by replacing the A-components by B-components and *vice versa*, we see at once that the first expression is equal to the second. The first may be shown to be equal to the third by replacing the B-components by A-components.

We thus obtain the following expressions for the electric and magnetic components of field intensity:

$$E_1 = a\,(A^{*m1}B^{m4} - B^{*m1}A^{m4}) = a\,(B^{*m4}A^{m1} - A^{*m4}B^{m1})$$
$$= ai\,(A^{*m2}A^{m3} - A^{m2}A^{*m3}),$$
$$H_1 = ai\,(B^{*m2}A^{m3} - A^{*m2}B^{m3}) = ai\,(A^{*m3}B^{m2} - B^{*m3}A^{m2})$$
$$= a\,(A^{*m1}A^{m4} - A^{*m4}A^{m1}),$$

with corresponding expressions for E_2, E_3, and H_2, H_3. The expressions for the current are

$$j_k = -aicY_k,$$

and for the density of charge

$$\rho = -aY_4;$$

while for the magnetic current we have

$$\sigma_k = aicX_k,$$

and for the magnetic density

$$\sigma = -aX_4.$$

The values of the X's and Y's have been indicated above and need not be re-written.

It is noteworthy that the same relations which are indicated by the analogy referred to and which lead directly to the results required in the quantum theory, should be exactly what is required in the present discussion. The components of field-intensity are derivable from the electromagnetic potential, which in this theory is a geometric quantity like the g's of the relativity theory. Through the field components the A's and B's are thus related to the electromagnetic potential and so to the geometry. The relation does not appear to be very simple but the result verifies the suggestion referred to at the beginning of this paper that A and B have a geometrical significance.

This is important from the point of view of a unitary theory, for Flint's work has shown that it is possible to include in a geometrical and metrical scheme the physical realms of gravitation, electromagnetism and the quantum theory. Moreover, in this scheme only one constant, e, occurs, positive and negative values of e

being related to a directional property while the asymmetry of electronic and protonic masses is accounted for by consideration of scales, which follow from an extension of the theory of Weyl and Eddington.

The present paper may be compared with those of other writers on this subject, especially with two papers by Proca† and v. Wisniewski‡.

In conclusion I take the opportunity of expressing my thanks to Dr H. T. Flint for suggesting this line of research to me and for help and advice in carrying it out.

† *J. de Phys.* **1**, 235 (1930).

‡ *Z. f. Phys.* **66**, 697 (1930).

SIR A. S. EDDINGTON'S RECENT THEORIES

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ABSTRACT. The significance of Sir A. S. Eddington's work on the fine-structure constant $2\pi e^2/hc$, the ratio of masses of proton and electron M_P/m, and on the cosmical constant, is outlined. The experimental verification of his theory of the packing effect and of the rate of recession of spiral nebulae is mentioned. His predictions that $hc/2\pi e^2 = 137$ and $M_P/m = 1847{\cdot}60$ are considered in detail. The only evidence against these predictions is that given by four recent experiments, which indicate that $e/m = 1{\cdot}761 \times 10^7$ e.m.u. Reasons are given for believing that this value is too low, and that Eddington's predictions are consistent with experiment. It is unlikely that *both* his predictions require correction terms of the order 1 part in 1847 and it is shown that if $M_P/m = 1847{\cdot}60$, then

$$hc/2\pi e^2 = (137 - 0{\cdot}02_7) \pm (0{\cdot}01_9 \pm 0{\cdot}00_2).$$

Hence it is shown that the theory of 137 is probably *exactly* correct. Similarly, it is shown that if $hc/2\pi e^2 = 137$, then

$$M_P/m = (1847{\cdot}60 - 0{\cdot}5_9) \pm (0{\cdot}5_1 \pm 0{\cdot}0_5),$$

and thus Eddington's prediction for M_P/m might be too great by as much as unity. (These final calculations do not depend on any experimental determinations of e/m.) Consequences of the theories are mentioned. The value of the electronic charge is about $(4{\cdot}777_5 \pm 0{\cdot}001) \times 10^{-10}$ e.s.u., differing from the Millikan-Birge estimate by about as much as might be expected when regard is had to the probable error of the latter.

§ 1. INTRODUCTION

During the last few years Sir Arthur Eddington has developed theories which predict relationships between some of the most important universal constants in physics. It is my purpose, in this paper, to discuss the experimental evidence supporting these theories, to point out their general significance, and to outline some of their consequences. Sir Arthur's theories, which concern the "meeting-ground of astronomy, relativity and wave-mechanics," are abstruse, and can only be properly studied by reference to the original papers*.

§ 2. THE GENERAL SIGNIFICANCE OF THE THEORIES

In order to understand the significance of these theories, we may consider briefly the way in which physics has developed. Progress has been made not only by observation and experiment, but also by the summarizing of the results in the form of "laws of nature." It is only by the formulation of general laws that we

* A. S. Eddington, *Proc. R.S.* A, **122**, 358 (1929); *Nature*, **124**, 840 (1929); *Proc. R.S.* A, **131**, 696 (1930); *Proc. Camb. Phil. Soc.* **27**, 18 (1931); *Proc. R.S.* A, **133**, 605 (1931); *Proc. Phys. Soc.* **44**, 1 (1932); *Proc. R.S.* A, **134**, 524 (1931).

are enabled to think rationally about such a vast amount of experimental knowledge. It has been claimed that there is a large *a priori* probability of the laws being simple in form*, and natural philosophers, such as Faraday and Joule, were almost convinced that there was a unity underlying all natural phenomena.

The amount of experimental knowledge of inanimate nature is continually increasing, but at present a large part of it can be correlated by general laws. In one respect, however, the unification lacked completion. Relativity interpreted gravitation, but did not predict the value of the universal gravitational constant. Indeed it introduced new constants, such as the number of protons in the "world." Similarly, the quantum theory introduced a new constant, Planck's constant of action h.

The only way in which the values of such universal constants can be predicted is by showing that they are related to other universal constants. We cannot, for instance, predict the mass of a proton in grams, for that would be equivalent to predicting the number of protons in the kilogram. But it may be possible to predict from general experimental knowledge the ratio of the masses of proton and electron. The matter may be expressed more generally by saying that non-dimensional products or quotients of the universal constants may be discovered to have numerical values that can be expressed in a simple mathematical way.

In recent years many attempts have been made to find such relationships†. Most of these attempts have had little theoretical basis, and many have definitely disagreed with experiment. Nevertheless, there was reason to expect that such relationships would eventually be found, because many similar ones had been established in the past. For example, Rydberg's, Stefan's and Wien's constants were known to be related to Planck's constant.

The situation is now changed. The theories that Sir Arthur Eddington has developed have predicted three further relationships. These predictions are supported by experimental evidence, and the theories constitute an advance in knowledge quite comparable with that brought about by relativity and the quantum theory. We may note, in passing, that the theories are concerned with the number of degrees of freedom of systems.

§ 3. SIR ARTHUR EDDINGTON'S PREDICTIONS

The three chief relationships deduced by Eddington may be stated thus:

Firstly, $$hc/2\pi e^2 = 137 \qquad \text{.....(1)},$$

where h is Planck's constant, c the velocity of light in a vacuum and e the electronic charge expressed in electrostatic units. h, c, e

Secondly, the ratio of the masses of proton and electron, M_P/m, is equal to the ratio of the roots of the equation M_P, m

$$10x^2 - 136x + 1 = 0,$$

or $$M_P/m = 1847{\cdot}599_6 \qquad \text{.....(2)}.$$

* H. Jeffreys, *Scientific Inference*, pp. 43–51.

† For instance, see G. N. Lewis and E. Q. Adams, *Phys. Rev.* **3**, 92 (1914); R. Fürth, *Phys. Zeit.* **30**, 895 (1929).

Thirdly, $\{M_P m/(M_P + m)\}\, c^2/e^2 = \sqrt{N}/R$ .....(3),

N where N is the number of electrons (or protons) in the universe, and the universe

R is of the Lemaître type, having an equilibrium radius R.

§ 4. THE PACKING EFFECT

One numerical test of Eddington's theories was the verification of the prediction, which he made incidentally, that if protons and electrons combined to form a rigid system the mass would be $(1 - 2\pi e^2/hc)$ of the sum of the masses of the constituents considered separately. This prediction of the packing effect in an α-particle, which had previously been suggested on a purely empirical basis by Lunn, was commented on favourably from an experimental point of view by Birge*, whose treatment I will follow.

The experimental value of the atomic weight of hydrogen, relative to a mixture of the oxygen isotopes as 16, may be taken as $1{\cdot}007{,}77 \pm 0{\cdot}000{,}02$, the relative weight of four hydrogen atoms being thus $4{\cdot}031{,}08 \pm 0{\cdot}000{,}08$. Assuming that the helium atom consists of a perfectly rigid α-particle together with two electrons, and applying Eddington's theory, we find the estimated atomic weight of helium to be

$$4{\cdot}001{,}66 \pm 0{\cdot}000{,}08.$$

The value found by chemical methods is

$$4{\cdot}001{,}8 \pm 0{\cdot}000{,}4,$$

and from Aston's work on mass-spectra, corrected for a trace of O_{18}, it is

$$4{\cdot}001{,}8 \pm 0{\cdot}000{,}2.$$

The agreement is thus entirely satisfactory, though the experiments are not accurate enough for this calculation to be used as a test of equation (1).

§ 5. THE THEORY OF 137

In discussing equations (1) and (2) it is best to state at once that the *only* experimental evidence against either prediction is based on certain recent estimates of the specific electronic charge, e/m. This estimate, which Birge† designated the "spectroscopic" estimate, is about $\frac{1}{2}$ per cent lower than the value suggested by the work of other experimenters (Birge's "deflection" estimate). The discrepancy between these two estimates will be discussed later in this paper.

Considering Eddington's theory of 137, equation (1), we find that the value of $hc/2\pi e^2$ was estimated in 1929 by Birge‡ as

$$137{\cdot}29_4 \pm 0{\cdot}11.$$

* R. T. Birge, *Phys. Rev.* **35**, 1015 (1930).
† *Phys. Rev. Supplement*, **1**, 1–73 (1929).
‡ *ad loc.* 71 (1929).

This estimate was based on Birge's estimate of the electronic charge, viz.

$$(4{\cdot}770 \pm 0{\cdot}005) \times 10^{-10} \text{ e.s.u.},$$

derived from a re-calculation of Millikan's experimental results and from the absolute determination of X-ray wave-lengths made by A. P. R. Wadlund. The value of h which Birge used was based on estimates obtained by six methods, but the method to which by far the greatest weight was attributed depended on the estimate of e/m. And Birge here assumed the low or spectroscopic estimate of e/m, to the entire exclusion of his deflection estimate.

Millikan subsequently discussed the problem*, and his final estimate of e was the same as that given by Birge. Millikan concluded that it was "highly improbable" that $hc/2\pi e^2$ could be exactly 137 (or exactly 136, the value Eddington originally suggested).

I have recently shown† that more reliable estimates of e and h can be obtained by an analysis of the experimental results that are usually used to estimate h. My calculation (which can be considered an elaboration of one originally used by Planck) resulted in the estimates

$$e = (4{\cdot}778_9 \pm 0{\cdot}002_9) \times 10^{-10} \text{ e.s.u.},$$

and

$$hc/2\pi e^2 = 137{\cdot}01_7 \pm (0{\cdot}05_9 \pm 0{\cdot}00_8).$$

Taking this estimate of e and the entirely independent estimate given by Birge and by Millikan, and weighting them according to their probable errors, I now obtain

$$e = (4{\cdot}776_4 \pm 0{\cdot}002_5) \times 10^{-10} \text{ e.s.u.},$$

$$h = (6{\cdot}553_5 \pm 0{\cdot}004_7) \times 10^{-27} \text{ erg.-sec.},$$

$$hc/2\pi e^2 = 137{\cdot}05_9 \pm 0{\cdot}05_4.$$

The agreement with Eddington's theory is again very satisfactory.

Prof. Birge has quite recently‡ discussed my calculations, and he agrees that a more reliable estimate of the electronic charge can be obtained by my method than by the direct methods. But he continues to adopt the lower ("spectroscopic") estimate of e/m, to the entire exclusion of the so-called deflection estimate. As a consequence his results disagree with two of Eddington's theories, equations (1) and (2).

I will give my reasons for believing that the value of e/m is not as small as Prof. Birge suggests. The lower or spectroscopic estimate was originally based on H. D. Babcock's work on the Zeeman effect and on that of W. V. Houston on the hydrogen and helium spectra§. Subsequently Perry and Chaffee‖ and Kirchner¶ obtained almost the same value by experiments on cathode rays. These four very

* R. A. Millikan, *Phys. Rev.* **35**, 1233 (1930).
† W. N. Bond, *Phil. Mag.* **10**, 994 (1930); **12**, 632 (1931).
‡ *Bulletin*, Am. Phys. Soc., Dec. 6th, 1931.
§ R. T. Birge, *loc. cit.*, 44–48.
‖ *Phys. Rev.* **36**, 904 (1930).
¶ *Phys. Zeit.* **31**, 1073 (1930); *Ann. d. Physik*, **8**, 975 (1931).

concordant estimates suggest that $e/m = 1{\cdot}761 \times 10^7$ e.m.u. There are, however, about 18 estimates of e/m ranging from $1{\cdot}763$ to $1{\cdot}775 \times 10^7$, suggesting a value of about $1{\cdot}769 \times 10^7$*. The disagreement is more than can be accounted for by the probable errors of the estimates. *Whichever* value we assume, the situation is peculiar. Moreover, we ought not to be guided by the probable errors in deciding between the two values.

We might be satisfied that the earlier determinations by the two spectroscopic methods were in each case too high, and by a similar amount. That the earlier results by the deflection method should have been in error in the same sense, and by about the same amount, seems strange. It has been suggested that the presence of traces of gas in the apparatus caused the results to be too large. But there seems to be no progressive decrease in the successive results, which might have been expected to occur as the technique of high-vacuum work developed. Again, the deflection of β-rays would be expected to give results little affected by this error: but the results so obtained indicate a value of about $1{\cdot}767 \times 10^7$. Further evidence for the higher value is given by Birge's† estimate, based on the Compton effect, of $(1{\cdot}772 \pm 0{\cdot}006) \times 10^7$. Yet again, J. A. Bearden‡ has recently deduced from the refraction of X-rays a value that may be stated as $(1{\cdot}769 \pm 0{\cdot}004) \times 10^7$. Finally, if I omit from my calculations *all* data depending on determinations of e/m, and do not assume either of Eddington's predictions in equations (1) and (2), I obtain the estimates

$$e/m = (1{\cdot}767_5 \pm 0{\cdot}003_1) \times 10^7 \text{ e.m.u.},$$

and

$$hc/2\pi e^2 = 137{\cdot}07 \pm 0{\cdot}12,$$

in good agreement with Eddington's 137.

For all these reasons we can continue to be satisfied with the accuracy of Eddington's theory of 137.

§6. THE RELATIVE MASSES OF PROTON AND ELECTRON

Prof. Birge in 1929 gave two estimates of M_P/m based on his spectroscopic and deflection estimates of e/m,

$$(M_P/m)_{\text{spect.}} = 1838._{26} \pm 1,$$

$$(M_P/m)_{\text{defl.}} = 1846._{61} \pm 2.$$

Birge pointed out that these two values definitely disagreed with one another, but he now adopts the spectroscopic or smaller value. I have discussed above my reasons for disagreeing with this estimate.

In my previous work I showed that the experimental evidence for Eddington's 137 was rather greater than that for any one of the other theoretical relationships between e and h, and therefore I assumed the value 137 in the subsequent parts

* See, for instance, Geiger and Scheel's *Handbuch der Physik*, **22**, 81.

† R. T. Birge, *loc. cit.* 58.

‡ *Phys. Rev.* **37**, 1228 (1931).

of my papers. Using Bohr's theory of spectra and Rydberg's constant, I then deduced

$$M_P/m = 1846{\cdot}5_7 \pm 0{\cdot}4_8.$$

This estimate, which agrees with Birge's deflection value, but claims a higher accuracy, is in reasonable agreement with Eddington's 1847·6, the difference being 2·1 times my probable error. It definitely disagreed with Eddington's former suggestion of $136^2/10$, differing from that value by 6·3 times my probable error.

Repeating the calculation, without assuming the 137, but taking into account the Birge-Millikan direct estimate of e as well as my independent estimate, I now obtain

$$M_P/m = 1845{\cdot}3_3 \pm 1{\cdot}3_3.$$

This estimate differs from Eddington's predicted value by 1·7 times my probable error: the agreement may again be considered satisfactory.

§ 7. ARE THESE EQUATIONS EXACTLY CORRECT?

The various estimates of $hc/2\pi e^2$ and M_P/m that I have deduced, and given above, differ from Eddington's predictions by between 0·3 and 2·1 times my probable errors: in other words, experiment and theory differ by from 0·01 to 0·12 per cent. The agreement is, therefore, entirely satisfactory. It may, however, be suggested that there is still the possibility of Eddington's 137 and 1847·60 being found to require correction by about 1 part in 1847. This can be tested further.

We have no experimental reason to expect either correction to be necessary, and it is relatively unlikely that *both* corrections will be needed. Let us therefore assume for the moment that M_P/m is exactly $1847{\cdot}599_5$. I am then able to deduce from the experimental data the estimate

$$hc/2\pi e^2 = (137 - 0{\cdot}02_7) \pm (0{\cdot}01_9 \pm 0{\cdot}00_2).$$

No experimental determinations of e/m are involved here. It follows that the chance of $hc/2\pi e^2$ being as small as $137\,(1 - 1/1847)$ is only 1 in 20, unless the 1847·60 *also* requires correction. Similarly, the chance of it being as great as $137\,(1 + 1/1847)$ is only 1 in 6000, unless *both* the 137 and 1847·60 require correction. As there is little reason to expect two corrections when theory and experiment are already in good agreement, we may conclude that the 137 is not in error by as much as 1 part in 1847. Smaller corrections are not to be expected, and therefore

$$hc/2\pi e^2 = 137$$

may be considered to be exactly correct.

Assuming this, and reversing the procedure, I obtain the estimate

$$M_P/m = (1847{\cdot}599_5 - 0{\cdot}7_9) \pm 0{\cdot}4_6,$$

or, omitting all the data that depend on experimental estimates of e/m (about which there has been discussion), the estimate becomes

$$M_P/m = (1847{\cdot}599_5 - 0{\cdot}5_9) \pm (0{\cdot}5_1 \pm 0{\cdot}0_5).$$

This discussion leaves open the possibility that the $1847{\cdot}599_5$ may be as much as 1 too large, but gives no reason for expecting this to be the case.

§ 8. THE EXPANDING UNIVERSE

It is only necessary to mention very briefly the numerical test of equation (3), as this has been dealt with by Sir Arthur Eddington himself in his Presidential Address to the Physical Society*.

Sir Arthur concludes, from the observed recession of the spiral nebulae, that the universe is of the Lemaître type, and, further, that it had at one time a stationary radius given by equation (3). At that stage the universe would have been of the Einstein form, its mass and radius being related by the equation

$$(\text{Total mass})\ G/c^2 = \tfrac{1}{2}\pi R,$$

where the total mass is approximately $N.M_P$.

According to Lemaître's theory, the rate of recession of the spiral nebulae per unit distance, uncorrected for any opposing gravitational effect, is given by the expression $c/R\sqrt{3}$. From these three equations Sir Arthur deduces the prediction for the rate of recession of the spiral nebulae:

528 km./sec. per megaparsec,

in good agreement with the experimental estimates, which range from about 430 to 550 in the same units.

§ 9. SOME CONSEQUENCES OF THE THEORIES

Having seen that Eddington's four predictions are in very good agreement with experiment, and that, in particular, his $hc/2\pi e^2 = 137$ is probably exactly correct, we may consider briefly some of the consequences of the theories. Whenever a new relationship is established between universal constants in physics, we can express the less accurately known in terms of the more accurately known and increase the precision of physical knowledge. In the table (p. 381) two sets of estimates are compared with those obtained using equation (1), which can be assumed with confidence, and with the values obtained when both equation (1) and (2) are assumed.

Millikan's determination of the electronic charge appears to be as accurate as he claimed it to be, but actually somewhat smaller than the true value.

Most of the determinations of e by the crystal and ruled-grating method appear to be definitely too large: and we may presume that the error has been correctly attributed to the mosaic structure that, according to Zwickey, occurs in crystals.

The lower or "spectroscopic" estimate of e/m seems to be quite definitely in error. It is not possible to decide whether the presumed error is experimental, or whether it is due to slight errors in the particular theories on which it is based.

The relationship involving G which was suggested by Fürth†, and which I formerly believed to be of significance, appears to be incorrect. Firstly, it was not in quite as good accord with experiment as might have been expected if it were valid; and the full account of Heyl's‡ recent determinations of G makes

* *Proc. Phys. Soc.* **44**, 1 (1932).

† *loc. cit.*

‡ *Bureau of Standards J. of Research*, **5**, 1243 (1930).

the agreement even less satisfactory. Secondly, in my previous work I found that Fürth's theory for M_P/m disagreed with experiment, and this throws doubt on the rest of his work. Finally, if Fürth's theory involving G were correct, and if Eddington's theories were true or at the most only subject to correction terms, then it would be possible to give a comparatively simple mathematical expression for the number of protons in the universe! It appears to be quite unlikely that the total number of protons in the universe is exactly expressible in a simple way, and hence the confirmation of Eddington's theory by experiment necessitates the abandonment of Fürth's theory.

Table 1. Values of universal constants.

	Birge (1929)	Not assuming equations (1) and (2)	Assuming equation (1)	Assuming equations (1) and (2)
$hc/2\pi e^2$	$137{\cdot}29 \pm 0{\cdot}11$	$137{\cdot}05_9 \pm 0{\cdot}05_4$	137	137
M_P/m	$1838_{\cdot 26} \pm 1$ $1846_{\cdot 61} \pm 2$	$1845_{\cdot 3} \pm 1{\cdot}3$	$1847{\cdot}0_0 \pm 0{\cdot}5_1$	$1847{\cdot}599_5$
$(e/m) \times 10^{-7}$ abs. e.m.u.	$1{\cdot}761 \pm 0{\cdot}001$ $1{\cdot}769 \pm 0{\cdot}002$	$1{\cdot}767_8 \pm 0{\cdot}001_3$	$1{\cdot}7694 \pm 0{\cdot}0004_9$	$1{\cdot}7699 \pm 0{\cdot}0001$
$e \times 10^{10}$ abs. e.s.u.	$4{\cdot}770 \pm 0{\cdot}005$	$4{\cdot}776_4 \pm 0{\cdot}002_5$	$4{\cdot}778_4 \pm 0{\cdot}001_2$	$4{\cdot}7768 \pm 0{\cdot}0004$
$h \times 10^{27}$ erg.-sec.	$6{\cdot}547 \pm 0{\cdot}008$	$6{\cdot}553_5 \pm 0{\cdot}004_7$	$6{\cdot}555_9 \pm 0{\cdot}003_4$	$6{\cdot}551_7 \pm 0{\cdot}001_0$

The acceptance of Eddington's treatment of the expanding universe indicates that the universe has passed through, or started from, a stationary state of minimum size. It is pleasing to find a theory of relativity which distinguishes so markedly between past and future! If the spiral nebulae are now some fifteen or so times as far apart as when packed in the stationary state (the reckoning being relative to atomic sizes which may be used to define an unchanging length) it appears that a period of the order 10^{10} years or more has elapsed since the closest packing. This interval of time is so near the estimates of the age of our own universe that it seems likely that the two periods will be identified. It is interesting to notice that, according to Eddington, the properties of cosmic or penetrating radiation cannot be used to predict directly the types of atomic change that gave rise to the rays. He points out that the light would have been reddened by age, relative to our standards, and might give a means of estimating the interval that has elapsed since the universe was of the stationary or Einstein form.

DISCUSSION

Sir A. S. Eddington said that he was naturally much interested in the paper. All the constants involved were known with sufficient precision except e/m, and the question in dispute was between the high and the low experimental value of this quantity. It was to be noted that the higher value agrees with both of the theoretical figures 137 and 1847·6, while the lower value is at variance with both. The speaker had hoped that his theory would give the number of protons in the universe—a result which the author seemed to consider improbable—but unfortunately it had failed to do so. Not much importance could be attached to the prediction of the age of the universe (10^{10} years) since there was at present available a choice between widely varying estimates.

Dr F. J. W. Whipple. Whilst the reasoning by which Sir Arthur Eddington succeeded in evaluating these natural constants is beyond the comprehension of most of us, I think we may, with advantage, dwell on the way in which two little pieces of algebra have found an application in physics. These are the investigation of the number of homogeneous products of known degree of so many variables and the discovery of an equation the roots of which are the ratios of the roots of a given equation.

We have ${}_{16}H_2 = \frac{1}{2} \times 16 \times 17 = 136$ and by adding unity (this seems rather arbitrary; may we say "and one for his nob"?) we get 137, the first of the mystic numbers, the number which determines how the scale of wave-lengths is related to the circumferences of electronic orbits.

Further if ρ be the ratio of the roots of the equation, $x^2 - bx + c = 0$, then ρ satisfies the equation $\rho + 2 + 1/\rho = b^2/c$. If $b = {}_{16}H_2 = 136$ and $c = {}_4H_2 = 10$ then $\rho + 2 + 1/\rho = 1849{\cdot}6$. The two roots of this equation are, we are told, the ratio of the masses of proton and electron and the reciprocal ratios. To a sufficiently close approximation $\rho = 1847{\cdot}6$; this approximation is obtained by ignoring the term $1/\rho$ in the foregoing equation. Taking account of that term we get the approximation quoted in the paper, $\rho = 1847{\cdot}5995$.

Mr Rollo Appleyard, referring to the author's remark that Faraday and Joule had sought the unification of all natural phenomena, said that Oersted before their time had devoted much thought to the same problem.

Author's reply. Sir Arthur Eddington points out that the higher experimental estimate of e/m agrees with both the theoretical figures 137 and 1847·6, whereas the lower estimate does not agree with either. I would emphasize that, without the assumption of *any* experimental estimate of e/m, it can be shown that the other experimental data and the two theoretical figures (137 and 1847·6) are in extremely close mutual agreement.

A VACUUM CALORIMETER FOR HIGH TEMPERATURES

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ABSTRACT. A form of the platinum-thermometer type of vacuum calorimeter has been developed which is suitable for the determination of true specific heats at high temperatures, since it is constructed without any organic insulating materials. The design is novel in that the heat is transferred from the platinum coil to the calorimeter by radiation.

The general principles of vacuum calorimetry are discussed and applied to the design of the present calorimeter, and a full description of the constructional details and experimental procedure is given.

§ 1. INTRODUCTION

The vacuum calorimeter was originally developed by Nernst and his collaborators, and in their hands attained to great precision as a means of investigating the thermal properties of matter at low temperatures. As designed by them it was restricted in its range on the high-temperature side by the limitations imposed by organic insulating material and by uncertainty as to the heat-loss by radiation, which increases very rapidly with increase of temperature.

With regard to high-temperature calorimetry, mention must be made of the extremely accurate work of Magnus who uses the method of mixtures. The heated substance is dropped into an aneroid calorimeter of solid copper, of which the rise of temperature is measured. While this method can be used up to very high temperatures, it yields, in the first instance, mean heat-capacities over a large temperature range. From this the curve of heat-capacity as a function of temperature can be constructed, and, by differentiation, the true specific heat at a given temperature can be deduced. The method requires, however, that the slope of the heat-capacity/temperature curve should be accurately known at every point. Also, since the substance is necessarily rapidly cooled from a high temperature to air temperature, it cannot be completely annealed.

Recently two attempts have been made to use the vacuum calorimeter, in which true specific heats of well annealed substances over a small range of temperature are measured, for work at high temperatures. The first of these was due to Sucksmith and Potter*, who measured the true specific heat of nickel over the range 20° to 410° C. They do not claim any great precision, however, for their calorimeter, which was in fact intended mainly to give information as to the shape of the specific heat/temperature curve of nickel in the neighbourhood of its Curie point. The other and more recent adaptation of the vacuum calorimeter for high temperature work was made by Klinkhardt†, who adopted the ingenious plan of supplying the

* *Proc. R.S.* A, **112**, 157 (1926).

† *Ann. d. Phys.* **84**, 167 (1927).

measured quantity of heat to the calorimeter by bombarding it at a known voltage with electrons from a hot filament. The precision of his results is rather low, however; even at temperatures below 500° C. individual values of the specific heat as experimentally determined differ from the mean curve by as much as 3 per cent in some cases. Also, his measurements of thermal capacity were made over temperature ranges of about 4° C., which are inconveniently large when investigations are being made in the neighbourhood of a transition point. The method could, no doubt, be developed further, and rendered more accurate.

As we wished to make a precise and detailed study of the atomic heats of elements in both the solid and liquid phases and especially in the neighbourhood of their melting points, an extensive series of experiments has been carried out to explore the possibilities of other methods of design for high temperatures*. The design finally adopted is very satisfactory and, so far as we know, novel, in that the heat supplied to the calorimeter from the heating coil, which also serves as a platinum thermometer, is transferred by radiation instead of by solid conduction, the advantage being that the coil, while perfectly insulated from the calorimeter electrically, nevertheless quickly comes into thermal equilibrium with it when the heating current is cut off. The calorimeter, when working over a temperature interval of only 1° C., will determine thermal capacities correct to within 1 per cent.

§ 2. GENERAL PRINCIPLES OF VACUUM CALORIMETRY

In order to make an accurate determination of heat-capacity by means of the vacuum calorimeter, it is necessary to know the amount of heat supplied electrically to the calorimeter, and the resulting rise of temperature. If it is desired to experiment on a substance in the liquid as well as in the solid phase, the heat-capacity of the necessary container must be determined separately.

We shall deal here only with the particular type of calorimeter in which the heat is supplied by means of a coil of platinum wire, the same coil serving as a resistance thermometer to measure the resulting rise of temperature.

The energy supplied is usually deduced from a knowledge of the time-integral of the watts developed in the coil during the time of input, which we will call t_i. This measurement presents no difficulty, provided that accurate ammeters and voltmeters are available and that t_i is not made unduly small. The question where this energy input is developed requires some consideration, as connexion to the platinum coil is necessarily made through leads. If these leads are of low electrical resistance, so that the wattage in them is negligible, then their thermal resistance is also very low and the heat-leak from the calorimeter is large. It is best, of course, to make the resistance of the coil so large that the thermal resistance of the leads may be adequate, and yet their electrical resistance small compared with that of

* We wish to acknowledge here the work done by Mr L. G. Stoodley who made some preliminary investigations.

the coil. There is, however, a limit in practice to the possible resistance of the coil, set by the maximum resistance which can be measured with the bridge available*. The matter is considered further in § 5.

The measurement of the rise of temperature produced by the heating-current is intimately connected with the question of correction for heat-loss. For the sake of simplicity we will assume that before the current is switched on the calorimeter and its surroundings are at the same temperature, which is maintained constant by a thermostat. The measurement of the initial temperature with the coil, used as a platinum thermometer, involves no difficulty. After this measurement has been made, the heating-current is switched on for the time t_i and a certain known amount of heat is supplied to the calorimeter. On switching off, however, the coil is found to be hotter than the calorimeter by an amount depending on the heating-current and on the design. Also the temperature inside the calorimeter is not uniform on account of the finite thermal conductivity of container and its contents. Before making any temperature measurements, it is therefore necessary to wait for a certain length of time, t_e, for thermal equilibrium to be re-established. During t_e
this time, however, heat is being lost by the calorimeter, so that, when the final temperature is actually measured it is lower than that corresponding to the energy input. Also heat is lost by the calorimeter during the time of input t_i. This heat loss is due to radiation from the calorimeter surface, to gaseous conduction, and to solid conduction along the leads and suspensions of the calorimeter. Its total magnitude can be reduced by suitable design, but it can never be made so small as to be neglected altogether, at any rate at high temperature where radiation is considerable. A correction is therefore necessary.

In order to make the correction small, the calorimeter must be designed so that the heat lost by the calorimeter during $(t_i + t_e)$ shall be small compared with that imparted to it during t_i.

Let A be the area of the emitting surface of the calorimeter, A

E the effective emissivity of this surface, assuming, for the purpose of calculation, that all the heat is lost from the surface and none by solid conduction up the suspensions and leads†; E

C_c the thermal capacity of the calorimeter; and C_c

β the rise in temperature of the calorimeter as a whole produced by the heating current during the time t_i. β

For the purpose of design we shall assume that, during the time t_e before thermal equilibrium is established between the coil and the calorimeter, the temperature-excess of the calorimeter above its surroundings does not differ much

* In our case, for instance, a Callendar-Griffiths bridge is used which is of course very convenient for platinum thermometry, but the maximum resistance which this particular instrument will measure is 50Ω.

† The justification of this procedure, which is admittedly a rough approximation, is that for small temperature-differences the total heat-loss due to radiation, and to solid and gaseous conduction depends on a single factor, viz. the temperature-difference between the calorimeter and its surroundings

from β. Actually the temperature will decrease slowly with time, but the difference will not be large in a reasonably well heat-insulated calorimeter. Then

$$\text{heat input during time } t_i = C_c\beta.$$

Now, we can assume that during t_i the calorimeter temperature rises linearly with time. Hence

$$\text{heat lost during } t_i = \tfrac{1}{2}EA\beta t_i.$$

Also

$$\text{heat lost during } t_e = EA\beta t_e.$$

Therefore

$$\frac{\text{heat lost during } (t_i + t_e)}{\text{heat gained during } t_i} = \frac{EA\beta\,(\frac{1}{2}t_i + t_e)}{\beta C_c}$$

$$= EA\,(\tfrac{1}{2}t_i + t_e)/C_c. \qquad \text{.....(1).}$$

Assuming that E is made as low as possible by radiation shields, good vacuum, and leads and suspensions of high thermal resistance, this means that the product of A/C_c and $(\frac{1}{2}t_i + t_e)$ should be as small as possible.

Assuming for a moment that C_c is fixed, the ratio A/C_c is of course a minimum for a calorimeter of spherical form. For constructional reasons, however, this form is not convenient, and hence the calorimeter generally approximates in practice a cylinder whose diameter is equal to its height. This, however, merely settles the proportions of the calorimeter, assuming a given value of C_c. The question of what is the best value of C_c remains open. W. P. White* has pointed out that if t is the time taken for the attainment of a given degree of temperature-uniformity throughout the volume of the calorimeter, then t will vary as the square of the linear dimensions of the calorimeter. A is, of course, proportional to the square of the linear dimensions and C_c to the cube. Hence it seems that At/C_c is proportional to the first power of the linear dimensions, so that the calorimeter should be as small as possible. This argument, however, is not applicable if t is governed not only by the time taken for the heat to diffuse through the calorimeter but also by the time taken for the coil to re-attain thermal equilibrium with the calorimeter, after the heating current has been switched off.

This latter time is fixed by the properties of the coil and by its thermal contact with the calorimeter, rather than by the dimensions of the calorimeter. Further, White's argument is not necessarily applicable in determining the accuracy of determination of the heat-capacity of the contents of a calorimeter as opposed to the heat-capacity of calorimeter and contents together. If the calorimeter is too small, its thermal capacity may become large compared with that of its contents, in which case the determination of the heat-capacity of the contents will be subject to large errors. Hence, although we can say quite generally that the calorimeter, if cylindrical, should have the proportions mentioned above, we cannot state the conditions that $(A/C_c)\,(\frac{1}{2}\,t_i + t_e)$ should be a minimum without reference to the particular type of construction contemplated. We shall explain later how we solved the problem in our particular case.

* *J. Amer. Chem. Soc.* **40**, 1894 (1918).

The thermal requirements of the design having been stated the electrical requirements must next be considered. This involves deciding what is the largest resistance which the bridge available will measure, and the highest temperature at which it is desired to work. These two data fix the upper limit to R_0, the resistance of the platinum coil at o° C. The next consideration is the percentage accuracy required in the determination of the temperature-rise. Having settled this, and knowing the possible experimental error of a bridge-balance, we arrive at a minimum permissible value for the increase in the resistance of the platinum coil due to rise in temperature of the calorimeter, which can be measured with the required accuracy. R_0,

If the calorimeter is to be used for examining in detail the variation of heat-capacity with temperature, the rise of temperature of the calorimeter corresponding to the heat imparted during time t_i (which we shall denote by β) must not exceed a definite amount, or the fine detail of the variation of heat-capacity with temperature will be lost. β

Let α be the temperature-coefficient of the platinum coil. Then we have α

$$R_0 \not> \text{a definite value} \qquad \text{.....(2)},$$

$$R_0\alpha\beta \not< \text{a definite value}^* \qquad \text{.....(3)},$$

$$\beta \not> \text{a definite value} \qquad \text{.....(4)}.$$

Also, if R_l is the resistance of the leads, the restriction that the watts developed in the leads shall be a small fraction of the total wattage imposes the further condition R_l

$$R_0 \gg R_l \qquad \text{.....(5)}.$$

A compromise between all these requirements will lead to the choice of a definite value for R_0 and give a guide as to the value of β, though this latter quantity will no doubt vary somewhat from experiment to experiment.

We have now indicated the method of arriving at the most suitable electrical resistance for the calorimeter and have formulated the thermal requirement that $A(\frac{1}{2}t_i + t_e)/C_e$ should be a minimum. So far the discussion has been quite general, but in order to carry it further we must describe the particular method of construction which we found best for high-temperature work.

§ 3. PRINCIPLES OF DESIGN OF PRESENT CALORIMETER

The difficulty of constructing a vacuum calorimeter for high temperatures largely consists in reconciling the two opposing requirements that the coil should have high electrical insulation (if the platinum thermometry is to be really reliable) and yet good thermal contact with the calorimeter (if t_e is to be reasonably small).

After considerable experiment, we have concluded that the best method of

* If it is desired that the increase in resistance due to the temperature-rise shall be measurable by moving the slider over the bridge wire without changing the bridge coils, there is a further condition

$$R_0\alpha\beta \not> \text{a definite value} \qquad \text{.....(3a)}.$$

construction is to insulate the coil from the calorimeter and allow the heat-transfer from coil to calorimeter to take place by radiation. The arrangement adopted is shown, purely diagrammatically, in figure 1.

A is the calorimeter round which is wound a coil *B* of blackened platinum, well insulated from it. This coil radiates to the surface of the calorimeter, which is blackened, and to the inner surface (also blackened) of the silver shield *C* which is in good thermal contact with the calorimeter. The blackening is shown in the figure by thick lines. *D* is an outer silver radiation-shield, which also makes good thermal contact with the calorimeter.

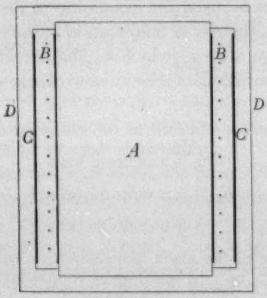

Fig. 1.

The theory of this method of design, showing, in particular, how t_c can be approximately calculated, is as follows: First assume that both the resistance R_0 and the gauge of the wire have already been settled. This latter quantity is generally fixed by mechanical rather than theoretical considerations, though a theoretical treatment has been worked out. R_0 is of course decided by using the conditions contained in equations (2), (3), (4) and (5). The two most important thermal characteristics of the coil are (i) the maximum temperature excess above the rest of the calorimeter to which it will attain with a given current flowing through it, and (ii) its time constant, i.e. the time taken for this temperature excess to diminish to e^{-1} of its original value, when no current is flowing.

We shall assume in our calculations that the heat-transfer from coil to calorimeter is entirely by radiation, gaseous and solid conduction being negligible. With the present method of design this is approximately true, at any rate at high temperatures, with which we are principally concerned.

According to the Stefan-Boltzmann law, the flux of radiant energy in
ϕ calories/cm^2-sec. given out by a surface at a temperature $\phi°$ absolute is

F $$F = B\phi^4 \qquad \text{.....(6),}$$

B where B is a constant which, for a perfectly black body, has the value of

$1{\cdot}36 \times 10^{-12}$*. It follows that the rate of net loss of heat ΔF when the surface is at a temperature excess $\Delta\phi$ above its surroundings (also assumed to be perfectly black) is

$$\Delta F = 4B\phi^3\,\Delta\phi \qquad \text{.....(7).}$$

Hence, provided that $\Delta\phi$ is small compared with ϕ, ΔF is proportional to the first power of the temperature-excess.

We may therefore write

$$\text{loss of heat} = K\theta \text{ cal./cm}^2\text{-sec.} \qquad \text{.....(8),}$$

where θ is the temperature excess, and K is a constant which is proportional to ϕ^3. K

Let r be the radius of platinum wire which forms the coil; r

θ the temperature-excess of the coil above the calorimeter; θ

R_1 the resistance per unit length of the coil at the particular temperature considered; R_1

l the length of the coil; l

ρ the density of material of the coil; ρ

σ the specific heat of material of the coil; σ

J the ratio of the calorie to the watt-second; J

I the current through the coil. I

We shall further assume that the time-constant of the coil is small compared with t_c, which is the case in practice. Now, when the heating-current is switched on, the temperature of the coil will rise very rapidly for a few seconds and then become nearly stationary, subsequently rising slowly at the same rate as its immediate environment, i.e. the calorimeter. If we denote by θ_0 the maximum temperature-excess of the coil, we have θ_0

$$2\pi r K\theta_0 = I^2R_1/J \qquad \text{.....(9).}$$

Equating the rate of loss of heat due to emissivity to the rate of loss of heat as calculated from its thermal capacity and rate of fall of temperature, we have

$$\pi r^2\rho\sigma\,.\,d\theta/dt = -\,2\pi r K\theta \qquad \text{.....(10).}$$

From which the time-constant T is given by T

$$T = r\rho\sigma/2K \qquad \text{.....(11),}$$

and

$$\theta = \theta_0\, e^{-t/T} \qquad \text{.....(12).}$$

It is of course evident, that, in theory, complete thermal equilibrium between coil and calorimeter is attained only after an infinite time. In practice, however, one considers thermal equilibrium to have been reached when the temperature excess falls to an assigned small value, say θ_e. We will denote the corresponding time by t_e. Then θ_e

$$t_e = T\,(\log\theta_0 - \log\theta_e) \qquad \text{.....(13).}$$

* As a matter of fact for some metals the power of ϕ is more nearly the fifth than the fourth. This does not, however, invalidate the linearity of the relation between heat-loss and temperature-excess for small values of temperature-excess.

In deriving this relation we have assumed the wire to be of circular cross-section, although actually it would be better to make it in the form of a flat ribbon, in which the ratio of radiating area to thermal capacity would be greater. This has not been done in the present calorimeter, though future instruments will probably be so wound.

In order to use (13)* to calculate t_e it is necessary to know θ_0 and θ_e. T is of course fixed by the gauge and physical properties of the platinum wire. θ_0 depends on I which is determined by the fact that the rise of temperature β is to be brought about by the energy supplied during the time of input t_i. Let us assume for the moment that the values of β and t_i have been fixed; θ_0 is then known. θ_e is decided by saying that the wire shall be considered to be in thermal equilibrium with the calorimeter as soon as θ_e/β falls to a specified small value. If, for instance, β is 1° C., which is the order of magnitude usual in these experiments, and if we put θ_e/β equal to $\frac{1}{400}$, the error introduced into the measured heat-capacity will not exceed $\frac{1}{4}$ per cent.

When the present calorimeter is used for work on metals, t_e, the time for the coil to come into thermal equilibrium with the calorimeter, is of a much larger order than the time required for the heat supplied to the surface of the calorimeter to diffuse uniformly through it. Hence the condition, mentioned in the previous section, that the calorimeter should be as small as possible, does not apply in this case, and indeed the question of size is decided by the requirement that the heat-capacity of the calorimeter should be small compared with the heat-capacity of its contents. The matter presents no difficulty in practice.

The requirement that $(A/C_c)(\frac{1}{2}t_i + t_e)$ shall be a minimum then reduces to requiring that $(\frac{1}{2}t_i + t_e)$ shall be a minimum. The problem may be stated as follows: We have a calorimeter with a given thermal capacity C_c wound with a coil of given length, gauge, and radiation characteristics, and we intend to use it in an experiment in which the rise of temperature is β. It has already been decided that the final temperature shall be measured at the time when θ has fallen to a certain predetermined fraction of β, say $p\beta$. During the time $(t_i + t_e)$, however, heat will be lost by radiation, and a correction must be made. It is known that this correction, expressed as a percentage of the total heat-input during t_i, is proportional to $(\frac{1}{2}t_i + t_e)$. What is the best value of heating-current to choose so that this percentage correction shall be a minimum?

Now, the product of the thermal capacity and the rise in temperature during input must be equal to the heat supplied during t_i.

Hence
$$\beta C_c = I^2 R_1 l t_i / J \qquad \text{.....(14).}$$

Also from (9)
$$\theta_0 = I^2 R_1 / 2\pi r K J.$$

Substituting in (14) we get
$$\theta_0 = \beta C_c / 2\pi r l K t_i.$$

* It must be understood that equation (13) is used merely as a guide for design purposes. In actual use of the calorimeter t_e is determined experimentally.

Hence from (13)

$$\tfrac{1}{2}t_i + t_e = \tfrac{1}{2}t_i + T\left(\log \frac{\beta C_e}{2\pi r l K t_i} - \log \theta_e\right)$$

$$= \tfrac{1}{2}t_i + T \log \frac{C_e}{2\pi r l K t_i \phi} \qquad \text{.....(15).}$$

Differentiating, one finds that $(\frac{1}{2}t_i + t_e)$ is a minimum, when

$$t_i = 2T \qquad \text{.....(16).}$$

This means that, during the input period of an experiment, a voltage must be applied to the coil such that the required temperature-rise β is produced during a time equal to twice the time-constant of the coil.

At first sight it might appear that the present method of construction, in which the heat is transferred from coil to calorimeter by radiation alone, would be useless for work at lower temperatures, since by equation (7) K is proportional to the cube of the absolute temperature, and hence at lower temperatures T and t_e will become very large. On the other hand, it must be remembered that if gaseous conduction and heat-leakage along the leads and suspensions are so low that radiation is the predominant factor in heat-transfer, then E of equation (1) will be affected by temperature in the same way as K of equation (8) and hence the correction for heat-loss during $(t_i + t_e)$ will tend to be at all temperatures about the same fraction of the total input.

§4. CONSTRUCTION OF PRESENT CALORIMETER

An upper limit to the dimensions of the calorimeter was imposed mainly by mechanical considerations, since the present form of calorimetry necessitates the enclosure of the calorimeter and certain other apparatus in an evacuated vessel. The vessel here used consists of a pyrex glass tube sealed at one end, and connected to a pump system to be subsequently described. The diameter of the tube, 80 mm., approaches the limits set by the adaptability of glass as a material for constructing apparatus of this type, though the use of metal would, no doubt, permit the adoption of larger dimensions.

The above consideration led to the requirement that the external diameter of the calorimeter should not exceed 60 mm. overall. The winding of the platinum wire coil and the fixing of a radiation-shield of silver foil permitted a diameter of not more than 50 mm. for the actual container of the substance under investigation. This container A, figure 2, consists of a hollow cylinder of mild steel, 50 mm. in diameter by 80 mm. long, turned from a solid rod to a wall thickness of about 0·5 mm. It was constructed with a screw-on lid B, of diameter equal to that of the cylinder, and a neck 6 mm. high by 15 mm. in diameter, closed by a screw-cap C, the former being purely for constructional reasons, the latter for the purpose of filling. In order to simplify certain silver-soldering operations in later stages of the construction, the whole steel container was nickel-plated to a thickness of about 0·05 mm.

It has been pointed out that, for the efficient working of a vacuum calorimeter, the platinum coil must quickly take up the temperature of the calorimeter, and it is therefore desirable in the case where the transfer of heat is by radiation that the coil should be completely enclosed in surroundings which are in good thermal contact with the main bulk of the calorimeter. To this end, the coil which consists of 370 cm. of 38-s.w.g. platinum wire was wound bifilarly in the annular space between the steel container and a case of silver foil 0·1 mm. thick, *D* in figure 2. Thermal contact between this case and the container is effected by means of sixteen 10-B.A. cheese-headed steel screws *E*, which were silver-soldered head downwards to the container, the silver-foil case being held down firmly with nuts.

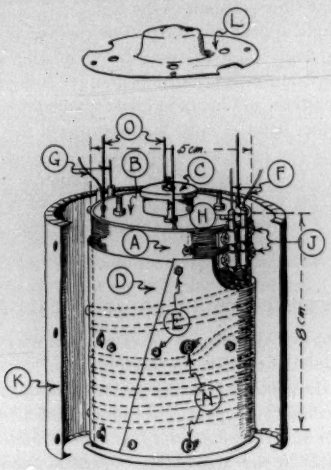

Fig. 2.

The outside of the steel container, the inside of the silver case, and the platinum wire were blackened by coating with Bakelite* varnish, which was carbonized by baking in a furnace at about 350° C. To remove any comparatively volatile substances that might still be present, a further baking was given in vacuo at 400° C. Tests show that the emissivity of the surface thus obtained approaches that of a full radiator, and also that the blackening withstands prolonged heating in vacuo at a dull red without deterioration. Moreover, the high-temperature coefficient of resistance which is characteristic of pure platinum was obtained when the coil was calibrated as a thermometer, which indicates that the blackening on the wire does not impair its quality as a resistance thermometer.

* We have no doubt that other varnishes would be equally satisfactory, and we have, in fact, tested a stove enamel sold under the name of "Namo" (heat-resisting grade) and find it quite suitable.

Electrical contact of the coil with either D or A was prevented by the interposition of thin sheets of mica, though, as a matter of fact, the presence of this mica unduly increases the time-constant of the coil, and will be avoided in future models. To prevent adjacent turns from touching each other, the wire was wound over six narrow strips of asbestos fabric* about 3 mm. wide, placed longitudinally along the container. The wire embedded itself in this tape, and risk of turns touching was eliminated without undue tension having to be applied to the winding. The ends of the coil were fixed by twisting them round pieces of small quartz capillary slipped over thin iron nails soldered head-down to the case.

The leads F to the coil and the compensating leads G consist of 26-s.w.g. platinum wire. They pass through quartz capillary tubing H, which has been coated with a film of silver by chemical deposition and then silver-plated to a thickness of about 0·25 mm. The space between the platinum and the quartz is filled with alundum cement, grade RA 562, and experiment showed that good thermal contact existed between the wire and silver coating. As it was desirable to make the temperature of the lower ends of the leads the same as that of the main bulk of the calorimeter, the tubes were bolted down to the steel container by means of silver-foil straps and the bolts J. In order that it might be possible to test the insulation of the platinum coil, the compensating leads were connected electrically at the lower end to the container. At 300° C. the insulation resistance between the platinum coil and the container was of the order of 200 MΩ.

Figure 2 shows the top L and the sides K of the radiation-shield partly removed from the calorimeter. The former was bolted down to the lid by 5 bolts, and another radiation-shield was fixed to the bottom of the calorimeter in a similar way. The nuts holding them down were shielded with a wrapping of silver foil. Twelve steel pillars N, 4·5 mm. long and 3 mm. in diameter, tapped through the centre and soldered to the container, served to carry the side shield K. The countersunk heads of the screws holding the shield down to these pillars were coated with silver solder to reduce their emissivity.

The calorimeter was hung up by means of the suspensions O, each consisting of 4 cm. of 26-s.w.g. platinum. Provision was made at the top of each suspension for adjustment in three dimensions in order that it might be possible to make the calorimeter hang symmetrically; and for reasons which will appear later, an insulating link in the form of a glass bead was incorporated in each suspension.

Figure 3 shows the calorimeter with radiation-shield in position hanging from the copper disc P. The latter is 13 mm. thick and machined with a screw thread to fit a corresponding thread in the copper cylinder Q. The purpose of the cylinder Q was to meet the requirement that the whole of the environment of the calorimeter, with which it exchanges heat, should be at a uniform temperature. This was ensured by giving the cylinder the wall-thickness of 3 mm., the bottom being closed by a copper disc covered with silver foil and fixed by four nuts and bolts to the

* This fabric was rendered a very good electrical insulator at high temperatures by boiling it for many hours in distilled water to remove soluble salts, and finally heating it to about 600° C. to remove its cotton content by carbonization and subsequent oxidation.

walls. The inside was electro-plated with silver to reduce radiation exchanges with the calorimeter, while the outside was blackened in order that its temperature might be governed by a thermostatically controlled furnace surrounding the pyrex tube in which the apparatus hangs. The copper disc *P* was covered with silver foil on both surfaces as it was undesirable to allow it to radiate appreciably to the cooler parts of the apparatus above. Four holes, *U*, about 1 cm. in diameter drilled in the disc aided the evacuation of the space between the calorimeter and copper

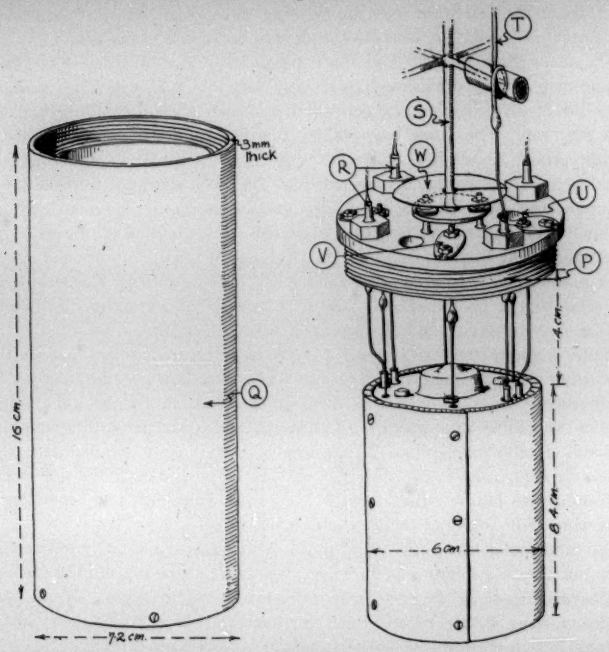

Fig. 3.

cylinder, but silver-foil cowls over them prevented undue increase in the radiation loss. The four leads from the calorimeter were brought through the copper disc by means of four bushings *R*, serving to insulate them electrically but not thermally. The bushings consisted of silvered quartz tubes which were hard-soldered into holes passing through four copper bolts, which, in their turn, were screwed into the copper disc *P*. These bolts together with the suspension adjustors, *V*, were covered with silver-foil shields, which are omitted, however, in the figure.

A steel frame *S* served to support the apparatus mechanically and also to carry the electrical leads to the top of the vacuum chamber. The lower end of the central

rod terminated in a steel plate which was bolted to the copper disc. To prevent a large thermal leak up the rod, mica was interposed between the metal parts, but a fine copper wire joined the frame and the copper disc electrically. This connexion, in conjunction with the insulating links in the suspensions, made it possible to test whether or not the calorimeter was hanging freely inside the copper cylinder. All that was necessary was to test the insulation between the frame and the compensating leads, since the former was connected to the copper cylinder, and the latter to the main part of the calorimeter. 20-s.w.g. copper wire was used for the leads T, which were insulated from the cross members of the frame S by quartz sheaths. W is a circular radiation shield of silver foil.

The top of the pyrex tube containing the apparatus was closed, with the aid of a wax seal, by a copper cap through which the leads were brought out by insulating glass bushings, wax again being used to effect a vacuum-tight seal. From here the compensating leads were taken to a Callendar-Griffiths bridge capable of measuring resistances up to 51 Ω, and the platinum-coil leads were taken to a two-way mercury-contact switch enabling the coil to be connected either to the bridge for temperature measurements or to an electrical supply for the energy-input. The latter was measured with an ammeter and voltmeter. The remainder of the vacuum apparatus consisted of a two-stage mercury-vapour pump, backed at about 1 mm. of pressure by a piston-type oil pump. As any contamination with mercury of the silver radiation-shields enclosing the calorimeter would have increased their emissivity, a mercury-vapour trap, containing sodium-potassium alloy, was placed between the pump and the main apparatus. A Pirani vacuum gauge near the pyrex tube measured the pressure, vacua of the order of 10^{-3} mm. being attained. A gauge of this type is particularly suitable, since it directly measures that characteristic of the residual gas which is peculiarly relevant, namely its thermal conductivity. Although 10^{-3} mm. is a comparatively poor vacuum, little would be gained by improving it beyond this value when the calorimeter is used at temperatures above 200° C., for at such temperatures the loss of heat by gaseous conduction is but a small fraction of the total.

The furnace surrounding the pyrex tube consisted of a thick-walled copper pipe on which was non-inductively wound 100 Ω of 34-s.w.g. nichrome wire, the electrical and thermal insulation consisting of asbestos paper and asbestos wool respectively.

The time-constant of the winding of the calorimeter calculated from equation (2) comes out to be about 3 sec. at a working temperature of 300° C. However, this equation is only meant to give the order of magnitude of this quantity, and assumes in fact that the coil radiates directly to the environment to whose temperature it is required ultimately to fall, without the interposition of any other material. Experiment showed the actual time-constant to be about four times the calculated value, the discrepancy being due, no doubt, to the mica insulation between the coil and the metal surroundings, to the emissivity of the radiating surfaces being somewhat less than that of a perfectly black body, and to the thermal capacity of the strips of asbestos fabric on which the coil is wound.

For 38-s.w.g. blackened platinum wire at a temperature of 300° C., and a heating current of 200 mA. (which is of the order usually employed), θ_0 as calculated from equation (9) comes out to be 25° C.

The heat-leak from the calorimeter at 300° C. was found experimentally to be about $6{\cdot}5 \times 10^{-3}$ cal./sec.-degree. This was of the same order of magnitude as the value obtained by calculating the leak up the leads, the radiation-loss, and the gas-conduction, the results of Heilman* being used to compute the loss due to radiation, and of Soddy and Berry† that due to gas-conduction.

Assuming all this heat to be lost by the side radiation-shield, the maximum temperature-difference which could exist between the ends of the steel pillars carrying the shield was about $\frac{1}{30}$° C., and the maximum temperature-difference between any two points of the silver shield was about $\frac{1}{250}$° C.

The platinum coil had a resistance at 0° C. of 20·6 Ω. It was calibrated as a thermometer at the ice point, the steam point and the melting point of bismuth. For the latter fixed point a temperature of 271·0° C. was accepted from the results of Adams and Johnston, and Mylius, Grosehuff and Holborn‡. The constants α and δ in Callendar's formulae for platinum thermometry had, within the limits of experimental error, the values usually obtained for platinum of high purity. A change of 1° C., the average amount by which the calorimeter was raised in temperature in a heat-capacity determination, corresponded to a change of resistance equivalent to 8 cm. of bridge wire. The time of heat-input was usually of the order of 200 sec.

Assuming a heat-capacity of 0·35 cal./cm^3 (an average value for common metals) the ratio of the thermal capacity of the full calorimeter to the empty calorimeter is about 4 to 1.

§ 5. CALCULATION OF RESULTS

The essential experimental observations necessary for the computation of the heat capacity of the calorimeter are represented graphically in figure 4. The graph (*a*) shows the resistance (and consequently the temperature) of the platinum coil plotted as a function of time; *AB* represents the behaviour of the coil before the heat-input and *CDE* its behaviour after the heating current has been switched off. The portion *CD* corresponds to the state of affairs when the winding is appreciably hotter than the calorimeter, temperature-equilibrium being re-established at *D*. The term "equilibrium" does not mean that the coil and the calorimeter are at the same temperature, as the current required for the bridge-measurements maintains the coil at a somewhat higher temperature than the calorimeter. However, this elevation of temperature is the same for *DE* as for *AB*; thus we can use the curves *AB* and *DE* to deduce the temperature-rise of the calorimeter due to the heat-input, although a correction must be applied to find the mean temperature of the calorimeter during the determination.

* *Trans. Amer. Soc. Mech. E.* **51**, 287 (1929).
† *Proc. R.S.* A, **83**, 254 (1909).
‡ Landolt-Bornstein, *Tables* **1**, 329 (1923).

The dotted lines *BF* and *FD* represent the temperature of the main part of the calorimeter (and hence the outer shield) during the periods t_i and t_e respectively. We make the following assumptions, for which justification has been found experimentally: (*a*) The temperature of the calorimeter rises linearly with time during the time interval t_i; and (*b*) the law of cooling for the portions *AB* and *FDE* is the exponential law of Newton, according to which both curves are asymptotic to the constant-temperature line *GH* corresponding to the temperature Δ of the copper cylinder surrounding the calorimeter. Δ

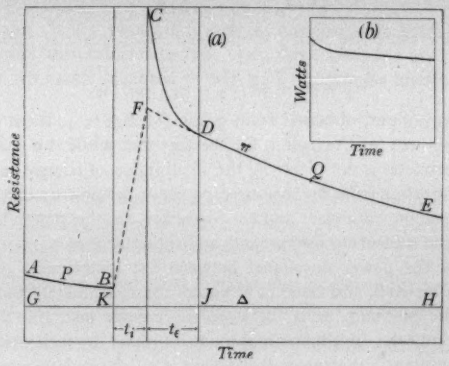

Fig. 4.

These assumptions require the condition, so far as the times t_i and t_e are concerned, that the thermal capacity of the parts of the calorimeter appreciably hotter than the main bulk be small compared with the total heat-capacity, which condition is satisfied in this calorimeter.

Let π be the temperature of the platinum coil, measured from the same zero as Δ. Then π

$$\pi - \Delta = (\pi_0 - \Delta)\, e^{-kt} \qquad \text{.....(17),}$$

Δ and k are found by taking the slopes of the curves at points P and Q well removed from each other in the temperature scale. If S_P and S_Q are these slopes, and the corresponding temperatures are π_P and π_Q respectively, then S

$$S_P = k\,(\pi_P - \Delta)$$

and

$$S_Q = k\,(\pi_Q - \Delta).$$

It follows that

$$k = \frac{S_Q - S_P}{\pi_Q - \pi_P} \qquad \text{.....(18),}$$

and

$$\Delta = \pi_P - S_P \frac{\pi_Q - \pi_P}{S_Q - S_P} \qquad \text{.....(19).}$$

The temperature π_D at D is, on account of the heat lost during the time $t_i + t_e$,

lower than that corresponding to the heat-input by an amount which is given by the product of k and the area $BFDJK$. In practice, the correction for this loss may be divided into two parts, each being calculated algebraically as follows.

The temperature π_F, which the calorimeter would have had if it had lost no heat during t_e is given by

$$\pi_F - \Delta = (\pi_D - \Delta)/e^{-kt_e} \qquad \text{.....(20),}$$

while the correction for the heat-loss during t_i is

$$\tfrac{1}{2}k\,(\pi_B + \pi_F - 2\Delta)\,t_i.$$

Hence the temperature-rise which would have taken place if there had been no heat loss is

$$\pi_F - \pi_B + \tfrac{1}{2}k\,(\pi_B + \pi_F - 2\Delta)\,t_i.$$

To the energy-input, obtained from graph (*b*), figure 4, three corrections are necessary. These are: (*a*) Correction for the fact that while the heating-current is flowing the calorimeter is not receiving the small influx of energy due to the bridge current; (*b*) subtraction from the measured power of the power necessarily liberated in either ammeter or voltmeter; and (*c*) correction for the power liberated in the leads between the measuring instruments and the calorimeter.

The part of the power developed between the instruments and the copper disc is easily computed, and none of it enters the calorimeter; but of the power developed in the platinum leads, between the copper disc and the calorimeter, about half goes into the calorimeter and must, therefore, be reckoned in the energy-input. In the present calorimeter this amounts to about $\frac{1}{6}$ per cent of the total energy-input.

§ 6. CONCLUSION

The design described in this paper is valid up to at least 500° C. and probably higher, though the maximum resistance measurable by the particular bridge used did not allow work above 370° C. on this occasion.

The sum of the greatest errors possible in the measurement of current, voltage and time are estimated at $\frac{1}{2}$ per cent, while those possible in the determination of the temperature-rise, due to inaccuracies in bridge-measurements and the heat-loss correction, do not amount to more than another $\frac{3}{4}$ per cent at the most in the case of the empty calorimeter, where they are likely to be most serious. A series of tests has been made of the heat-capacity of the calorimeter, both empty and full, over a range extending from air temperatures to over 300° C., and the greatest deviation of an individual determination from the mean smooth curve has not exceeded 1 per cent. In order to test the validity of the method of calculating results, determinations were made at the same temperature with varying values of t_i; also, in deducing the temperature-rise, different values of t_e were taken, but no systematic deviation of the final results from the mean curve was found. High accuracy in the determination of the thermal capacity at individual temperatures

is, of course, valueless unless corresponding precision is obtained in measuring the temperatures themselves, but the platinum thermometry is quite adequate for this.

The calorimeter here described is at present being used to investigate the heat-capacities of a series of pure metallic elements at high temperatures.

§ 7. ACKNOWLEDGMENTS

In conclusion, it is a pleasure to express our thanks to Prof. H. Stansfield, in whose laboratory this calorimeter has been developed, for his kind interest and support. We are much indebted to Mr E. E. Mann for kindly drawing figures 2 and 3.

DISCUSSION

Mr J. H. Awbery said that the authors had made a valuable contribution to the equipment of workers engaged on heat problems. He would look forward to seeing the results obtained with various materials.

Mr C. R. Darling asked what method was employed for measuring the temperature of the silver shield, which appeared to be of considerable importance.

Dr Ezer Griffiths. Could the authors give fuller information concerning correction for radiation-loss? Is it assumed that the temperature of the sheet of metal *D*, figure 1, is identical with that of the resistance-thermometer coil? I would suggest that information as to the uniformity of the temperature could be obtained by means of thermo-elements soldered to the surfaces.

Authors' reply. We are grateful to Mr Awbery for his kind remarks about the calorimeter.

With regard to the points raised by Dr Ezer Griffiths and Mr Darling: The temperature of the outer silver shield *D*, figure 1, is assumed to be the same as that of the main part of the calorimeter. The validity of this assumption and of the further one that the temperature of *D* is uniform is supported by the figures for the maximum possible temperature-differences which are quoted in the antepenultimate paragraph of § 4.

The temperature of the main part of the calorimeter during $t_f + t_c$, which is shown dotted in figure 4, is deduced as described in figure 5 of the paper.

A NEW TILTED ELECTROMETER

By HUGH CARMICHAEL, B.Sc., University of Edinburgh

Communicated by Prof. C. G. Barkla, F.R.S., January 15, 1932
Read and discussed March 4, 1932

ABSTRACT. A description is given of the construction and performance of a new electrometer on the principle of the Wilson tilted gold-leaf electrometer. The instrument has a quartz fibre "leaf" which moves in hydrogen to reduce sluggishness. There is complete protection of the instrument from atmospheric disturbance, and considerable simplifications are made in the construction and in the method of adjustment. Typical sensitivities of 3 mm. and 30 mm. fibre movement per volt (linear for 2 mm. and 1 mm. respectively) have been obtained, and a reliable eyepiece scale sensitivity of 1000 divisions per volt is within the range of the instrument.

§ 1. INTRODUCTION

The tilted gold-leaf electrometer was devised by C. T. R. Wilson[1]. It has been investigated and redesigned by G. W. C. Kaye[2], who gives a useful account of its behaviour, and many manipulative details.

The instrument depends for its sensitivity on the fact that a hanging gold leaf, maintained at or near zero potential, can be attracted out of the vertical by an inclined high-potential metal plate and can be brought near to a critical unstable state of deflection by adjusting the tilt of the plate, its proximity to the leaf, and the plate potential. Near this unstable state the deflection of the leaf is very sensitive to change in its potential. The sensitivity curve given by Wilson and by Kaye is of the type shown in figure 1 (*a*). The deflection of the leaf is not a linear function of the leaf-potential except approximately for a short range on each side of the point of inflection. It is of course possible so to adjust the electrometer that the position of the leaf when at zero potential corresponds to the point of inflection on the sensitivity curve, figure 1 (*b*). The electrometer then has a linear range for small positive and negative leaf-potentials.

In this paper an account is given of a new electrometer designed with the object of eliminating the disadvantages which have been found in electrometers made in this laboratory from the drawing in Kaye's paper. The disadvantages are:

(i) The case of the instrument is not airtight so that the leaf is liable to erratic movement if atmospheric conditions in the laboratory are not steady.

(ii) The electrometer is not protected from draught and stray heat radiations which disturb the uniformity of the temperature of the case and cause convection in the air inside it, while a protecting box makes adjustment of the instrument troublesome.

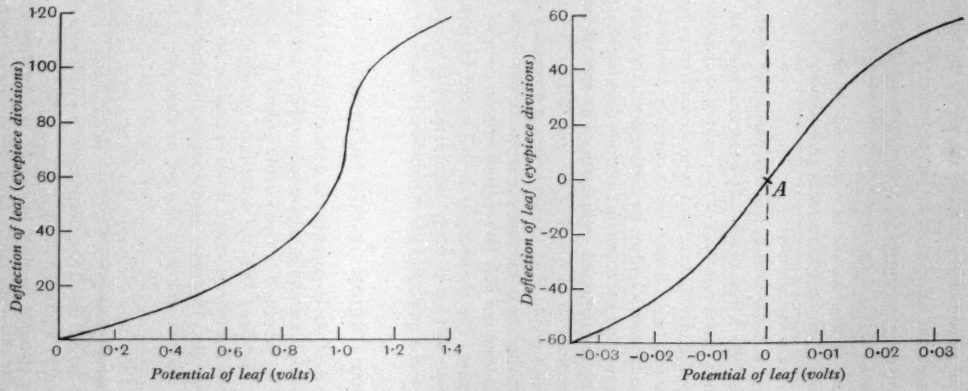

Fig. 1. Typical sensitivity curves.

(iii) The gold leaf itself is not satisfactory, as a sudden or too great deflection often causes a change in its shape which affects the zero reading and the sensitivity. Moreover the leaf is not a good object to focus sharply in the telescope.

(iv) The leaf is sometimes liable to swing out of focus.

(v) At high sensitivity the leaf moves very slowly through the air in the chamber to a new position of rest.

(vi) The instrument is troublesome to adjust and the degree of skill required has always been a serious drawback to its more general use.

These will be referred to by number.

The new instrument is being used as a detector in a null method of comparing the ionizing powers of two X-ray beams. It is connected to an insulated zero-potential electrode which passes into each of two ionization chambers. The cases of the chambers are at approximately equal and opposite saturation potentials, so that the ionization currents due to the beams are in opposite directions with respect to the electrode. They can thus be equalized by cutting down the more powerful beam till the electrometer shows that the common electrode does not change rapidly from zero potential on being disconnected from earth. The rate of leaf-movement is measured for several settings on each side of the true balance point, and the latter found by plotting a rough graph. With the apparatus at present in use, which has a variable aperture controlled by a micrometer screw to cut down one beam, the determination of a balance point may be repeated with an error of less than 0·0001 of the maximum aperture. The electrometer is well suited for making those measurements as it is compact and has an easily varied sensitivity with a high maximum value, a low capacity, and a quick and linear response to small potential changes.

§ 2. DESCRIPTION OF THE ELECTROMETER

The case A, figure 2, of the electrometer is a short circular cylinder with plane ends. It is made of thick brass and is supported by the bearing B inside a larger cylinder C of brass. The bearing B is tapered and ground to fit, and is made gas-tight with tap grease. The front D of the outer case, on which the telescope E is supported, is removable, but when screwed down in position so as to compress the rubber gasket F is gas-tight. The outer case protects the electrometer from non-uniform temperature changes and from changes in atmospheric pressure; cf. (i), (ii) above.

To the outer end of the supporting axle B is fitted a lever G which is moved by a worm screw S_1 and serves to alter the tilt of the electrometer. It is arranged that the point of support of the leaf inside the electrometer is on the axis of the bearing B, so that it is not affected by alteration of the tilt.

The telescope E is clamped on to V blocks V_1, V_2 fixed to a rod R which is supported by bearings M, N on the continuation of the axis of the bearing B. Thus the telescope is always perpendicular to the plane of movement of the leaf, and swings in an arc about the point of suspension of the latter. Movement of the telescope by

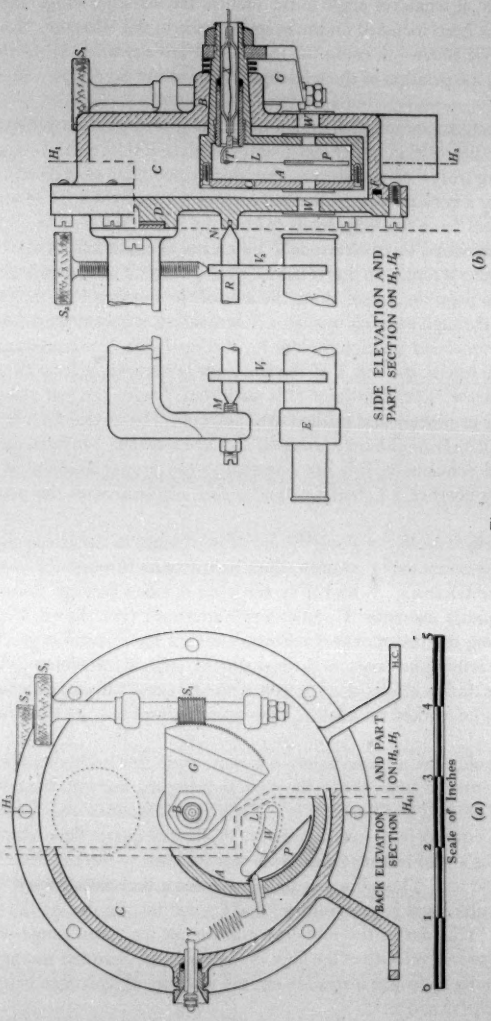
SIDE ELEVATION AND PART SECTION ON H_3, H_4
(b)
BACK ELEVATION AND PART SECTION ON H_5, H_6
(a)
Scale of Inches
0
1
2
3
4
5

Fig. 2.

a single screw S_2 brings the image of the same part of the leaf on to any eyepiece scale-division, at whatever angle to the vertical the leaf may hang. After the image of the leaf has been focussed on the eyepiece scale in the telescope, the only adjustments—cf. (vi) above—in using the instrument are alteration of (*a*) the tilt by the screw S_1; (*b*) the position of the telescope by the screw S_2; (*c*) the value of the plate voltage.

In the electrometer itself, of which the front Q of the case is removable and not gas-tight, the disposition of the leaf L and the plate P is shown. To insure that the leaf will swing truly—cf. (iv) above—the broad tilted plate of Wilson's electrometer is replaced by a very narrow plate consisting of the straight edge of a piece of thin brass. The field from this edge tends to keep the leaf in the middle of the case. The leaf itself is attached to an electrode T by means of small knife-edge[3] jaws closed by a watchmaker's screw, so that it has a definite hinge. The electrode, which should be as small as possible, passes along the axis of the bearing B to the outside of the electrometer through a quartz insulator X into which it is cemented. A compressed rubber gasket makes a gas-tight holder for the insulator. For convenience in fitting and replacing leaves, the end T of the electrode is removable from the part passing into the insulator. The snap joint U is such that T with the leaf attached may be held in a pair of pincers and pushed into position. The device for varying the distance of the plate from the leaf, provided in Kaye's design, has been dispensed with since, though convenient, it is not essential to the proper working of the electrometer. In its absence a potentiometer for fine adjustment of the plate voltage is necessary.

Narrow windows W cut along the arc of movement of the telescope, and closed with thin glass cemented in position, allow light to pass through the instrument past the leaf to the telescope. A lead from the plate is taken through the outer case on a gas-tight quartz insulator Y. Inlet and outlet taps (not shown in drawing) are fitted for filling the instrument if necessary with a more suitable gas than air (see below). An earthing-key also, on the instrument itself, is convenient. An important feature of the design of this electrometer is the fact that the metal castings are such that they can be worked on a lathe. This greatly simplifies the construction of the instrument.

With a gold leaf approximately 0·5 mm. wide the instrument was found to require a potential of 260 volts on the plate to bring the leaf into the critical state of deflection. The gold leaf however was not very satisfactory and it was decided to replace it by a quartz fibre—cf. (iii) above. A silvered quartz fibre which must be at least as light as a gold leaf 0·25 mm. wide was attached to the electrode by means of a hinge of gold leaf. This fibre was found to form a very satisfactory "leaf" and it provided the additional great advantage that a potential of only 80 volts was required on the plate. This makes for economy and for an increased range of sensitivity. Finally the terminal velocity of the fibre in the gas in the chamber was approximately doubled by using hydrogen instead of air, the viscosity of hydrogen being about half that of air—cf. (v) above.

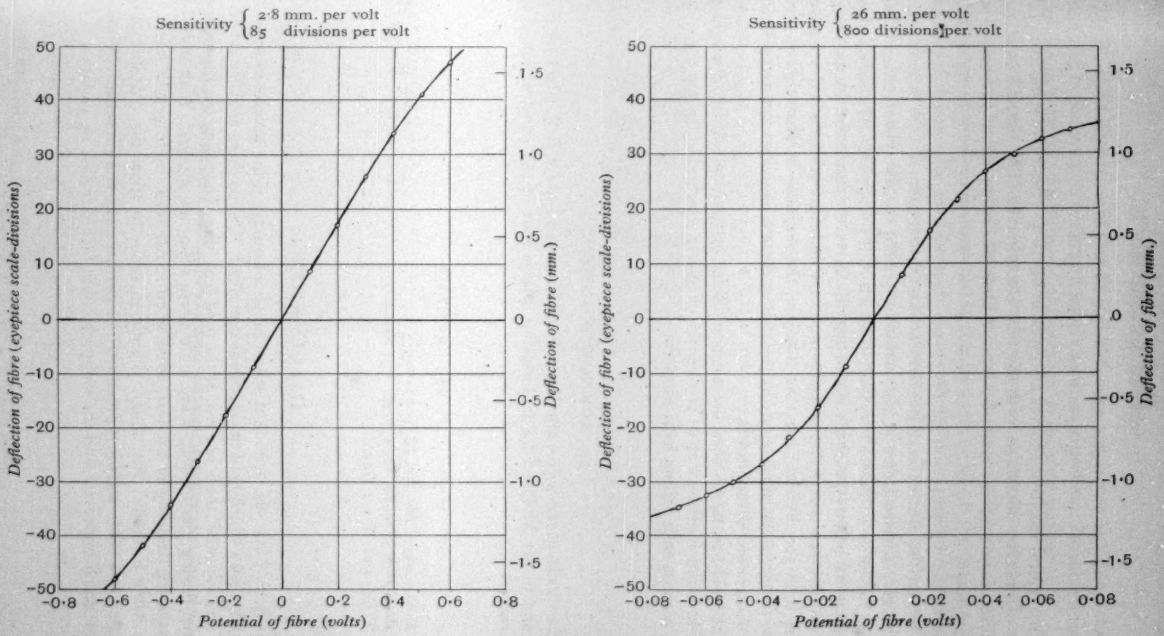

Fig. 3. Calibration curves showing range of linearity at 2·8 and 26 mm./volt.

§ 3. PERFORMANCE

Two typical sensitivity curves are given in figure 3. These give some idea of the range of linearity obtainable with the new electrometer. The magnification is 30 divisions of the eyepiece scale to 1 mm. of fibre movement, about one-half of that used by Kaye.

No erratic movements of the fibre were observed even after handling the outer case, so that the protection of the electrometer is adequate.

The only adverse factor at high sensitivity, given that the plate voltage is constant, is found to be the viscous drag of the gas in the chamber. At a sensitivity of 1000 divisions per volt, with the above magnification, the time taken by the fibre to come to a new position of rest is over 2 minutes, even with hydrogen in the chamber. At less than 500 divisions per volt, however, the response is relatively rapid, and such is the precision of the electrometer that at these sensitivities the magnification may be increased several times.

The gold-leaf hinge does not seem to exert undue control, but it might be possible to replace it by a short cross-piece of quartz attached to the top end of the fibre, with its ends rolling in U supports.

§ 4. ACKNOWLEDGMENTS

The thanks of the writer are due to Prof. Barkla and to Dr R. J. Clark for their help, and to Mr Mollison for undertaking a large part of the construction of the instrument.

REFERENCES

(1) *Proc. Camb. Phil. Soc.* **12**, 135 (1903).
(2) *Proc. Phys. Soc.* **23**, 209 (1911).
(3) *J. Sci. Inst.* **5**, 113 (1928).

DISCUSSION

Mr S. B. Fulford. I think it would be an advantage if the precise form of the knife-edge jaws which hold the leaf were described. Unless these are very carefully made and adjusted, there is a risk of the gold-leaf hinge becoming cut through, by repeated flexing, particularly if the knife edges are at all opposed. With regard to the excessive damping of the leaf, even when it moves in hydrogen, it might be worth while to modify the design slightly in order to allow the chamber to be evacuated to a moderately low pressure. The performance at high sensitivity would then be much improved, and the damping could be controlled by variation of the pressure.

Mr W. V. MAYNEORD said that he had had some experience of Wilson electrometers used in conjunction with small γ-ray ionization-chambers. He recommended evacuation as a means of getting rid of ionization in the electrometer.

AUTHOR'S reply. By knife-edge jaws I do not mean that the edges are in the least opposed. The leaf is held between the flat surfaces of a slot in the electrode, and the edges of the slot are sharpened so that no metal to which the leaf can easily adhere is left near the hinge. I have of course considered the possibility of reducing the damping of the fibre by evacuating the electrometer. This may be done without modifying the design. Since, however, the viscosity of a gas is independent of the pressure until comparatively low pressures are attained, a pump capable of producing a pressure of at least 10^{-2} mm. of mercury would be necessary. For ordinary uses, since the instrument may be filled with hydrogen by displacement, this would be an expensive complication.

I think that a moderate evacuation, especially of the hydrogen-filled electrometer, would be a very effective method of reducing the ionization due to radiation passing through the electrometer itself.

ON THE SYMMETRICAL MODES OF VIBRATION OF TRUNCATED CONICAL SHELLS; WITH APPLICATIONS TO LOUD-SPEAKER DIAPHRAGMS

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ABSTRACT. It is shown that in general the stresses in a vibrating conical shell are so complicated that the problem is unsuited to analytical treatment. If the stress were purely extensional, the frequency would be independent of the thickness t and vary inversely as the major radius a. This, however, would necessitate a much larger potential energy at the small end than practice indicates. The expression for the frequency of a shell of constant minor radius is of the form $f = kt^{n_1} a^{-n_2}$ where, for certain conditions, $n_1 = 0{\cdot}22$ and $n_2 = 0{\cdot}67$.

Experimental work with paper, glass and aluminium shells is described. The modes crowd together as compared with the segregation which occurs in the case of a disc. With thick glass or aluminium of comparatively low loss, the nodal frequencies are very clearly-defined peaks. In the case of paper cones driven by coils of small mass, the peaks disappear and the nodal region is indicated by a broad rounded contour. The influence of thickness, apical angle and the mass of the driving coil is considered. Vibrations of the air column within the shell and the general requirements for loud-speaker diaphragms are discussed and illustrated by practical examples.

§ 1. INTRODUCTION

THERE are two salient types of vibration pertaining to truncated conical shells: (*a*) radial modes as in a bell, (*b*) symmetrical modes as in a disc. Radial modes of the shell are associated with bending, whilst the symmetrical modes depend upon bending and extension. This paper deals exclusively with the latter modes, more particularly in their application to loud-speakers of the hornless type, where radial modes are invariably of secondary importance.

So far as the vibrational modes of cylindrical and spherical shells are concerned, one has only to turn to the works of the great applied-mathematicians to find an answer. But the vibrational modes of conical shells are secrets closely guarded from the inquisition of modern analytical methods. This is indeed unfortunate, for although one can obtain an answer empirically in any special case, the apparatus involved is costly and the time vastly in excess of that required for a mere computation. As the matter stands at present, it appears that to establish a general empirical formula embodying the two radii, apical angle, thickness, and Poisson's ratio for various edge conditions, a concentrated or mass attack would be required by a number of investigators. This would undoubtedly be a protracted and laborious process.

The present paper is mainly an account of a number of experiments in this direction, and the general consequences of the results in modern acoustic apparatus.

Although a general empirical formula has not been constructed, the information is adequate to predict the main vibrational frequency band of conical shells whose structure and dimensions obtain in modern horniess moving-coil loud-speakers.

Only a free-edge condition has been considered throughout the paper, since this agrees closely with the elastic suspension of a loud-speaker cone so far as the main vibrational modes, namely 2000 to 3500 ~, of the shell per se are concerned.

§ 2. THEORETICAL CONSIDERATIONS

The principal stresses involved in any form of shell are bending and extension, positive and negative. When the vibrations are associated mainly with one or other of these stresses, it is usually possible to determine the frequencies analytically. The analysis, however, becomes very formidable when both types of stress are of primary importance*.

So far as the bending of a shell of revolution is concerned, the frequencies depend directly upon the thickness and inversely upon the square of the radius, e.g. for the flexural modes of a cylindrical ring $f \propto ta^{-2}$, where t is the thickness and a the radius. If the governing stresses are tension and compression, the frequency is independent of the thickness but varies inversely as the radius, e.g. for the extensional modes of a cylindrical shell $f \propto a^{-1}$. f, t, a

It is clear that in an extremely thin conical shell the main factor is bending, the circumferential tension or hoop stress being of secondary importance. Consequently, if the radii and apical angle are constant, the frequency ultimately decreases with the thickness without limit. When the thickness exceeds a certain value, the circumferential stress becomes of comparative importance, and the frequency increases slowly with thickness to an upper limiting value—within the limitations of the case, since the thickness must always be finite.

For the thickness of paper and the apical angles used in the construction of loud-speaker diaphragms, there is both bending and circumferential stress, the latter being of considerable importance. Consequently, one would expect an expression for the frequency of vibration to be of the form $f \propto t^{n_1} a^{-n_2}$ where $n_1 < 1$ and $n_2 > 1$. n_1, n_2

In practice, however, there are three essential conditions which complicate the problem. Firstly, the radius of the driving-coil for any class of speaker is fixed; secondly, there is the mass of this coil; and thirdly, there is the former on which the coil is wound and the degree of stiffness it imposes at the minor radius of the cone. If the coil former is several times the thickness of the cone, the extensional deformation at the joint will be reduced appreciably.

In practice, therefore, we are concerned not merely with the case of a pure conical shell, which in itself appears to defy mathematical analysis, but with the more complicated structure due to the addition of a driving coil. In the experimental work described herein, the coil formers were of the same order of thickness as the paper

* Rayleigh, *Sound*, **1**, 395 (1894); E. Spenke, *Wissenschaftliche Veröffentlichungen Siemens-Konzern*, **10**, 128 (1931).

cones, so that considerable alteration of frequency would not be expected from this source. A greater source of disturbance is the mass of the coil, but its influence can be estimated by extrapolation.

These disturbing factors modify the values of the indices n_1 and n_2. Apart from the coil-mass etc., the influence of keeping the minor radius constant, instead of making it proportional to the major radius, is to make n_2 less than unity. As the major radius a is increased, the frequency decreases more slowly owing to the enhanced stiffness near the apex. The stiffness resides mainly in this region. This follows from the fact that the potential energy per unit mass of shell required to cause a definite deformation increases with decrease in radius.

If in a cone complete to the vertex it is *assumed* that the stress is purely circumferential*, the frequency varies inversely as the radius. This is easily proved as follows. Taking the vertex as the origin, the axis being coincident with that of y,

x, y let the radial extension be given by $\triangle x = \phi(y)$. Then the kinetic energy is

A_1, B_1 $A_1 \int_0^h \phi^2(y)\, y\, dy$ and the potential energy is $B_1 \int_0^h \frac{\phi^2(y)}{y}\, dy$ where A_1 and B_1 are con-

h, m stants and $h = ma =$ axial length of shell.

Thus the expression for the kinetic energy contains terms whose order in h and therefore in a exceeds that for the potential energy by two (a^2). Consequently, in finding f by equating the kinetic energy to the potential energy, the factor a must occur in the denominator.

If we take the first nodal circle to be $0{\cdot}8a$†, assume a deformation curve and proceed on these lines, we can obtain an expression for the frequency which gives values considerably in excess of those found by experiment.

For the thick‡ conical shells described herein, the vibrational frequency corresponding to one nodal circle *happens* to be given approximately by $f = \frac{m}{2\pi a}\sqrt{\left(\frac{E}{\rho}\right)}$

a, m where a is the major radius, $m = \cot \psi/2$, and ψ is the apical angle.

In other words, the frequency is nearly m times that of the purely radial mode of a cylindrical ring whose radius is equal to that at the base of the cone. As a case in point, take the glass cone of § 4, where

$$a = 12{\cdot}7 \text{ cm.}, \quad \psi = 107^\circ, \quad m = 0{\cdot}74, \quad \sqrt{(E/\rho)} \doteqdot 5 \times 10^5 \text{ cm./sec.}$$

Thus $f = 4630 \sim$ whilst the experimental value is $4500 \sim$. In the case of a 90° aluminium cone of radius 19 cm., the values are $4260 \sim$ and $4400 \sim$ respectively§. In the latter case, the frequency of a cylinder having the same radius as the truncated end is $4260 \times 19/2{\cdot}5$ or $3{\cdot}24 \times 10^4 \sim$. If the vibration were purely radial, the frequency of the conical shell would obviously be much in excess of the observed value, owing to the large potential energy in the vicinity of the minor radius. Consequently, bending, etc. cannot be neglected.

* Actually this cannot exist alone since it implies additional stresses.

† Table 1, column 2; a = radius of base.

‡ For very thin shells the formula would not be so accurate.

§ The values of $\sqrt{(E/\rho)}$ have been taken from physical tables for both of the preceding calculations.

§ 3. EMPIRICAL FORMULA FOR VIBRATIONAL FREQUENCY

In a former paper* it was found that the main frequency of a series of 90° paper cones of varying major but constant minor radius and thickness, driven by a coil of mass 7·8 gm. was given by the expression $f = k_1 a^{-0.67}$.

Starting with a very thin shell of constant radii, the frequency increases rapidly with the thickness at first, but ultimately settles down to an almost steady value. Over a certain range of thickness the frequency is given by $f = k_2 t^{0.22}$.

If the action were one of pure bending, the index of t would be unity. The departure therefrom is caused mainly by circumferential stress, which in practical diaphragms is at least as important as bending. The index depends upon various factors such as coil mass, but it usually lies between 0·5 and 0·8 for cones used as loud-speaker diaphragms.

§ 4. SYMMETRICAL MODES OF VIBRATION

When an annular disc is driven quite symmetrically by a circular coil, the nodal figures corresponding to the centre-moving modes are concentric circles. With an annulus having inner and outer radii of 2·5 and 12·2 cm. respectively, the ratios of the first three modes are roughly 1 : 4 : 9, provided the mass of the driving coil is negligible. In a practical case the latter condition would only be realized approximately, the corresponding ratio being $1 : p : q$ where $p < 4$ and $q < 9$.

Owing to the different nature of the stresses in a disc and in a conical shell, the nodal ratios of the one bear no resemblance to those of the other. Accurate inferences cannot be made regarding these ratios from experiments on paper cones, owing to four pertinent factors: (i) heterogeneity, (ii) internal transmission loss, (iii) radiation loss, and (iv) low density of paper. By taking a heavy conical cast glass shade illuminating results can be obtained. Figure 1 (*a*) shows three resonances of an ordinary glass lampshade. These were obtained in a large highly damped room with a special microphone situated on the axis of the shade at a distance of about 180 cm. To avoid the influence of standing waves, the note supplied to the driving coil was varied $\pm$ 50 ~ continuously throughout the range of frequencies. The resonances were so sharp that the frequencies could readily be determined by ear, although actually the output curve was recorded. At each frequency the nodal figures were found and data relating thereto are given in the table.

Table. Data for glass cone.

Major radius a of shell = 12·7 cm., $\psi = 107°$, radius of driving coil = 2·5 cm., minor radius b = 1·4 cm., thickness $t \doteqdot 1{\cdot}65 \times 10^{-1}$ cm. Mass M_c of driving coil = 4 gm.

Nodal pattern	Radii of nodal circles	Frequency (~)
One circle	$0{\cdot}8a$	4500
Two circles	$0{\cdot}6a$, $0{\cdot}92a$	5700
Three circles	$0{\cdot}44a$, $0{\cdot}78a$, $0{\cdot}95a$	7500

* *Phil. Mag.* **12**, 771 (1931).

Although nodal circles are specified, neither the drive nor the cone was sufficiently symmetrical or homogeneous to obtain perfect circles.

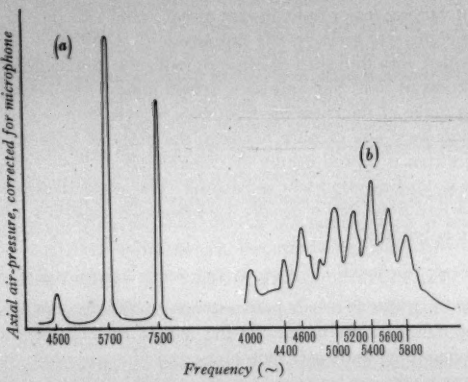

Fig. 1. (*a*) Axial air-pressure curve for free-edge conical glass shell corrected for microphone characteristic.

(*b*) Axial air-pressure curve for free-edge spun conical aluminium shell corrected for microphone characteristic. $a = 19$ cm., $b = 2{\cdot}5$ cm., $t = 4{\cdot}5 \times 10^{-2}$ cm., $\psi = 90°$, $M_c = 6{\cdot}$gm.

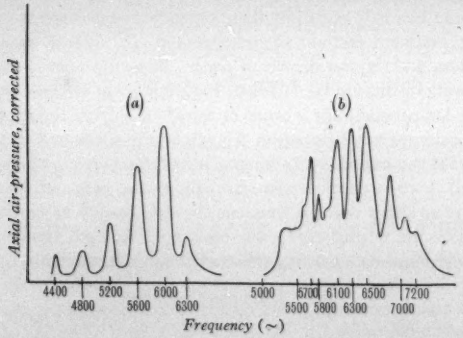

Fig. 2. (*a*) Axial air-pressure curve for free-edge spun conical aluminium shell as in figure 1 (*b*); but $t = 8 \times 10^{-2}$ cm.

(*b*) Axial air-pressure curve for free-edge conical aluminium shell with seam (seccotined). $a = 10{\cdot}9$ cm., $b = 2{\cdot}5$ cm., $t = 7{\cdot}5 \times 10^{-2}$ cm., $\psi = 90°$, $M_c = 1{\cdot}3$ gm.

Whereas the first mode is by far the most powerful with an annular disc*, it is the second mode which is the more powerful in the above case. There may have been

* *Phil. Mag.* **12**, 115 (1932).

modes above 7500 ~, but the apparatus did not permit investigation beyond 9000 ~.

Experiments have also been conducted with aluminium cones, and some of the results are portrayed in figures 1 (*b*), 2 (*a*), (*b*). There are more than three peaks, but as the two thick aluminium cones were spun, whilst the thin one had a seam, the homogeneity cannot be expected to be as good as that of *annealed* cast glass. It should also be observed that here the second peak is by no means the maximum.

As a matter of interest and comparison, curve 1 of figure 3 shows the behaviour of a typical paper cone. The preliminary notch on the curve at 1800 ~ is an indication of the first mode. There are signs of others, whilst the main mode is fairly clearly

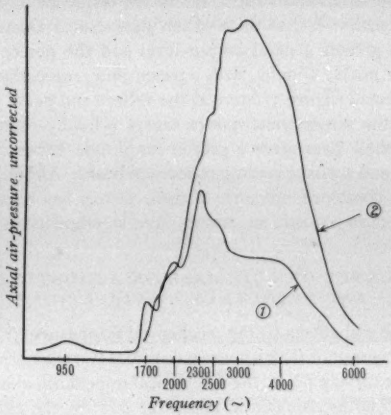

Fig. 3. Curve 1: Axial air-pressure curve for free-edge conical paper shell uncorrected for microphone characteristic.

$a = 12$ cm., $b = 2{\cdot}5$ cm., $t = 2{\cdot}1 \times 10^{-2}$ cm., $\psi = 90°$, $M_c = 7{\cdot}8$ gm.

Curve 2: as for curve 1, but $M_c = 2{\cdot}7$ gm. The reduced coil-mass results in increased and more uniform output above 2300 ~.

shown. But the modes of paper and aluminium do not stand out in the same conspicuous manner as those of the glass shade. This applies particularly in the case of paper when a light driving-coil is used.

The occurrence of the modes in cluster formation, as compared with the segregation that obtains in the case of a disc, is very striking. This is due to the different nature of the two vibrational systems. When the frequencies of the various modes cannot be definitely allocated, we shall refer to the frequency corresponding to the greatest output. At frequencies exceeding that of the main mode the output falls away, doubtless owing to losses.

§5. INFLUENCE OF THICKNESS OF SHELL ON ACOUSTIC-OUTPUT CURVE

When a curve delineating the acoustic output is taken on the axis of a coil-driven conical shell of given radii and apical angle, the profile depends upon the thickness and mass of the diaphragm. The general effect is clearly exhibited in figures 1, 2 and 3, which we shall now analyse. The curve for the thick heavy glass shade consists of three precipitous peaks separated by deep valleys. Owing to its great mass the general output-level is almost evanescent and it is only at resonant points, where the mass is offset by elasticity, that the output is readily audible. The thickest of the three aluminium shells shows the influence of considerable mass, but the valleys are shallower than those of the glass shade. Decrease in thickness is accompanied by a greater general output-level and the perceptible rising of the valleys towards the peaks. Finally, with a paper cone, especially when the mass of the driving-coil is small (figure 3, curve 2) the valleys and peaks coincide. Nothing is left to indicate the symmetrical modes except a boldly-rounded contour. The reduced coil and shell mass gives a greater amplitude between peaks, whilst the frictional damping and acoustic loading reduce the peaks. Although not fully understood hitherto, the combined influence of these factors has been invaluable in the construction of modern acoustic apparatus. And so empiricism has been justified.

§6. INFLUENCE OF COIL-MASS ON ACOUSTIC OUTPUT AND FREQUENCY OF VIBRATION

The mass of the coil affects (i) the mechanical impedance, (ii) the amplitude of vibration, (iii) the contour of the response curve, and (iv) the frequency of vibration.

ω

B, M_e

Referring to the driving point, the mechanical impedance at a frequency $\omega/2\pi$ is $B + j\omega M_e$ where B is the mechanical resistance and M_e the effective mass of the complete structure which may be positive, negative or zero. Since the coil can be considered as a rigid structure when vibrating axially, M_e can be written

M_e', M_e

$M_e = M_e' + M_c$, where M_e' refers to the shell and M_c to the coil. The value of M_e' is a small proportion of the natural mass in the nodal region*, so that the impedance is reduced appreciably when a coil of small mass is used. It follows that for any given driving force the amplitude of vibration and, therefore, the output increases. In fact, to preserve a good balance between the upper and lower registers, the mass of the coil must lie within prescribed limits. The latter are, of course, determined experimentally and depend to an extent upon the taste of the listener. If the mean frequency of the main nodal group is too low, the reproduction is woolly and lacks interpretational qualities. When it is too high we are wafted back to the juvenile days of playing tunes with paper and a comb. For any particular diaphragm there is a certain coil mass giving maximum output. The mass, however, varies with frequency owing to inconstancy of the diaphragm impedance†.

* *Proc. Phys. Soc.* **44**, 88 (1932).

† Letter to *Wireless Engineer*, p. 151, March 1932; *Proc. Phys. Soc., loc. cit.*

The profound influence of coil-mass on the contour of the axial pressure curve, and therefore upon the frequency characteristic, is displayed in figure 3. Whereas in curve 1 of figure 3, with a 7·8 gm. coil the notches and peaks are an indication of "modes," the same cannot be said of curve 2, where the coil-mass is 2·7 gm. The increased amplitude, sound-radiation and transmission loss concomitant with reduced mass-reactance culminates in a substantially uniform output-level over a certain frequency band. In fact, from an axial pressure viewpoint we have a good mechanical band-pass-filter effect.

So far as the influence of mass on the frequency of vibration is concerned, we cannot derive much inspiration from circular discs, since the vibrational conditions are dissimilar. One usually associates added mass with a reduction in frequency, especially when the mass does not contribute to the stiffness of the vibrator. Since

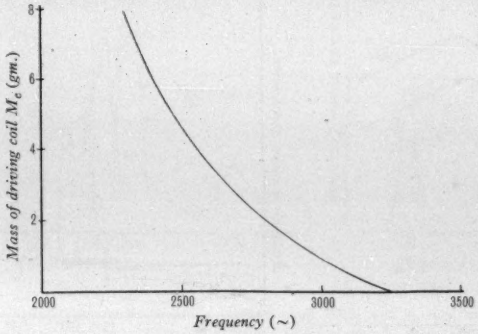

Fig. 4. Curve showing influence of coil-mass on frequency of main symmetrical mode of cone used for figure 3.

the modes of vibration are partly extensional, the influence of mass is less prominent than it would be under a bending regime.

To illustrate this feature a series of experiments was performed on a coil-driven shell, the frequency of the combination being determined for various coil-masses. Variation in mass was accomplished by the simple expedient of removing turns from the coil until a practical limit was reached. The results are plotted graphically in figure 4. By extending the curve until it intersects the horizontal axis, the frequency for a coil of zero mass is obtained. Owing to the stiffness imposed by the cylindrical paper former, this value may be on the high side. The frequency does not alter seriously with reduction in coil-mass until the latter is quite small.

In the ideal case of a lossless disc driven in vacuo where the coil does not contribute any stiffness, the influence of variations in coil-mass can be found by the simple expedient of shifting the frequency axis to the point where $M = M_c$, M_c being the coil-mass. Owing, however, to variation in acoustic loading and transmission loss with frequency, this artifice cannot be accurately applied to paper

cones. This will be evident from figure 3, which shows the influence of variation in coil-mass.

§7. INFLUENCE OF APICAL ANGLE ON FREQUENCY OF VIBRATION

The effect of the apical angle is exhibited graphically in figure 5. The rise in frequency with reduction in ψ is quite rapid from 180° to 100°. Beyond this the rise is curbed, whilst below 90° it falls away very slowly indeed. The general appearance of the air-pressure curves for various angles is shown in figures 3, 6, 7, where the resonances are in the neighbourhood of 2000 ~.

The relative power-output throughout the frequency range is not given by the ordinates of the axial air-pressure curves. This is due to (i) the microphone cha-

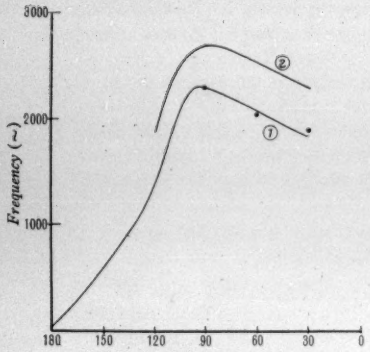

Fig. 5.

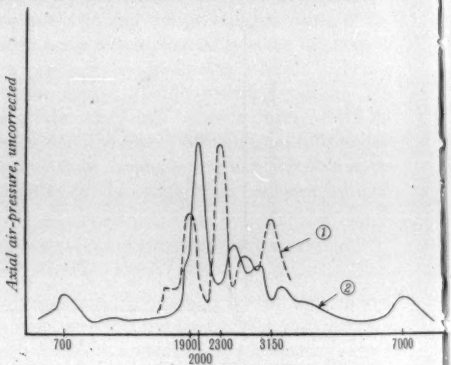

Fig. 6.

Fig. 5. Curves showing variations with apical angle in frequency of main mode.
$a = 12$ cm., $b = 2{\cdot}5$ cm., $t = 2{\cdot}1 \times 10^{-2}$ cm. Curve 1, $M_c = 7{\cdot}8$ gm. Curve 2, $M_c = 2{\cdot}7$ gm.

Fig. 6. Axial air-pressure curves, for free-edge conical paper shell with ψ equal to 60°, other dimensions as for figure 5, curve 1. Curve 1, with re-entrant cone 2·5 cm. in radius at apex. Curve 2, without re-entrant cone. The resonance at 700 ~ is due to the air column within the cone.

racteristic not being uniform; (ii) the current through the driving unit not being constant, since constant voltage was applied to the grid of the power valve; (iii) the spatial distribution of sound not being spherical*; (iv) variation in interference on the axis according to the number of nodal circles, i.e. the dynamic deformation curve of the shell. This is immaterial since the problem of the moment concerns the *frequencies* of vibration. In curve 2 of figure 3, where the frequency is indeterminate, an impulse method is used†.

* *Wireless World*, March 23, 30 and September 21 (1927); *Proc. R.S.* A, **122**, 604 (1929); *Phil. Mag.* **7**, 1026 (1929).

† *Wireless World*, April 3, 1929; *Phil. Mag.* **11**, 48 (1931); **12**, 140 (1932).

It has already been shown that when ψ is 90° the main frequency is clearly defined with a heavy coil, but loses definition when a light coil is used, figure 3. As the angle decreases, the definition is much improved, as will be seen from figure 6 for $\psi = 60°$. Two curves are given, one of which (curve 1) applies when there is a re-entrant cone at the minor radius. An auxiliary resonance is introduced at 1900 ~ and the main peak is shifted to 2300 ~. In curve 2 there is a series of minor resonances at 7000 ~.

The change caused by alteration in coil-mass is indicated in figure 7, where $\psi = 30°$. The main frequency is raised and the output enhanced, also the mode before the main peak becomes more prominent. The acuteness of the main peak

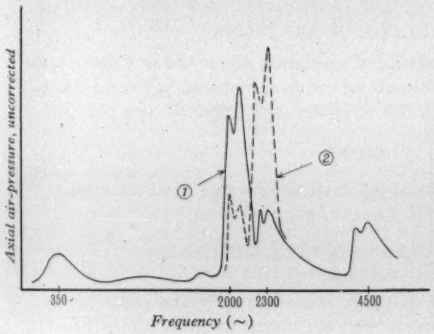

Fig. 7.

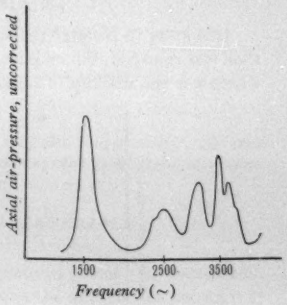

Fig. 8.

Fig. 7. Axial air-pressure curves for free-edge conical paper shell with ψ equal to 30° and other dimensions as figure 5, curve 1. Curve 1, $M_e = 4{\cdot}4$ gm. Curve 2, $M_e = 2{\cdot}7$ gm. The resonance at 350 ~ is due to the air column within the cone.

Fig. 8. Axial air-pressure curve of free-edge cylindrical paper shell. The output was very small and considerable amplification was required to obtain the record. Radius = 2·5 cm., $t = 2{\cdot}1 \times 10^{-3}$ cm. (with seccotined seam) length 56 cm. Driven by coil of 7·8 gm.

is in striking contrast with figure 3, where $\psi = 90°$. Moreover, a 90° cone is better suited for a loud-speaker than one of 30°. It is inferred that, owing to the small angularity of a 30° cone, bending is relatively of less importance and the transmission loss is reduced.

When the angle becomes appreciably greater than 90°, the main modes occur at too low a frequency to be of much value for loud-speaker work. This ought to be self-evident from figure 10, where $\psi = 135°$. It so happens, therefore, that for general reproduction an angle about mid-way between a disc and a cylinder gives the best results. I came to these conclusions regarding the angle ψ over seven years ago, entirely by aural observation. The scientific corroboration of one's sense of musical values is quite reassuring.

The slow change in frequency with reduction in ψ below 90° arouses curiosity regarding its ultimate value when ψ approaches zero and the axial length tends to

infinity. From the aspect of a nodal circle, the frequency ought also to be zero; but when a certain angle is reached, the cone becomes long enough for relatively important longitudinal vibrations to occur. It might be thought that the acoustic energy associated with such vibrations is very small. This is undoubtedly true, but the extension and compression of the shell give rise to an important auxiliary effect. By virtue of lateral expansion and contraction (Poisson's-ratio effect) portions of the cone vibrate radially and this augments the general output. Tests on a paper cylinder were made with the result given in figure 8. So far as could be ascertained by ear, a large portion of the sound was generated in this manner. Thus at a longitudinal node there is a radial antinode.

§ 8. AIR-COLUMN VIBRATIONS

In a letter to *Nature** the occurrence of resonances due to the air column in the shell was reported. An empirical formula for the main frequency is $f = c/2\,(l + ka)$,

c, l where c is the velocity of sound in free air, l the axial length of cone plus part of

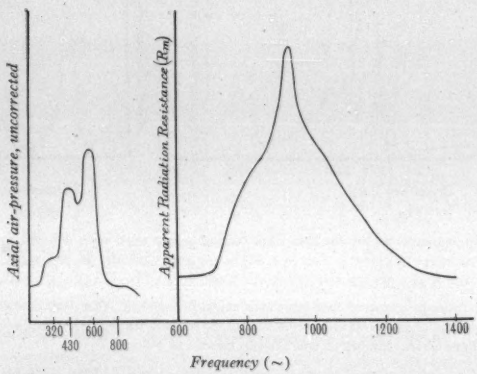

Fig. 9. (*a*) Axial air-pressure curve for cone of figure 1 (*b*) corresponding to air-column resonance band. The amplification required was considerably in excess of that for figure 1 (*b*) owing to the large mass of the cone.

(*b*) Apparent radiation resistance (*Rm*) for free-edge conical paper shell corresponding to air-column resonances. Dimensions as in figure 3, curve 1, but $M_c = 4{\cdot}7$ gm.

a, k coil former, a the major radius and k the end-correction coefficient, which varies between 0·6 and 0·8.

The form of the air-pressure curve in the vicinity of the resonance is of interest, and typical examples are given in figures 3, 6, 7, 9, and 10. In figure 10 the column resonances occur *above* the main nodal group, since ψ is 135° and the air column short. A curve for a standard-size paper cone is portrayed in curve 2 of figure 9. The air-column resonance is then quite serious, being about 50 per cent of the

* February 6, 1932.

main nodal group which occurs just above 2000 ~. With heavy cones, e.g. of thick aluminium or glass, the amplitude of vibration is reduced considerably owing to large mass-reactance; and the column vibrations are of little importance in comparison with the symmetrical modes of the shell itself.

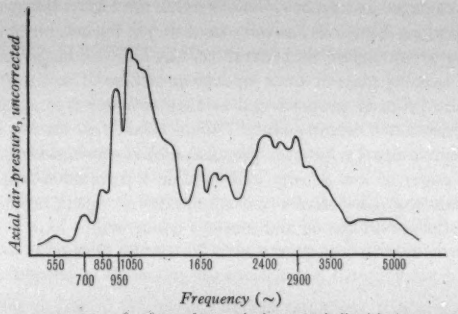

Fig. 10. Axial air-pressure curve for free-edge conical paper shell with ψ equal to 135°. The shell modes centre round 1000 ~, whilst the air-column resonances reside in the region of 2600 ~. $a = 9{\cdot}5$ cm., $b = 2{\cdot}5$ cm., $t = 2{\cdot}1 \times 10^{-2}$ cm., $M_e = 2{\cdot}7$ gm.

§ 9. SUITABILITY OF VARIOUS MATERIALS FOR LOUD-SPEAKER DIAPHRAGMS

In a former paper* experiments with thin aluminium cones were described. Although quantitative measurements were not made at the time, it was shown that such cones were valueless as loud-speaker diaphragms. Figure 11 shows an

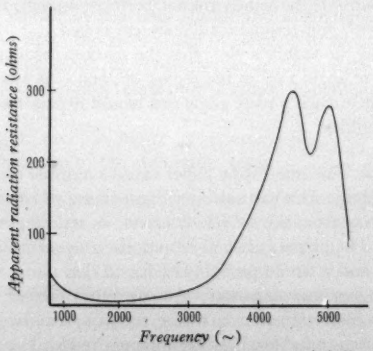

Fig. 11. Apparent radiation resistance (*Rm*) for elastically supported shell made from $7{\cdot}5 \times 10^{-3}$ cm. Aluminium sheet and having a seccotined seam*.

$a = 12$ cm., $b = 2{\cdot}5$ cm., $\psi = 90°$, $M_e = 4{\cdot}7$ gm.

* A report on the quality of broadcast reproduction with this diaphragm was given in *Phil. Mag.* 12, 804 (1931).

output curve of one of the diaphragms where diaphragm loss is included. This curve is of a similar nature to that described from aural observation on broadcast reproduction.

The column resonances occur at about 800 to 900 ~, but the symmetrical modes lie between 4300 and 5300 ~. This is much too high a frequency and causes great aural irritation. Moreover, as was stated in § 8, for comfortable audition the main frequency group should lie between certain limits. This is accomplished by using a coil of suitable mass or some equivalent artifice. The output in the upper register must be properly proportioned to that at lower frequencies. In fact, a curve of the form shown recently in the *Philosophical Magazine** is desirable.

So far as the investigation has been pursued, such results have only been achieved by the use of paper of low density and suitable transmission loss. It is hardly necessary to state that glass shades and aluminium are quite beyond the pale, so far as the reproduction of speech and music is concerned.

DISCUSSION

Major W. S. Tucker. It is interesting to compare the cone employed as a transmitter with that employed as a receiver of sound. If the cone could be fitted with a stethoscope attachment, its responses to an external source of sound could be detected and the resonance peaks described in the paper could be found. The existence of these resonances in the walls of thin cones prevents their adoption for receivers in a binaural sound locator, where a sense of direction is derived by matching the phases of the received sound. The phase-distortion produced by resonance in the walls causes uncertainty if the cones are not perfectly matched. Hence in sound-locators, receivers with non-resonant walls are employed.

I am surprised by the author's view that the marked resonance shown in figure 10 for a paper cone of angle 135° in the region of 2600 ~ is attributable to the air-column. With a cone of such a wide angle one would expect resonances due to the air to be quite negligible.

Mr D. A. Oliver. This interesting paper raises a number of questions. I should like to ask why a warbling tone was used in a highly damped room when the receiver was only 180 cm. away from the source of sound, as standing-wave effects should then be very small. The physics of shell vibrations are preferably examined with a pure searching tone and it would probably be found that the peaks of figures 1 and 2 would then be higher and narrower. Among other factors, is it justifiable to neglect microphone corrections since, owing to cavity resonance alone, peaks as large as 3 : 1 above the general level can be obtained at about 3000 ~ with ordinary commercial condenser microphones; and if such a correction were necessary, the shapes of some of the curves (e.g. figures 3 and 10) would be considerably affected?

* *Phil. Mag.* **12**, 771 (1931), figure 9.

I have found that with paper cones, spun-metal coil-formers are better than paper formers, as the boundary condition at the cone-coil junction is then more definite. In § 4 it is said that the cast glass shade is expected to be more homogeneous than the spun aluminium cones. I should have thought that internal stresses in the glass would have reversed the order. Also from dynamical measurements which I have made of the velocity of sound in certain papers, I hardly think heterogeneity in the paper itself is a large factor, especially as the vibrations of a cone are not very dependent on small variations in thickness. In § 6 does Dr McLachlan mean that he would recommend increasing the mass of a light and satisfactory driving-coil to control the tonal balance? Would he agree that tonal balance is more efficiently obtained by using a light coil and making a suitable choice of cone angle, diameter and thickness?

I agree as to the general superiority of paper as a diaphragm-material—commercial practice is, of course, almost unanimous—but would add that celluloid and similar materials fall into an interesting class, having properties intermediate between paper and sheet-metal so far as density and internal damping are concerned.

Mr A. G. Warren. Some of the results which the author has obtained are particularly interesting in view of the inconsistent operation of loud-speaker diaphrams. Reproduction with a fair balance over the musical scale and with the resonances reasonably spread and not unduly exaggerated is possible, but unfortunately it is rarely obtained in practice. Resonances are essential; the difficulty arises in taming them without suppressing them. The behaviour of the paper diaphragm has been very puzzling. The author's method of approach has considerably clarified the problem. With annealed glass (which is reasonably isotropic) in a geometrically symmetrical form, the resonances are extremely sharp. The subsidiary resonances shown in the thin aluminium diaphragm figure 1 (*b*) are an interesting comment upon the known lack of isotropy in such material. Paper not only is anisotropic but varies in properties in any one direction. Further, geometrical irregularities are almost impossible to avoid. In consequence the resonances are much subdued.

It is interesting to note, in figure 11, scientific justification for one's natural dislike of the reproduction of the aluminium cone.

Mr G. A. V. Sowter. As the author points out at the beginning of this paper, the mathematical treatment of conical shells has been neglected. This is not unexpected in view of the great difficulty of devising analysis for such a complicated structure. Nevertheless, his experimental results enable him to give a complete explanation of the behaviour of the hornless loud-speaker. The good quality of reproduction obtainable from such a system, where resonances abound, was a mystery until Dr McLachlan's researches indicated the reason. It has been my pleasure to be associated with him in some of his work and I can confirm the accuracy of his conclusions regarding the various diaphragm-dimensions at which he arrived from a purely aural standpoint. A further proof lies in his design of the moving-coil loud-speaker which has been operating for some years (1926) in the

Science Museum. It is also interesting to observe, in connection with the thin aluminium cone mentioned, that the preliminary aural tests on broadcasting coincide very closely with the results to be expected from the curve of figure 11 measured subsequently. The performance of paper as a vibrational material is remarkable. Although measurements of Young's modulus sometimes reveal peculiarities, the main frequency with a given type of paper does not vary a great deal. This is rather striking, since paper cannot be regarded as homogeneous. With a paper cone the seam and the small undulations on the surface modify to some extent the acoustic output and generally broaden the frequency-response round the main resonance, which is all to the good. In curves taken by the author without a warble note, the profile over the resonant region resembled a small-tooth comb. Although the warble note smoothes out these serrations, the resulting curves are more easily interpreted, and the main effects, particularly with regard to frequency, are not masked in any way. The influence of the apical angle and the elastic modulus upon the output curves is very striking and shows what must not be done in loud-speaker design. The mass of the coil is an important factor, and in a number of commercial designs the powerful upper register is due to the radius of the diaphragm being too small and the coil too light.

Mr F. D. Smith. The author's measurements have been made at a point on the axes of the cones in a highly damped room. The method has many advantages in a study of the vibrational properties of cones; but, since the results are ultimately to be applied to the moving-coil loud-speaker, it needs perhaps to be supplemented by measurements in a moderately damped room at points off the axis. The use of a frequency of supply variable over $\pm$ 50 $\sim$ in combination with one or more microphones maintained in continuous motion should lead to a measure of the total energy emitted from the cone; supplementary measurements of reverberation time over the experimental range of frequency would further enhance the value of the results. It is interesting to note, in this connection, that a room can be designed to have a reverberation period constant to within about 20 per cent for all frequencies between about 40 $\sim$ and 10,000 $\sim$.

The lack of sufficient data relating to the current flowing in the moving coil in the various experiments and to the scale of the diagrams of axial air-pressure makes comparison of the diagrams difficult. Measurements of this kind can be placed on a more metrical basis by comparing the received e.m.f. from the microphone with an e.m.f. of known magnitude derived from the current flowing in the moving coil via a potentiometer, resistive attenuator, or mutual inductance.

It would be of interest to know the range of ψ over which the empirical formula in § 2 for the vibrational frequency of a conical shell is applicable. With reference to the table of data for a glass cone in § 4 it is noteworthy that the mode of vibration having one circular node produces relatively little axial air-pressure. Is this an indication of a small total output of sound-energy arising from a circulation of air between the two oppositely vibrating areas of the cone; or is there a cancellation by interference, at points on the axis, of the sound originating from these two vibrating

areas? When the spacing of antinodes on a cone, measured along the axis of the cone, happens to approach a regular series of half-wave-lengths, a relatively large axial pressure is to be expected. It seems possible to account for the largest peak at 6000 ~ in figure 2 (*a*) in this way. The effect on axial air-pressure of the mass of the moving coil is shown in figures 3 and 7. Were these curves taken with the same circuit density in the copper of the coils, or, in other words, is it legitimate to take the driving force as proportional to M_c?

AUTHOR's reply. I fully appreciate Dr Tucker's surprise regarding air-column resonances. The magnitude of this effect is exaggerated in figure 10 owing to focussing of the sound and to the rising characteristic of the microphone which is partly attributable to the same cause, cavity resonance. The air-column resonances in 90° paper cones are quite important as has been shown recently*.

Mr Oliver's remarks arise very largely from a misapprehension regarding the purpose of the experiments. I started with a pure tone, but was not absolutely satisfied that the results were always entirely free from standing waves. The object was to determine vibrational frequencies, not the power-output at such frequencies, and a warble note was employed. No fundamental resonances were missed. This was confirmed by bridge measurements of the resonances in figure 10. Incidentally this curve agreed well with that for a pure tone. So far as sharpness of definition is concerned Mr Oliver should surely be satisfied with figure 1 (*a*) unless he desires Shakespearean sharpness, "Keen as a razor's edge invisible." In certain cases with a pure tone a series of serrations was superposed on the main resonance band, which Mr Sowter compares to a small-tooth comb. This was probably due to local perturbations and general heterogeneity of the shell. Mr Oliver vetoes heterogeneity, but he cannot have it both ways. The majority of published acoustic measurements on loud-speakers are *axial* pressure curves with a plethora of paltry peaks. Analyses of these would be like looking for a particular proton in Persia—impossible! The warble note was employed to avoid this dilemma. Whilst I am on the subject of axial pressure curves, it is relevant to remark that owing to increase in focussing of the sound with frequency they are quite meaningless so far as sound-reproduction is concerned. An output or power curve is required. The curves in figures 1 and 2 are corrected for the microphone. There would have been little use in correcting all the curves, since the magnitude of a resonance depends on power output, not axial pressure. In any case I am not fond of hours of uninteresting arithmetic. A fair idea of the shape of the output curves can be gained from measurements of apparent radiation resistance†.

The results with paper formers depend upon the skill with which these are glued to the cone. I have never had difficulty so long as the apex of the cone was removed after the coil had been put into place. If Mr Oliver gets a close enough coupling between the coil and the metal former on which it is wound, the combination will

* *Nature*, *loc. cit.*

† *Phil. Mag.* **12**, 771 (1932).

simulate a transformer with short-circuited secondary. Then he will inaugurate an era of dumb and distortionless loud-speakers! I think he is under a misapprehension regarding the glass cone. The results indicate uniformity of structure, due to annealing as suggested by Mr Warren.

I feel sure that a number of commercial models would not be put on the market if their designers had any sense of musical values. Quality is a matter of personal taste and is not amenable to the laws of physics. Loud-speaker design is a compromise. If perfect reproduction could be obtained there would still be dissentients, so bizarre is the psychology of musical appreciation. So long as the public has an unsatiable appetite for jazz and the reception of foreign stations in the present congested state of the ether, high-class reproduction will not be a success. But that is no reason why we should not strive to improve loud-speaking apparatus.

The interesting contributions to the discussion by Messrs Warren and Sowter confirm my general views on the subject. This applies in particular to homogeneity. The velocity of propagation as found from $\sqrt{(E/\rho)}$ is not an acid test of homogeneity. If E and ρ each vary in proportion, $\sqrt{(E/\rho)}$ is constant, but it does not follow that the material is homogeneous. The fact that nodal figures on paper discs or conical shells are very irregular points clearly to heterogeneity*. Also the seam precludes any possibility of homogeneity.

Mr F. D. Smith will find quantitative measurements on the effect of coil mass in the October *Phil. Mag.* paper. In curve 3 of figure 12 of that paper, the resonance at 2000 ~ is due to a re-entrant cone. Without this auxiliary cone the main resonance occurs at 3000 ~†.

The scope of the formula in § 2 can be found by applying it to the data in this and other papers to which reference is made. Beyond this I have no experience. The small axial pressure with one nodal circle on the glass cone was due to reduced power. The increase in output with two nodal circles was very marked, whether one listened in the damped room or outside it down the corridor with the door open. Theory does not indicate the maxima and minima along the axis unless one is close to the diaphragm. The measurements were made beyond 180 cm. where such effects are absent. The increase in power above the first nodal circle is corroborated by measurements of motional resistance and this is clearly shown in the October *Phil. Mag.* paper.

The precise reason is rather elusive, but we can examine possibilities. Firstly the effective mass M_e is expected to vanish at a centre-moving mode. If for some reason M_e is not quite zero, the amplitude and therefore the output will be reduced. Secondly, although with one nodal circle the amplitude may be large, so also may be the interference in space due to the inner and outer portions of the shell vibrating in opposite phase. Ultimately the amplitude is curbed by transmission loss.

Figures 3 and 7 were obtained by applying constant voltage to the grid of the power valve, the output transformer being unaltered but the coil turns reduced.

* *Phil. Mag.* **12**, 115 (1932).

† See curve 1 in figure 6 of present paper.

The experiments were conducted some time ago and I cannot now guarantee that the amplifier-attenuator was unaltered. Consequently I should not like to advise Mr Smith to assume the force to be proportional to M_e.

My main object was to locate vibrational frequencies and to find the general influence of coil mass on the *profile* of the axial-pressure curve. The apparatus was at my disposal for a limited time during which it was only possible to do what has been described in the paper. I hope to publish analyses of the axial and spatial pressure-distribution and of the power radiated from flexible discs and conical shells in the near future.

PRESENTATION OF THE DUDDELL MEDAL FOR 1931

to CHARLES THOMSON REES WILSON, F.R.S.,

Jacksonian Professor of Natural Philosophy, Cambridge

Address by the President

PROF. C. T. R. WILSON'S contributions to physical science fall mainly into two groups: firstly those associated with the investigation of the tracks of ionizing particles, and secondly those associated with atmospheric electricity. But there is a connecting link between the two groups furnished by the word *cloud*.

I shall only deal very briefly with his work in atmospheric electricity. He has opened up new fields of inquiry along which great progress is being made. His method of studying rapid changes of the earth's electric field has led in his own hands and in those of others to a great increase in our knowledge of the electric fields associated with thunderstorms. By studying the changes of electric field at short distances from discharge channels, Wilson made the first trustworthy determination of the magnitude of the electric charge dissipated in a lightning flash. Appleton and Schonland working at greater distances confirm his view that thunderclouds are predominantly bipolar, the positive charge being elevated above the negative.

This work, together with Wormell's study of point-discharges in thunderstorm fields, gives the strongest support to Wilson's solution of what is perhaps the most important of all problems in atmospheric electricity, namely the maintenance of the earth's electric charge. The persistence of the earth's negative charge in fair-weather regions, notwithstanding the current of positive ions constantly flowing from the air to the ground, is attributed to the action of thunder and shower clouds, the charges of which are so disposed as to send negative electricity into the ground in sufficient quantity to balance the loss.

The other main line of Wilson's work branched off at a very early stage from his meteorological interests, but it was nevertheless his study of cloud-formation—the condensation of small drops of water out of supersaturated air—that led him to invent the expansion chamber, which is one of the most potent instruments of physical investigation ever devised. In his Nobel Lecture, delivered at Stockholm in December, 1927, when he received the Nobel Prize, he has told us how his attention was directed to condensation phenomena. During a short visit in September, 1894, to the Observatory which formerly stood on the summit of Ben Nevis, he became interested in the coronas and glories which the sun and clouds conspired to produce. Originally the optics of the phenomena engaged his attention; but when he set to work in 1895 to attempt the reproduction of the phenomena in the laboratory, the optics soon fell into the background and the investigation was

directed to the conditions of cloud-formation. He discovered that under certain conditions clouds could be formed in a supersaturated atmosphere free from dust particles; it had previously been supposed from Aitken's researches that dust was essential to provide nuclei for the drops to form round. Wilson found that there was a critical expansion ratio v_2/v_1, equal to 1·25, corresponding to 4-fold supersaturation; no drops were formed in dust-free air unless the expansion exceeded this limit, but when it was exceeded a shower of drops was seen to fall. The result was published in May, 1895. With more elaborate apparatus Wilson discovered a second critical expansion corresponding to 8-fold supersaturation. When this was exceeded dense clouds, consisting of very fine drops and associated with beautiful colour phenomena, were produced. Similar results were obtained with various pure gases.

The dense clouds due to the second critical expansion were explained by supposing that the gas molecules themselves acted as nuclei for the condensations, and interest became directed especially to the sparser shower formed at the first critical expansion.

The year 1895 of which I am speaking is the one which we usually name as the birth-year of the "new physics." Abroad Roentgen was discovering X-rays, the discovery being announced in the autumn. Close at hand J. J. Thomson was discovering the electron. Wilson was able to make use of a new X-ray tube made by Mr Everett in the Cavendish Laboratory. He found that when the moist air was exposed to X-rays, the first critical expansion, instead of producing a sparse shower of large drops, formed a dense fog consisting of very small drops which fell very slowly. In other words the action of X-rays greatly multiplied the number of the mysterious nuclei which operated at the first critical expansion.

The ionic character of these nuclei was next proved by their behaviour in an electrostatic field. Later study showed that it was the negative ions (i.e. the electrons) that gave the first condensation; for condensation on positive ions a 6-fold supersaturation was required.

In this way the whole phenomenon was disentangled. Owing to the large surface-energy involved, a drop cannot begin to form about nothing. If solid particles are lacking, the next most favourable nucleus is an electron. There are always a few of these present in a gas, due to various radiations including cosmic rays; but the number can be greatly increased by artificial methods of ionization. With higher expansion-ratios drops can be formed on positive ions or on molecules. Wilson now knew the proper conditions to bring each type of nucleus into operation. The next development was to use the line of drops to trace the track along which an ionizing ray or particle had passed.

We jump 15 years. Meanwhile many things had been discovered, including radium and its various radiations. Wilson, with his untiring pursuit of perfection, had spent infinite pains in making a suitable form of expansion apparatus and arranging efficient means of instantaneous illumination of the cloud particles for the purpose of photographing them. Whilst this was still incomplete it occurred to him to take a photograph with the rough apparatus already constructed to see

if he could detect with it the tracks of ionizing particles. In his Nobel Lecture he says:

"The first test was made with X-rays, and with little expectation of success; and in making an expansion of the proper magnitude for condensation on the ions while the air was exposed to the rays, I was delighted to see the cloud-chamber filled with little wisps and threads of clouds—the tracks of the electrons ejected by the action of the rays. The radium-tipped tongue of a spintharoscope was then placed inside the cloud-chamber, and the very beautiful sight of the clouds condensed along the tracks of the α-particles was seen for the first time."

His final improvement of the expansion method consisted in taking stereoscopic pictures, so as to enable the tracks to be plotted in three dimensions.

I will not attempt to enumerate the manifold results which have been obtained from the use of Wilson's cloud method of studying rays and particles. I will only say that broadly speaking the expansion chamber is to the atomic physicist what the telescope is to the astronomer. But it is perhaps appropriate to mention that at the moment we have chosen for honouring the inventor, all the expansion chambers in Cambridge (and probably throughout the world) are working overtime on a new discovery. The facts appear to be that *something* gets through a thickness of lead which no kind of matter hitherto known could penetrate, and on entering the expansion chamber buffets the atoms with momentum much too great to be carried by a quantum of radiation. Subject to further tests, some of which are now being made with Prof. Wilson's own apparatus, it is concluded that the invisible agent is a new element, *neutron*, of atomic number zero and therefore preceding hydrogen in the sequence of the elements.

Wilson is the man who first enabled us to see the atoms. By steps extending over nearly a century physics had been arriving at a theoretically conceived world of minute particles—a strange, almost incredible world, well-nigh transcending the imagination. Wilson made that world visible. With our own eyes we see the atoms deflected by a magnetic field; we see them collide, and verify that momentum is conserved. We see them disintegrate, an α-particle shooting off in one direction and the transmuted atom recoiling in the other direction. That is—provided we have an expansion chamber.

Do not be misled by those captious persons who will tell you that Prof. Wilson does not really show us the atoms but only remote indications of the atoms. Is there anything in Nature that we are aware of except by remote indication? If Wilson has never seen an atom, then I have never seen a star. I know that what he actually sees is a chain of little drops not in the least resembling a real atom, but from which an atom may be inferred. But it is the same with me; what I see is a system of diffraction rings not in the least resembling a real star, but from which a star may be inferred.

The amazing consequences which have sprung from Prof. Wilson's discovery, and the part which he himself has played in developing them, have tempted me away from the instrumental side of his work which is the primary ground of this award. I would like, however, to emphasize that his wonderful instrumental

technique, his perseverance and accuracy, and the standard of perfection which he set himself to reach, have been prime factors in his success. The following incident was related to me by an authority whose stories I do not always trust, but I am inclined to believe him in this instance. My informant who was going abroad went into Wilson's room in the Cavendish Laboratory to say good-bye and talk of various matters. Wilson at that time could not get his expansion chamber to work as well as he thought it should, and all the time they were talking Wilson had the plunger in his hand, grinding, grinding, grinding. Six weeks later the traveller returned and went to see Wilson again and talk to him of the latest scientific news. He found Wilson with the plunger in his hand, and all the time they were talking Wilson was grinding, grinding, grinding.

I do not know whether Prof. Wilson will confirm or deny this. If it should prove to be apocryphal, I may remind you that the character of a man may be revealed not only by the incidents which happened to him but by the stories which are told about him.

Nowadays, of course, expansion chambers are in use all over the world; and many of the foremost physicists have added their own skill and brains to obtain more results from Wilson's instrument. Nevertheless, it should be added that though he has many followers, for delicacy of work he is still unrivalled. The instruments elsewhere are no doubt adequate for most purposes; but I believe that if some point of special delicacy arises, nowhere in the world is there such perfection of technique as in the work of Wilson and his department.

I will end by quoting Lord Rutherford. When I told him of the duty I had to perform to-day, his words were "You cannot say too much about Wilson."

Prof. Wilson, in your expansion chamber we see some tracks that twist about, easily deflected. Others go straight as a die not to be turned aside by any obstacle in the path. I know of one physicist who resembles the latter. For 37 years your work has gone unwaveringly towards the goal of perfection at which you aim; and your determination has overcome all obstacles. I do not doubt that the portion of your track which is still ahead will afford further demonstration of your "penetrating power." It might seem inappropriate to wish a future of sunshine and fair weather to one whose delight is in clouds and thunderstorms. So, abandoning dangerous metaphor I ask you to accept this medal with our admiration and friendship; and may you have health and strength to bring further advance to our knowledge.

DEMONSTRATION

"Some lecture experiments." *Demonstrations given by* W. BENNETT, A.R.C.S., B.Sc., *on March* 4, 1932.

The electric bell. The insufficiency of the simple explanation of the action of the electric bell, as it is given in elementary text-books, is not always realized, though it is obvious when pointed out. Work is done on the armature only while it is moving toward the magnet with the contact made. The armature is drawn by the magnet and retarded by the contact spring. But it has to go back the same distance against what are at first sight the same forces. The action of the bell is made possible by the inductance of its windings, which delays the growth of the current when the contact is made and its decay when the contact is broken. Attention was called to this by M. Ch. Féry in May 1919* when he demonstrated before the French Physical Society an experiment similar to the one shown here. The present Lord Rayleigh has pointed out, however, that the explanation was given by his father no less than fifty-five years ago†. A bell of small inductance is made by removing as much copper and iron as possible from an ordinary bell. It will ring only when an iron core is introduced into a solenoid in series with the bell. Domestic-pattern bells which have been tested have inductances of 25 to 60 mH., and time-constants of 0·01 to 0·02 sec.

Fig. 1.

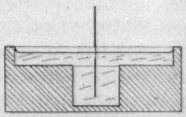

Fig. 2.

The electrostatic effect of a high-frequency current. The two leads of a galvanometer are joined together and taken to an insulated metal plate, standing vertically. One of the leads is cut, and a crystal rectifier is placed in it. A simple valve oscillator is set up at a distance of some feet, facing the metal plate. The galvanometer is deflected when the oscillator is working. The deflection increases with the frequency and with the amplitude of the oscillating voltage.

A simple magnetometer needle. A safety-razor blade is magnetized, floats on water, and carries two galvanometer mirrors attached, one below the blade and one above, to a light-stem passing through the centre of the blade. The water is contained in a shallow circular pit turned in the upper surface of a brass block. A central deepening accommodates the mirror, which is immersed. There is only a small clearance between the ends of the blade and the walls of the pit, and the blade keeps itself centred perfectly. The use of two mirrors in the same plane allows them to be set normal to the magnetic axis by the usual method of reversal.

* *Soc. Franç. de Phys. Procès Verbaux.*
† Rayleigh, *Sound*, 1, 58 (1877); 1, 68 (1894).

REVIEWS OF BOOKS

Recent Advances in Physics (*Non-atomic*), by Prof. F. H. NEWMAN. Pp. x + 378. (London: J. and A. Churchill.) 15*s*.

Prof. Newman has had the courage to label his book as "non-atomic" physics, and the good sense to waive the implications of this sub-title whenever he has found it convenient to do so. The first two chapters are on the "Wave-like character of matter" and "Statistical mechanics" respectively. The remaining six correspond with the conventional subdivisions of classical physics—heat being represented mainly by an account of low-temperature work.

Although the style is clear, the book is adapted less to the needs of the general reader than to those of the young serious student of physics. To the latter, the scope of the work can be most easily conveyed in the statement that almost any one of the 112 sections of the book, taken at random, could form the nucleus of a discussion suitable for a physical colloquium. References are given to recent and very recent books and original papers; the bibliography has wisely been made representative rather than exhaustive, so that the reader is deprived of one excuse for not following up the references. The work should prove of much service to students reading for honours degrees, and possibly at times to their seniors. It will probably be of interest to physical chemists as well as to physicists.

H. R. R.

Smoke, A study of Aerial Disperse Systems, by R. WHYTLAW-GRAY, O.B.E., Ph.D., F.R.S., and H. S. PATERSON, B.Sc. Pp. viii + 192. (London: E. Arnold and Co.) 14*s*.

This book, while doing full justice to earlier workers in the same field, is largely an account of long-continued and painstaking investigations carried out at Leeds. The authors have considered smoke "as a special type of disperse material and have tried to indicate how far the parallelism between aerial and aqueous suspensions implied in the term 'aerosol' really extends." This term, first put forward by Prof. Svedberg at the discussion on colloids in 1920, is indeed shown to be a picturesque label rather than a description justified by any real analogy. The comparative instability of smokes, the normal presence of particles with both positive and negative charges, and the general tendency to aggregation and chain-formation constitute profound differences between smokes and hydrosols. In an exhaustive discussion the authors find the cause of these differences in the nature of the dispersion medium, which, in comparison with water, has low viscosity and dielectric constant and contains a very small number of ions. The methods used in the investigation of sols accordingly require very considerable modification, and the procedures developed at Leeds are fully described in chapters on early investigations on smokes, errors in counting the particles in smokes, present methods of counting particles, the coagulation of smokes, and the determination of weight-concentration and size-distribution. Many of the methods described are of great ingenuity, like that of condensing water on the particles by adiabatic expansion, or the micro-filtration method of determining weight-concentration, and the chapters named will be read with profit by anyone interested in experimental technique.

The smokes investigated cover a wide range and include classical examples like ammonium chloride, smokes dispersed at low temperature (generally by "blowing a stream of dust-free and carefully filtered air at a known rate over the material heated in a metal boat") and smokes produced by making an arc between electrodes of the metal to be dispersed, and blowing a stream of air across the arc. A number of excellent photomicrographs illustrate the various types of particles occurring in these smokes; the oxide particles are particularly striking both for their markedly crystalline character and their tendency to aggregation.

A theory of coagulation is developed by introducing into Smoluchowski's deduction the modifications called for by a medium in which the mean free path is large compared with the dimensions of the particles, and the effects of polydispersity are fully discussed. The agreement with experimental results is as good as that found when Smoluchowski's original formula is tested for aqueous sols.

As regards the electrification of smoke particles, conditions are far more complicated than in hydrosols. In some smokes, such as ammonium chloride, all particles carry charges of the same sign, while in others, such as stearic acid, the numbers of positively and negatively charged particles are equal. In smokes dispersed at low temperature a high percentage of charged particles is present from the beginning and remains constant for some hours, while smokes dispersed at high temperature initially contain a small number of charged particles which rapidly increases with time.

While conditions are thus more complicated than in hydrosols, the possible explanations are more restricted, since the formation of such complexes as are postulated by Pauli and his school in all aqueous sols is ruled out. "If an aerosol is uncharged initially, it can only become electrified by catching ions from the air." The behaviour of a number of smokes is investigated experimentally and discussed from this one point of view.

It will be seen from this brief synopsis that the authors present (in very clear and simple language) a very complete survey of the properties of disperse systems in gaseous dispersion media, as far as they are known at present. They are quite aware that much remains to be done and devote to "Further problems" a final chapter which should stimulate further research. The book is a valuable contribution to our knowledge of disperse systems and will be welcomed by all who feel, with the present reviewer, that water is the most complicated of all dispersion media and that progress is to be achieved by getting away from it.

E. H.

The Donnan Equilibria and their application to chemical, physiological and technical processes, by T. R. BOLAM, D.Sc., M.Sc. Pp. vii + 154. (London: G. Bell and Sons.) 9*s*.

It was a simple—almost innocently simple—observation that Donnan and Harris made some twenty years ago. Take an aqueous solution of congo red (the sodium salt of a complex sulphonic acid), dissolve sodium chloride in it and make an osmotic system by separating this solution from pure water by means of a parchment membrane. It will be found when equilibrium is attained that the concentration of the sodium chloride is greater on the water side than on the dye-stuff side of the membrane.

This modest beginning may be expanded into the general theorem that "the presence in any system of electrolytes of a species of ion which is restrained in *any* way from diffusing to all parts of the system will give rise to unequal distribution of every species of diffusible ion present. This *particular state of unequal ionic distribution* is the characteristic feature of Donnan equilibria."

Dr Bolam has built up the structure of the story extremely well and lucidly. His theoretical outline is specially good, beginning as it does with simple kinetic illustrations, and proceeding by nicely graduated stages through an elementary thermodynamical treatment to a general theory dealing with non-ideal solutions and taking account of activities. He then discusses some simple chemical illustrations, afterwards proceeding to physico-chemical, technical and biological applications. These last-named include an account of equilibria in the blood, and a very clear résumé of Duke-Elder's work on intra-ocular fluids.

The volume is compact in size, is well produced and forms a noteworthy contribution to the series of "Monographs on Modern Chemistry."

A. F.

The Physical Properties of the Soil, by BERNARD A. KEEN, D.Sc., F.Inst.P. Pp. vi + 380. (London: Longmans, Green and Co., Ltd.) 21*s*.

Millennia have slipped by since Adam digged outside the garden; but no systematic study of the soil appears to have been made until the work of G. Schübler was published at Leipzig in 1838. It is true that Fitzherbert's *Boke of Husbandry* had appeared in 1523 "dealing expressly with practical agriculture" and deserving "also to be remembered for the acuteness of many of the observations on soil cultivation. Fitzherbert possessed a keenly observant mind and his facts are given with surprising accuracy." Also an isolated *experiment* was made by J. Houghton (1681)—"important because it was the first recorded account of any attempt to classify soils on the basis of particle-size." "Not only did Houghton obtain a measure of the relative proportions of fine and coarse material in different soils, but he was able to observe differences in their properties from one soil to another." Small (a Scotsman) had experimented on, and Jefferson (third President of the United States, with an observant and mathematical mind) had thought out designs for, improved ploughs. These are mere incidents however compared with the work of Schübler, who studied apparent and real specific gravity, cohesion and plasticity, shrinkage on drying, moisture-holding capacity, evaporation and uptake of water, heat of wetting, specific heat, thermal conductivity, absorption of radiation, electrical conductivity and absorption of oxygen. He correlated the various properties with each other. Their names have been modernized but the subjects might have been taken out of a present-day volume on physical chemistry. The advanced character of these investigations can be in part realized by recalling that Ohm had published his electrical law only ten years previously and Joule had scarcely begun work on the conservation of energy.

The present volume is a lineal descendant of that of Schübler, but with all the advantages that are brought to it by the tremendous advances of physical science in the last century.

The soil is a composite thing, a loose aggregate of many components, whose function it is to bring food and water to the roots of plants. One of the main problems discussed is how to grade it and specify the properties required to carry out its functions. This is not a simple problem because soil is of so many kinds—fine clay, sand, pebbles and intermixtures of these. Houghton separated them by stirring the soil in water, allowing the mixture to settle, and collecting the sediment from time to time: the finest particles come down last in accordance with Stokes's law. Odén and Keen and others have sought to improve this process and have devised special balances for the purpose. Unfortunately any scale-pan immersed in the suspensions disturbs the fall, and these balance methods have had to be discarded owing to this inherent error (p. 66). The present mode employed internationally (though with minor national differences) is to pipette a sample from a given layer at depth x at the end of a given time t; this sample is assumed to contain no particles whose velocity of fall exceeds x/t; all particles with less velocity are present in the sample in the same concentration as in the original suspension. The bulk suspension is then thoroughly shaken and a fresh sample is taken corresponding to a different x/t and so on (p. 71). The method is not free from defect but it is the best devised so far, and international rules have been drawn up for carrying it out. In Switzerland and Czechoslovakia an exception is made in the preparation of the suspension. Small gravel is first separated out by sieving, but no attempt is made to disperse the remaining particles thoroughly; it is considered that the material should be left as nearly as possible in the state in which it is found in the ground (p. 76). Much can be said in favour of this contention; it leaves the finer particles cemented together (with lime, etc.) when they are so cemented in the ground. Fortunately the Odén-Keen balance can still be used for other problems (e.g. evaporation), though discarded as regards the purpose for which it was designed, and it is fully described (p. 82, etc.). Much work has been done on the way water distributes itself in such an assemblage of particles. The old method of replacing the

interstices between the particles by equivalent capillary tubes is far too crude to be of use. If water is contained in the interstices between spheres it may not be sufficient in amount to fill these interstices; the free surfaces of the water near each contact will then be determined by surface tension. Much of the work on this question has been elaborated at the Agricultural Station, Rothamsted.

The plasticity of soil and clay pastes has also led to fruitful experimentation (Chap. v). But more such work of a critical nature on well-defined systems is needed. It is interesting to learn that such work—as exemplified by the experiments shown at the recent Physical and Optical Societies' exhibition—is being conducted at Rothamsted. Much has also been done on the significance of the vapour-pressures above the soil (p. 212) and the information it may give concerning water-content. All the properties are being studied with a view to correlating to one another the biological and physical properties of soils. Another class of physical properties is concerned with dynamometer measurements of the resistance offered to the passage of cultivation implements. A full account of this work is to be found in Chap. VIII, while the succeeding chapters deal with soil temperature and atmosphere.

The volume gives a stimulating account of the work that is being carried out both at home and abroad. The term "soil" is given, however, to very varied materials, and in a climate like that of England it is subject to very varied circumstances; so that to the workers themselves the results obtained may sometimes give rise to disappointment. Nevertheless the subject is now in a totally different position from that which obtained at the beginning of the present century. We should have liked to have dealt with it more fully, and especially with the great contributions that have been made by British workers.

The book itself is well presented. It is fully illustrated with diagrams exhibiting the experimental results. There appear to be very few typographical errors. "Kirchhoff," p. 86, "W. Thomson (Kelvin)," p. 297 and "analytique," p. 307, should be spelt as here shown.

A. W. P.

Vision and Colour Vision, by R. A. HOUSTOUN, M.A., D.Sc. Pp. viii + 238, 102 illustrations. (Longmans, Green and Co., 1932.) 15*s*. net.

A proper appreciation of the laws of vision, including those relating to colour, is becoming of much importance in various branches of industry and technology, and a book of moderate length giving a sound exposition of what is known on this subject would receive a warm welcome. Dr Houstoun is known among physicists as the author of papers on some aspects of vision, and the table of contents of his book suggests that it may well meet a real want. The chapters include discussions on the discrimination of intensity, dark-adaptation, acuity of vision, the visibility of the spectrum, the laws of colour-mixing, theories of vision and colour-blindness, and many other not less interesting topics. The first chapter is entitled "Units" and begins

"Light can be measured in three kinds of unit...the erg...the metre-candle...the brightness unit. This unit is wholly subjective and has hitherto found very little use. It is one of the functions of this book to develop it. The possibility of a brightness unit has been denied in some quarters. As much of the confusion at present existing in colour-vision theory arises from the fact that observers have not attempted to give numerical values to their sensations, we shall proceed to elucidate...."

After so direct a challenge to his views the more or less orthodox physicist anticipates a discussion in which accurate experiment and observation, combined with forcible logic, will compel him at least to reconsider and possibly to modify some of his views.

This expectation is not realized: the book in fact is very disappointing. Dr Houstoun has clearly read widely in the subject, and this book is largely composed of extracts from and comments on his readings, together with a description of experimental work he has

carried out either alone or in collaboration with others. A reader can hardly escape from the conclusion that the author has at times failed to grasp the real significance of what he has read. Many of the chapters are by no means satisfactory. The text often requires correction; at times the meaning is obscure, and examples of direct contradictions in different passages are not wanting. The author exhibits a weakness for introducing (sometimes on very doubtful evidence) speculations which apparently lead nowhere, and in at least one place he leads an argument in a particular direction with an "undoubtedly" when there is a complete lack of evidence whether that way or another is correct. Many of the experimental conclusions he accepts appear to the reviewer not to have been established owing to the neglect of important conditions. In general the work suffers from an insufficiently critical attitude on the part of the author.

Detailed criticism of the bulk of the book is unnecessary. It will suffice to comment briefly on the last chapter, to which some of the earlier parts lead up. This chapter is entitled "The theory of colour-vision," by which we are to understand the theory Dr Houstoun advocates—other theories have been mentioned in an earlier chapter. In the opening section, entitled "The argument against the Young-Helmholtz theory," it is claimed that this theory (i) violates common sense and (ii) leads inevitably to mathematical contradictions. Objection (i) is unsubstantial, and it is sufficient here to say that the criticism applies equally to Dr Houstoun's own theory. On (ii) the author says "I have known for years that the Young-Helmholtz theory was mathematically untenable, but did not arrive at the above proof until two years ago." In this "proof" an essential part is played by the measures of sensation. Had Dr Houstoun looked back to the opening section of his book he would have been reminded that many—certainly a large number of adherents of the Young-Helmholtz theory—do not admit numerical scales of sensation. Thus the proof, if otherwise valid, would not have been relevant to the Young-Helmholtz theory: it can only apply to a theory, such as Dr Houstoun's, which incorporates measures of sensations not linearly related to the measures of the stimuli which evoke them. If the argument of this "proof" is examined it will be seen that it rests upon the *assumption* $2S(x) = S(2x)$, where $S(x)$ is the measure of the sensation resulting from the stimulus measured by x. The author has put his statement in such a way that he has failed to see what he has assumed, and by including unessential colour terms has deceived himself into believing that it is of significance to the theory he is attacking. He has, in fact, completely failed to establish the existence of any inconsistency in the Young-Helmholtz theory.

It is clear that in the reviewer's opinion the book is not to be recommended to those who are setting out to learn the fundamentals of vision and colour vision. Those who already have a good knowledge of these subjects will not necessarily find its perusal unprofitable; their views are likely now and then to differ enough from the author's for the conflict to be stimulating, and they may be led to realize more fully the outstanding importance to the progress of this science of precise measurements of indisputable significance. For example, our knowledge of colour-blindness would be immeasurably advanced if investigations comparable with those carried out in recent years on normal subjects were to be undertaken on a few selected persons.

In conclusion attention may be called to a small error which is repeated often enough to irritate—the use of the contraction $\mu\mu$ where $m\mu$ is meant. T. S.

Mathematical Tables, Vol. 1, by THE BRITISH ASSOCIATION COMMITTEE FOR THE CALCULATION OF MATHEMATICAL TABLES. Edited by J. HENDERSON. Pp. xxxvi + 72. (London: British Association.) 10*s*.

The books of tables to which we are most accustomed give the values of functions to not more than 7 figures, and interpolation is by proportional parts; but when a table is first calculated, things are quite different. Briggs's *Arithmetica* (1624) and Vlacq's so-

called *Editio secunda* (1628) gave logarithms to 14 and to 10 figures respectively, whilst Rheticus (1596) calculated sines every 10″ to 10 decimals, and extended the accuracy to 15 places years later. Nearer to our own day we find Glaisher tabulating $1/x!$ to 28 significant figures, Hayashi giving its square to 23 figures, and Stieltjes giving the sum of the series $\sum_1^\infty r^{-x}$ to 32 figures for every integral value of x from 2 to 70. Manifestly the entries in such tables cannot be separately calculated at sufficiently close intervals of the argument to make linear interpolation possible, and they are only regarded as the basis of other, more convenient, tables. Normally the high-accuracy table now finds its home in one of the mathematical journals, and is only consulted on the rare occasions when it is imperative to employ such accuracy—chiefly for the calculation of tables of some other function.

The British Association Committee which has been sitting since 1873 (but not with the same personnel throughout) to deal with the subject of mathematical tables has now broken somewhat fresh ground by collecting together all the tables save those of Bessel functions and those connected with the theory of numbers which have been published in its reports. The tables have been re-arranged, since their first publication, so as to present them in a form suitable for use with Everett's interpolation formula and, to assist in this, the differences of even order are tabulated also.

Of the functions given, we single out firstly the sines and cosines up to 50 radians, at every tenth of a radian, together with a table up to 1·6 radians at every thousandth of a radian. Of these, the first table is given to 15 figures, and is only of use at the arguments actually tabulated. It cannot be interpolated to its full accuracy. The other table goes to 11 figures, and may, with the assistance of the table given for $n\pi/2$, be used to obtain the sine and cosine of any angle. The introduction to the tables mentions that a 23-figure table calculated by Van Orstrand is available, and it may be useful to mention that there is also one to 21 figures, using the sexagesimal division of the degree in *Abh. Berlin* for 1911. It was computed by Peters.

Next, there are tables of $\sinh x$ and $\cosh x$, as also of $\sinh \pi x$ and $\cosh \pi x$, all to 15-figure accuracy, and of the functions

$$\int_0^x \sin x\,dx/x, \quad \int_\infty^x \cos x\,dx/x \quad \text{and} \quad \int_{-\infty}^{-x} e^{-x}\,dx/x.$$

These will all be of great help, since none of them has a competitor in respect of both range of the argument and accuracy. Incidentally, the introduction implies that the Committee have not been aware of the table given by Miller and Rosebrugh in *Trans. R.S. Canada* (1903) where $Ei(x)$ is given from 0·1 to 2, with 9-figure accuracy.

The gamma function, which is here called the "factorial function," is tabulated with 12-figure accuracy from 0 to 1 at intervals of 0·01; a table of this range (with a table of factorials), of course supplies the value for any argument. In addition to this are tabulated the integral of $\log \Gamma(x+1)$ and its first, second, third and fourth derivatives, which may also be written as the sums from $r = 1$ to $r = \infty$ of certain convergent series, the typical terms being $x/r(r+x)$, $(r+x)^{-2}$, $-2(r+x)^{-3}$ and $6(r+x)^{-4}$ for the four derivatives in order. Similar series with x equal to 1 are also tabulated in the introduction, viz. $\sum_{r=1}^\infty (1+r)^{-p}$ for all values of p from 2 to 35, the results being given to 16 decimal places. This table is regarded as subsidiary, and is not quoted in the list of contents. In connexion with this set of functions, it should be realized that although Emde's 3-figure table from $x = 0$ to $x = 1$ is not a substitute for the fine table here given, yet it suffices to nullify the claim in the introduction that only one table had been given previously. Actually, Emde's appears to have been the earliest, since it was published in 1906, and Miss Pairman's in 1919.

A word as to the terminology employed may not be out of place. The gamma function

is called the "factorial function," presumably in view of the fact that $\Gamma(x+1) = x\,\Gamma(x)$, so that x is a factor of the function. The change seems to have much in its favour but, if it is to be adopted, let us at least be consistent. In these tables, the derivatives are styled the "polygamma functions" (the 1st derivative is digamma, the 2nd trigamma, and so on) which seems to be pointless if the primary function is not the gamma function.

The last group of tables gives a set of functions which have so far been studied only by the mathematical statistician but will certainly find their way into general mathematics sooner or later. They are here called the Hh_n functions, where n is an integer. Hh_{-1} is $e^{-\frac{1}{2}x^2}$, and the others are such that $Hh_{n-1} = -d(Hh_n)/dx$. Consequently those with negative n are, save for a factor $e^{\frac{1}{2}x^2}$, the Hermite polynomials which have recently become interesting to the student of wave mechanics. Another member of this set of functions, viz. $Hh_0(x)$, gives in effect a table of the error function, since $erf(y) = 1 - (2/\pi)^{\frac{1}{2}}\, Hh_0(y\sqrt{2})$. In this particular group of tables, interpolation is naturally by Taylor's series, because the functions of the successive orders supply the terms $f(x), f'(x), f''(x), \ldots$ required in that series.

The volume is extremely well produced, and the paper and type are good, whilst the price is distinctly low. A great deal of tedious labour must have been expended on the production, but this will be repaid in the gratitude of the users, for it cannot be denied that a single volume of tables of quarto size is vastly more convenient than the whole shelf full of B.A. reports which it renders superfluous. And anyway, we do not all possess copies of the reports for our very own, even if we should like to!

J. H. A.

Tables Annuelles de Constantes et Données Numériques, Vol. **8**, 1927–8. Pp. xl + 1101. (Paris: Gauthier Villars et Cie.)

Prof. Marie's magnificent publication pursues its majestic course. The present volume contains data for the years 1927–28, and covers the usual ground—elasticity, viscosity, density, surface tension, expansion, specific heats, thermal conductivity and other topics, heat, acoustics, electricity and magnetism, radioactivity and certain branches of optical science, including emission spectra, are represented in this volume. The limits of selection are very wide indeed, and it would be difficult to find a relevant experimental paper of any value which has been overlooked. The volume is indispensable and is destined to be used heavily; that being so, it is unfortunate that it should be covered only, and not properly bound.

A. F.

Proceedings of the Institution of Mechanical Engineers, Vol. **120**, January to June, 1931. Pp. viii + 781. (London: Institution of Mechanical Engineers.)

The present volume contains a number of papers which deal with some important developments in modern engineering. The paper by Mr H. N. Gresley on high-pressure locomotives gives an account of the first high-pressure locomotive to be built in this country. In this there is a water-tube boiler of the Yarrow type, a completely new departure from the fire-tube boiler which has now been in use for a hundred years. In accordance with developments in other modern steam plant the object has been to generate steam at much higher pressures. In Mr Gresley's locomotive the boiler pressure is 450 lb./in.^{2}, but he describes another locomotive working on the German State Railways under a pressure of 1700 lb./in.^{2}

Messrs R. S. Allen and W. E. W. Millington have contributed an interesting paper on modern methods of raising water from underground sources. Here the centrifugal pump has been applied to the pumping of water through a bore-hole as deep as 300 ft. The pump, which runs at 1000 r.p.m., is placed in the bore-hole and is connected to the

motor at the surface by a long vertical shaft. In this connexion considerable mechanical difficulties have been surmounted, and the paper is a tribute to the skill of the engineer.

In his paper on post-war land-turbine development Mr C. D. Gibb reveals some interesting facts relating to the modern steam turbine. Here, as in the locomotive, one is impressed with the use of steam-pressures over 1000 lb./in^2 But probably the most important development is in the construction of the blades. Blade speeds have now been increased from 600 ft./sec. to 1200 ft./sec., and blades are being made as long as 34 in.

There are three papers devoted to the internal-combustion engine, of these the paper by Messrs Farmer and Alcock on fuel injection systems for high-speed oil engines is perhaps the most important. In this, airless injection systems are dealt with, and in particular an analysis of the mechanics of the timed-pump fuel-injection system is given. Diagrams derived from the theory developed are shown to agree reasonably well with diagrams obtained by the authors from engine tests. The paper is an important contribution to knowledge of a rather specialized character relating to the high-speed oil engine.

The fatigue of metals is dealt with in another three papers; these record the results of much work on the failure of spring steel by fatigue. Two of the papers relate to helical springs and the third to laminated springs, and the importance of the effects of surface conditions of the materials on resistance to fatigue is shown clearly in one of these papers.

Prof. Coker gives another contribution to his work on photo-elasticity, and this with some other shorter papers make up a very large but important volume of the *Proceedings*.

G. A. W.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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THE MEASUREMENT OF REFLECTION COEFFICIENTS FOR OBLIQUE INCIDENCE

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ABSTRACT. A method is described whereby the effect of obliquity of incidence upon the reflection coefficients of certain materials can be examined. The material is spread upon the metal receiver of a thermopile which, when exposed to radiation, yields an e.m.f. proportional to the absorption coefficient of the surface. Results obtained with black and white paints and with polished copper are discussed.

§ 1. INTRODUCTION

IN a previous paper* a determination of the reflection coefficients of a number of rough surfaces with the aid of a hemispherical integrating mirror has been described. The apparatus used in that investigation was designed to allow the incident beam of radiation to fall normally upon the specimen and cannot readily be adapted for measurements at oblique incidences.

§ 2. METHOD

With certain materials, notably those which can be spread as a thin coating upon a metal sheet, the effect of obliquity of incidence can be investigated in the following simple manner. The material is painted upon the metallic receiver of a thermopile mounted so as to be rotatable about a vertical axis lying in the plane of and bisecting the receiver. With a pencil of radiation falling upon the test surface the deflection of a galvanometer connected to the thermopile is proportional to the fraction of the incident energy absorbed, and will in general vary as the surface is rotated. The information obtainable in this way, together with an absolute measurement of the reflection coefficient of the surface for normal incidence with the same type of radiation, enables the variation of its reflection coefficient to be examined over a wide range of incidence.

* H. E. Beckett, *Proc. Phys. Soc.* **43**, 227 (1931).

§ 3. APPARATUS

A convenient form of thermopile, whose single thermo-electric element is constructed with fine wires (46 s.w.g.) of copper and constantan, is shown in figure 1. Parallel constantan wires stretched across an ebonite frame support two receivers of copper foil, at the centres of which the copper leads are attached.

Both receivers are coated with the test material and, being of equal size (2 cm. × 1 cm. × 0·1 mm.), are similarly controlled by the radiative and convective effects of their surroundings. An extremely stable zero is thus assured. It has been shown experimentally that, as a result of the high thermal conductivity of the copper foil, there is no appreciable variation in sensitivity over the surface of the receiver.

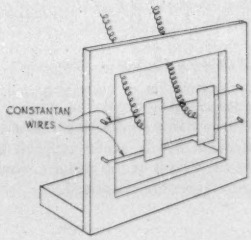

Fig. 1. Thermopile for reflection tests.

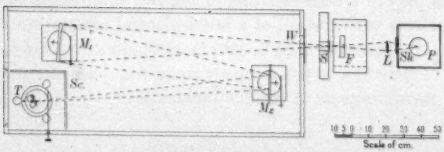

Fig. 2. Plan of apparatus.

P, pointolite lamp; *Sh*, shutter; *L*, lens; *F*, filter; *S*, slit; *W*, window; M_1, concave mirror; M_2, plane mirror; *Sc*, blackened screen; *T*, thermopile.

The apparatus must be enclosed within a well-blackened box to avoid the exceedingly variable effects of draughts and to ensure that no radiation reflected from the receiver is returned to it by reflection from surrounding surfaces. With the thermopile mounted upon a worm-driven turn-table, rotation can be effected by means of a key inserted through a small hole in the side of the box, while the angular position of the turn-table can be read through a window.

The radiation is conveniently produced and controlled by means of the apparatus described previously. Tests can then be made with the artificial sun (Pointolite and gold filter) or at various wave-lengths determined by the filters adopted. It is

essential that the image projected upon the test surface should be in the form of a narrow rectangle, so that even with large angles of incidence the entire image is intercepted by the test surface. With an image measuring 0·8 mm. in width the limiting angle of incidence is 85°.

The general lay-out of the apparatus is shown in figure 2.

§ 4. CALIBRATION OF RADIATION-RECORDER

Apparatus of the above type has been employed in the calibration of the spherical receiving surfaces of an instrument designed by Mr Dufton of the Building Research Station for recording the intensity of solar radiation. Such instruments are commonly provided with receiving surfaces lying in a horizontal plane and, with a low sun, must give very erroneous readings owing to the rapid increase of the reflection coefficient of a blackened surface as grazing incidence is approached. Mr Dufton's instrument embodies two spherical receiving surfaces, painted black and white, whose effective reflection coefficients should be independent of the height of the sun.

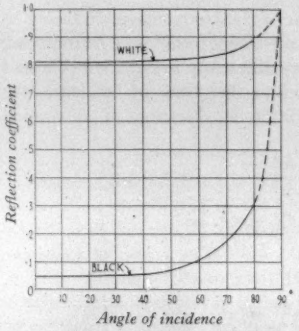

Fig. 3. Reflection coefficients of black and white painted surfaces.

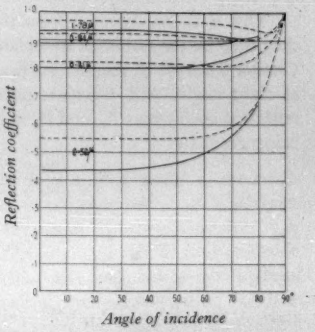

Fig. 4. Reflection coefficients of polished copper.

It may be shown that the energy of radiation absorbed by a sphere of radius r is equal to

$$\pi r^2 I \int_0^{\pi/2} (1 - R_\theta).\sin 2\theta . d\theta,$$

where R_θ is the reflection coefficient of the surface for the angle of incidence θ, and I is the intensity of radiation. Since πr^2 is the projected area of the sphere, the value of the integral may be termed the effective absorption coefficient of the spherical surface.

With the artificial sunlight of the Pointolite and gold film combination the reflection coefficients at normal incidence of the black and white cellulose paints used in the sunshine recorder were found to be 0·051 and 0·813 respectively.

The variation of the reflection coefficients with obliquity of incidence is shown in figure 3. Each point represents the mean of observations on each side of normal

incidence, so that any slight inaccuracy in the assumed position of normal incidence should not lead to appreciable error. From the curves the effective absorption coefficients of the black and white spherical surfaces have been computed as 0·899 and 0·175 respectively.

§ 5. OBSERVATIONS ON POLISHED COPPER

In view of the considerable theoretical interest attaching to metallic reflection, the effect of obliquity of incidence upon the reflection coefficient of polished copper has been investigated. For this purpose suitable bands of radiation were separated by means of filters from the spectrum of a Pointolite lamp, and were allowed to fall directly on the polished copper receiver of the thermopile.

The results obtained are shown in the full curves of figure 4, the means of readings on both sides of normal incidence being again used. One characteristic should be noted; at the longer wave-lengths a dip in the reflection curves is found which becomes more pronounced and at the same time shifts towards the angle of grazing incidence as the wave-length increases.

This is to be expected from the application of the electromagnetic theory to the problem, the results of which are quoted in convenient form in the *International Critical Tables**. A metallic surface behaves very differently to radiation polarized at right angles and radiation polarized parallel to the plane of incidence, the

$R_\perp$, $R_\parallel$ reflection coefficients $R_\perp$ and $R_\parallel$ of the surface for the two directions of polarization being given by

$$R_\perp = \frac{(m_\theta - \cos\theta)^2 + m_\theta^2 . k_\theta^2}{(m_\theta + \cos\theta)^2 + m_\theta^2 . k_\theta^2}$$

and

$$R_\parallel = R_\perp \cdot \frac{(m_\theta - \sin\theta . \tan\theta)^2 + m_\theta^2 . k_\theta^2}{(m_\theta + \sin\theta . \tan\theta)^2 + m_\theta^2 . k_\theta^2},$$

m_θ where $2m_\theta^2 = [(n^2 - n^2k^2 - \sin^2\theta)^2 + 4n^4k^2]^{\frac{1}{2}} + (n^2 - n^2k^2 - \sin^2\theta)$

and $2m_\theta^2 . k_\theta^2 = [(n^2 - n^2k^2 - \sin^2\theta)^2 + 4n^4k^2]^{\frac{1}{2}} - (n^2 - n^2k^2 - \sin^2\theta)$

and θ is the angle of incidence;

n_θ n_θ the index of refraction for angle of incidence θ;

n n the index of refraction for normal incidence;

k_θ k_θ the index of absorption for angle of incidence θ; and

k k the index of absorption for normal incidence.

From the above expressions it is found that whereas the reflection coefficient $R_\perp$ rises smoothly from its value at normal incidence to the value 1·0 at grazing incidence, the coefficient $R_\parallel$ first falls to a minimum value and then rises steeply to the value 1·0. The reflection coefficient of the surface for unpolarized incident light is $\frac{1}{2}(R_\perp + R_\parallel)$ and the graph obtained by plotting this quantity against angle of incidence may or may not show a dip according to the values of the constants n and k.

* *International Critical Tables*, 5, 248.

At wave-lengths in and near the visible spectrum it is not practicable to determine n and k for a metal except by a reverse application of the theoretical expressions. Usually these constants are computed from observations of the ellipticity of polarization of light reflected from the metal surface at various angles of incidence.

A number of such determinations for copper are quoted in the *International Critical Tables*, and it is of interest to compare the reflection coefficients which can be derived from them with the present measurements. The constants n and k vary somewhat according to the manner of preparation of the copper specimen, but the values given in table 1 seem to be fairly representative of copper which is thick enough to be opaque. The wave-lengths chosen are those about which the energy transmitted by the various filters used in the present investigation is equally shared.

With these values the variation of the reflection coefficient with angle of incidence is shown in the dotted curves of figure 4. The curves show the same general characteristics as those determined by the thermopile method, although the quantitative agreement is poor.

Table 1. Optical constants of polished copper

Wave-length (μ)	n	k
0·50	1·17	2·03
0·61	0·56	5·65
0·84	4·35	10·4
1·78	0·73	13·0

Some of the difference is certainly due to the finite spectral width of the bands of radiation transmitted by the filters. In order to compute reflection coefficients which shall be strictly comparable with those measured experimentally, it is necessary to take into account the spectral distribution of the filtered radiation and the variation of n and k with wave-length. The data given in the *International Critical Tables* enable this to be done at the two shorter wave-lengths, and it is found that for normal incidence the correction lowers the computed coefficients appreciably, in one case below that found experimentally (see table 2). At the two longer wave-lengths the correction can only be small, since the reflection coefficient of copper does not vary rapidly in this part of the spectrum.

Table 2. Reflection coefficients of polished copper at normal incidence

Filter		Reflection coefficients			
			Computed from n and k		
No.	Nominal wave-length (μ)	Measured	At nominal wave-lengths	Corrected for spectral distribution	Standard, corrected for spectral distribution
I	1·78	0·94	0·969	—	0·943
II	0·84	0·89	0·922	—	0·886
III	0·61	0·80	0·821	0·782	0·699
IV	0·50	0·43	0·548	0·529	0·418

Even so there remain appreciable differences between the measured and computed values, which would appear to indicate that the standard values of n and k do not hold accurately for the specimen of copper used. Unfortunately facilities for checking these constants by the ordinary methods have not been available.

Tool* has stated that n and k can be affected by the presence of films of impurity derived from commercial metal polishes, one of which was used in the present experiments. The negligible smallness of such an effect has, however, been shown by further reflection tests with rouge-polished specimens, which have yielded coefficients almost identical with those previously found.

In conclusion it is noted that the standard reflection data for copper quoted in the *International Critical Tables* (see table 2) show considerably better general agreement with the present measurements than with the computed figures.

* A. Q. Tool, *Phys. Rev.* **31**, 1 (1910).

A DIRECT-READING γ-RAY ELECTROSCOPE

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Communicated by B. L. Worsnop, March 14, 1932. *Read and discussed April* 15, 1932.

ABSTRACT. A dead-beat direct-reading γ-ray electroscope having a linear scale graduated in milligrams of radium is described. It is a combination of a special ionization chamber, Lindemann electrometer and high resistance, and allows the estimation, in less than three seconds, of the activity of small γ-ray sources of the order 1 mgm. Ra with an accuracy of $\frac{1}{2}$ per cent on a full scale deflection.

§ 1. INTRODUCTION

THIS instrument was designed for the rapid estimation of the γ-ray activity of the small radio-active appliances, usually called "radon seeds*," which are employed therapeutically in large numbers in hospitals and radium institutions.

A high degree of accuracy is not called for in an instrument for this purpose; the chief requirement is speed of operation, so that the time of measurement and exposure of the operator may be reduced to a minimum.

The well-known circuit of figure 1 has been applied in the convenient form shown in figure 2: the ionization current set up in a chamber C of special design is passed through a high resistance R, and the steady potential difference between the ends of the resistance is measured by means of a Lindemann electrometer L. When certain precautions are taken, the deflection of the electrometer needle is proportional to the γ-ray activity of the source, and if the constants of the instrument are suitably adjusted the deflection can be read off directly in millicuries or milligrams of radium on the eyepiece scale of the observing microscope.

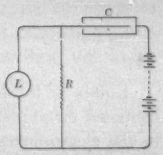

Fig. 1. Circuit.

The Lindemann electrometer is particularly suitable for measurements of this kind, because it is dead-beat and has a high sensitivity, a short period, a linear scale and a stable zero.

* Radon seeds consist of small tubes of gold, platinum, or silver, into which are inserted fine sealed glass capillaries containing a few millicuries of radium emanation. The emanation and its active deposit emit α-, β-, and γ-rays. The α-rays and most of the β-rays are absorbed by the glass and metal containers, so that a seed furnishes a compact γ-ray source, the activity of which decays at approximately the same rate as that of radon, the half-value period being 3·86 days. A typical radon seed would be a platinum tube 5 or 6 mm. long, 1·5 mm. in diameter, with a wall-thickness of 0·5 mm., and containing from 0·5 to 2·5 millicuries of radon.

§2. DESIGN OF THE IONIZATION CHAMBER

In order to avoid the difficulties associated with the use of very high resistances, it is desirable to work with currents as large as possible. Experiments were therefore made to find out what sort of chamber best utilized the feeble ionizing power of the small γ-ray sources concerned. So far, the investigation has been confined to air chambers at atmospheric pressure, because the use of heavy gases for increasing the currents was considered unsuitable for routine work.

Whatever the shape of the chamber, the currents must be approximately saturated and the insulated electrometer system must be carefully protected against ionization leakage to surrounding earthed conductors.

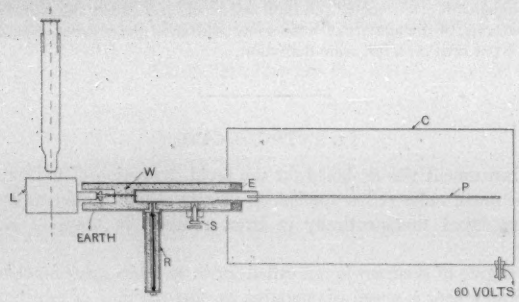

Fig. 2. Constructional details.

C, metal casing; *P*, plate electrode; *L*, Lindemann electrometer; *R*, high resistance; *W*, paraffin wax; *S*, spring earthing plunger; *E*, ebonite collar.

Losses of the latter kind are avoided in the apparatus shown in figure 2: an insulated metal plate *P*, totally enclosed by a charged metal casing *C*, is connected to the electrometer by a lead which passes down the centre of an earthed tube filled with paraffin wax; the latter is recessed at its outer end so as to enclose the electrometer terminal. The whole arrangement was tested for ionization leakage by measuring the rate of increase of potential of the insulated system, with the high resistance removed, when exposed to a weak γ-radiation. The absence of ionization losses is proved by the linearity of the curve so obtained (full line, figure 3) which shows that the current to the electrometer is constant and independent of the potential of the system. When leakage is present a falling-off of current is observed at the higher potentials (dotted line, figure 3). Under the latter conditions, when the high resistance is replaced, the current flowing through it will always be less than that arising in the chamber itself, and no proportionality can be expected between the potential across the ends of the resistance and the γ-ray activity of the source.

When a chamber satisfactory in this respect had been obtained, a properly shielded high resistance of suitable magnitude was made up in the manner detailed

below, and fitted to the lead-in tube, figure 2. Thereafter this part of the circuit was not changed, so that the currents obtained from different chambers under various conditions could be conveniently compared by observing the steady deflection of the electrometer needle.

First of all, the saturation voltage was determined for two rectangular chambers measuring respectively 21·5 × 23 × 12 cm. and 22 × 23 × 24 cm., each provided with a rectangular inner electrode whose dimensions were a little smaller than those of the outer casing. The size of the inner plate was found to be unimportant,

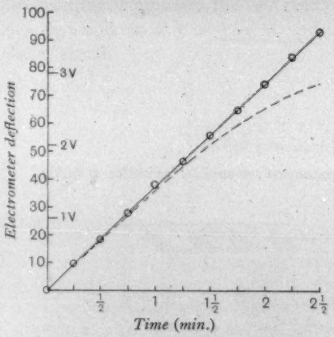

Fig. 3. Test for presence of ionization leakage.

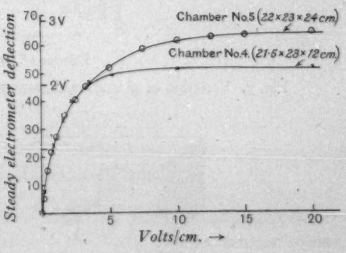

Fig. 4.

as might be anticipated, since all the ions generated in the air space are swept to the plate. The γ-ray source, a platinum needle 25 mm. long and containing 5 mg. of radium, was placed where it gave the strongest ionization current, viz. in the centre of the top of the metal casing *C*. As will be seen from figure 4, a field of 10 to 15 V./cm. brings the currents sufficiently near to saturation for readings proportional to the strengths of the sources to be obtained.

The influence of the volume of the chamber on the magnitude of the saturation currents was next investigated by fitting rectangular chambers of different sizes. In each case the 5-mg. needle was placed on the outer casing, in the centre. Figure 5 and the table show how the saturation current depends upon the dimensions of the chamber.

Table. Variation of saturation current with dimensions of chamber

No. of chamber	Dimensions* (cm.)	Volume (cm.^{3})	Relative saturation currents (scale divisions)
1	13 × 11 × 12	1,720	22
2	21·5 × 23 × 6	2,960	36
3	16 × 17 × 12	3,260	33
4	21·5 × 23 × 12	5,920	39·5
5	23 × 22 × 24	12,200	46

* The last figure gives the height of the chamber.

Evidently there is not much to be gained by using a volume greater than 5 or 6 litres.

Now in all of the above measurements only half of the radiations issuing from the radioactive substance actually passed through the ionization chamber, and an obvious way of augmenting the current is to place the source inside the chamber so that all its radiations are utilized. The change was found to double the current approximately, but in practice this procedure is inconvenient and has also the disadvantage that it restricts the use of the instrument to the measurement of pure γ-ray sources.

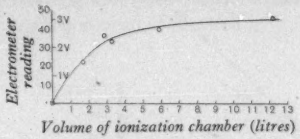

Fig. 5. Variation of saturation current with volume of rectangular ionization chamber.

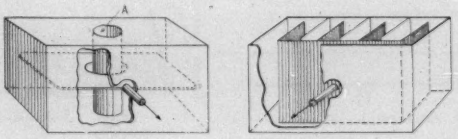

Fig. 6.

A type of chamber which is free from these defects and yet makes use of nearly all the rays emitted by the source is shown in figure 6 (*a*). The radium needle was placed half-way down the axis of the tube *A*, which had a diameter of 4 cm.

With this arrangement, the saturation current was only slightly greater than that obtained from a simple rectangular chamber of the same dimensions with the same radium needle on the outer casing; it would appear from this and the foregoing experiments that the bulk of the ionization effects arise in the immediate vicinity of the source.

This particular form of chamber was found, however, to possess the useful property that the currents obtained from it were practically independent of the position of the source within the limits of a cylindrical zone (about 4 cm. high and 1 cm. in diameter) in the middle of the tube *A*; consequently the activity of all sources, whatever their size and distribution, can be directly compared, so long as they can be contained by a cylinder of these dimensions. The small gain in current obtained with this pattern did not warrant its adoption for ordinary work, and its use was therefore confined to the comparison of sources of unequal size. Further efforts were made to improve on the simple closed box type.

An increase of current was sought by making use of the well-known fact that a large proportion of the ionization is due to photo-ions liberated from the metallic

walls of the chamber. Successful attempts were made to increase the photoelectric emission in two days, firstly by fixing a multiplate electrode system, figure 6 (*b*), designed to present a large surface to the radiations, and secondly by lining the chamber with different metals.

The saturation currents recorded with the multiplate electrode were about 20 per cent higher than with a single-plate electrode, and the effect of partially lining the chamber with aluminium, brass and lead was to give currents in the ratio 38 : 42 : 46. Thus as far as can be seen from these experiments on air chambers at atmospheric pressure, the best results may be expected from a lead-lined chamber having a volume of 5 or 6 litres, and provided with a multiplate electrode of some heavy metal.

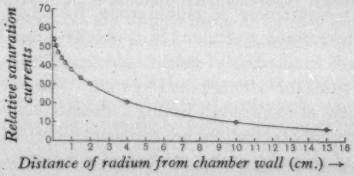

Fig. 7. Variation of current with position of radium.

The maximum γ-ray ionization current yielded by such an arrangement is roughly 5×10^{-11} A. per mg. of radium, although this value may be reduced somewhat by the filters necessary to cut off β-radiation from unscreened radon tubes.

The variation in the electrometer reading as the radium is moved about on the top of the casing of a simple rectangular chamber is remarkably small. In fact, within a radius of 2 cm. from the centre no difference at all can be detected. There is, however, a marked reduction of current as the source recedes from the chamber (figure 7).

§ 3. CONSTRUCTION OF THE HIGH RESISTANCE

For the purpose of this instrument a resistance of the order $10^{10}\ \Omega$ is required. A satisfactory graphite leak can be constructed in the following manner: a heavy pencil line is drawn between two terminals on a strip of some hard highly insulating material such as orca or ambroid. The graphite streak is polished vigorously with cotton wool until most of it has been removed; the resistance is tested from time to time by connecting it to the ionization chamber, and the polishing process continued until a suitable value is obtained. Formed in this way, the graphite layer is firm and durable and is not easily damaged. Finally the resistance unit is mounted in a metal tube and sealed up with paraffin wax, which serves the double purpose of eliminating ionization losses and also of protecting the graphite film from the atmosphere. The waxing process sometimes alters the value of the resistance slightly, especially if the wax be poured in too hot.

§4. GENERAL REMARKS ON THE USE OF THE INSTRUMENT

The instrument is calibrated by laying a γ-ray source of known value on the top of the chamber and adjusting the sensitivity of the electrometer until the steady deflection of the needle corresponds to the numerical strength of the radium standard. The activity of other γ-ray sources of similar filtration placed in the same position can then be read directly in milligrams or millicuries on the eyepiece scale. An accuracy of about $\frac{1}{2}$ per cent on a full scale deflection can be obtained if the calibration process is repeated at frequent intervals.

Sources of different diameters may be mounted in a V-shaped trough at the side, so that their centres are at the same distance from the wall of the chamber, or they may be compared by using the special chamber of figure 6 (*a*).

To expedite the measurement of radon seeds, for which this apparatus was primarily intended, the screening of the radium standard and its distance from the chamber are simulated, in the case of naked radon capillaries, by the insertion of an aluminium disc of a thickness determined by trial. Discs of other thicknesses are used in the measurement of radon seeds having a filtration different from that of the radium standard. Screening corrections are automatically eliminated by this device.

It should be noted that the group reading of a number of radon seeds will be less than the sum of the individual strengths, unless the seeds are set sufficiently far apart (about 5 mm.) for the mutual screening of the oblique rays to be negligible.

The range of measurement can be extended by varying the electrometer sensitivity, by changing the value of the high resistance and by altering the position of the radio-active source. The effects of stray γ-radiation can be nullified by displacing the zero of the scale so that no corrections are needed for measurements carried out near other radium. Adequate protection for the operator can be provided, for since the instrument functions on current, the lead-in tube can be made as long as desired without loss of sensitivity.

Further experiments are at present in progress on high-pressure air-chambers, the results of which will be published in the *Journal of the British Institute of Radiology*.

DISCUSSION

Prof. F. L. Hopwood emphasized the remark made at the end of the paper as to the importance of protecting the operator. In the absence of suitable protection the latter would be constantly exposed to radiation which, though its intensity might be too small to produce early effects, would be very harmful if endured during a long period.

Prof. A. O. Rankine suggested that the necessity for frequent calibration with a sub-standard comprising a known γ-ray source could be avoided by the use of a standardized e.m.f.

Mr R. S. Whipple suggested that a potentiometer might be incorporated in the instrument for the purpose mentioned by the last speaker.

Author's reply. I am not sure whether the high resistance remains sufficiently constant for the method suggested by the last two speakers to be reliable.

STUDIES IN INTERFEROMETRY

(1) A NEW TYPE OF INTERFERENCE REFRACTOMETER

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Received March 7, 1932. Read and discussed April 15, 1932.

ABSTRACT. The importance of accurate refractive-index measurements in relation to spectroscopy is emphasized and the various methods hitherto employed are compared. A new type of interferometer is described in which the beam from the central part of the collimator objective is divided and each half laterally displaced. After passing through the gas tubes, the beams are again reunited so that the interference pattern at the focal plane of the telescope objective is similar to that from a two-plate transmission échelon. The brightness of the fringes is approximately twenty times greater than with the usual Rayleigh interferometer, so that the instrument can be combined with a spectrograph for dispersion measurements. A new method, based on Benoît's "fractional-part" principle, is developed for this purpose.

§1. INTRODUCTION

The accurate determination of the refractive indices of gases has become very important in spectroscopy. Within recent years, very considerable progress has been made in the standard of accuracy within which the wave-length of a spectral line can be determined in terms of that of the standard red cadmium line. This increased accuracy is in part due to improved technique and knowledge as to the best methods of exciting the source, and in part to improved apparatus and methods for effecting the intercomparison. For example, the wave-lengths of some of the sharp satellites in the line spectra of certain heavy elements can be determined to within one or two units in the fourth decimal place in Ångstroms, yet even when such a line is close to that of red cadmium so that the conditions of pressure and temperature during exposure are of little account, an uncertainty of only one unit in the seventh decimal place of the refractive index of dry air at 15° C. and 760 mm. pressure may introduce an error of six units in the fourth place in Ångstroms in the vacuum wave-length required for theoretical calculations. It is fairly safe to say that the probable error in the tables of the refractive index of air in the range of 2000 to 8000 Å. is at least three or four units in the seventh place, so that an experimental accuracy of $\pm$ 0·001 Å. at 6000 Å. will involve an uncertainty of $\pm$ 0·002 Å. when a conversion to vacuum wave-length is made. In the ultra-violet, the uncertainty measured in Ångstroms will be proportionally smaller, but the uncertainty in frequency or wave-number (which is the quantity required for theoretical work) becomes much greater.

§2. METHODS

The various interferometric methods that have been employed to determine the refractive indices of gases divide themselves sharply into one of two classes: (A) Methods in which the total path-difference involved is due to change of refractive index obtained by exhausting or filling one tube; and (B) Methods whereby only a change in an already large path-difference is determined, corresponding to the change of the refractive index from its normal value to unity as the vessel is exhausted.

The various methods in which a Fresnel bi-prism or the Jamin or Rayleigh interferometers are used form examples of the (A) class. Rentschler's* method of using a Fabry-Perot étalon is the only example of the (B) class that has hitherto been used, although other methods could be devised. Thus Köster's† application of the Twyman and Green‡ interferometer for wave-length measurements could be easily adapted for refractive index measurements by enclosing the gauge or reference block in a chamber which could be exhausted.

The most complete set of data on the refractive index of air is the now classical work of Meggers and Peters§ at the Bureau of Standards. They employed Rentschler's method, and their tables are those in general use for converting wave-lengths determined in air to the corresponding vacuum values.

The Fabry-Perot method, used in combination with a grating, enables the determination to be made simultaneously on a large number of lines, and it can be extended by suitable means to the near infra-red and ultra-violet. For convenience and rapidity the method could hardly be improved upon, but for obtaining the greatest accuracy it seems fundamentally weak. The refractive index μ is given in this method by the ratio of the orders of interference n_{air} and $n_{\text{vac.}}$ determined by the photographs taken respectively with an air-filled and an exhausted Fabry-Perot interferometer.

It is generally accepted that under very favourable conditions of a very sharp line and a high reflection coefficient for the silver films, the fractional part can be determined to even better than 1 per cent. When, however, the fringes become broad, either because the line is in a region where the reflection coefficient of the films is reduced, or because the actual width of the line itself becomes an appreciable fraction of the range (in Ångstroms) between successive fringes, the accuracy within which the fractional part can be determined is considerably reduced. In Megger's and Peter's experiments the value $(n_{\text{air}} - n_{\text{vac.}})$ varied between 5 and 25 with interferometer-plate-separations varying between 3 and 25 mm. A final accuracy of 1 per cent in the determination of the fractional part means an uncertainty of between 1 and 6 units in the seventh decimal place for μ_{air}. The inherent defect of the method is that if larger plate-separations of the interferometer were

* *Astro. J.* **28**, 345 (1908).
† *Z. f. Feinmech.* **34**, 55 (1926); and see also Weber, *Phys. Z.* **29**, 233 (1928).
‡ *Phil. Mag.* **35**, 49 (1918).
§ *Bull. Bur. Stand.* **14**, 697 (1918).

chosen, there would be a corresponding reduction in the accuracy within which the fractional parts could be determined, so that increased accuracy could only be obtained by using more homogeneous light-sources. While lines of far greater homogeneity than the iron lines are well known, these do not suffice to yield representative values throughout the spectrum. It is therefore desirable to consider the possibilities of methods of the first class, the final accuracy of which do not depend on the homogeneity of the spectral line which is used.

A Jamin interferometer with tubes 100 cm. in length will show a displacement of about 560 fringes with a wave-length of 5000 Å. when one of the tubes is exhausted from atmospheric pressure. If the displacement be measured to within a tenth of a fringe, the possible error in μ is approximately half a unit in the seventh decimal place. Owing to the greater length, the difficulties of temperature- and pressure-measurements are now much greater, but they should not be insuperable since this very much greater accuracy only requires that the initial temperature and pressure be accurately measured to within 0·05° C. and 0·1 mm. respectively. The main difficulty with the Jamin interferometer is that it cannot be conveniently combined with a spectrograph. It is dependent, even in visual work, on an artificial fiducial mark or cross wire in the eyepiece, which would in no way take account of any small instrumental change (such as the tilting of a Jamin block or a small twist of the framework) that might occur during or between the two exposures.

The Rayleigh interferometer has a fiduciary set of fringes formed by the light traversing that part of the interferometer which is not occupied by the tubes. Any accidental disturbance or deformation of the interferometer itself would cause an equal displacement of both sets of fringes. By using the method in which coincidence of the upper and lower fringes is adjusted by means of a calibrated compensation plate, a far higher standard of accuracy can be attained than by estimating the position of a cross-wire between two fringes. It is generally accepted that an accuracy of about $\frac{1}{80}$ to $\frac{1}{40}$ of a fringe can be obtained by means of this coincidence method.

While the Rayleigh interferometer forms the most accurate means of determining the refractivity of gases, it can only be employed with the very strongest monochromatic radiations, such as the lines from a mercury arc. White light fringes can be employed for many industrial applications of the instrument provided the path-difference introduced is not too great, and it is now customary in such cases to use a Pointolite lamp to obtain the fringes with maximum clearness or visibility and comfortable brightness.

In order to obtain dispersion curves, the method of selecting a narrow range of the white-light spectrum has been suggested. This narrow band is then to be passed into the Rayleigh interferometer, and the fringe-displacement is supposed to give the refractive index for the mean wave-length. This method is open to the very serious objection that it may not be the ordinary phase refractive-index μ_0 that is measured but a group index μ_g defined as (velocity in vacuo)/(velocity of group in medium), in which case μ_g

$$\mu_g = \mu_0 \left(1 - \frac{\lambda}{\mu_0} \cdot \frac{\partial \mu_0}{\partial \lambda}\right).$$

The reality of this effect is shown by the abnormal displacement of the fringe system obtained by R. W. Wood* with sodium vapour, when the Zeeman component of the helium line used as the source was drawn into the anomalous region of sodium by a suitable magnetic field. In this region $\partial\mu/\partial\lambda$ was so great that the effect of this term caused a displacement twice as large as that which would be obtained by the phase index alone. The dispersive power of air is so low that $\left(\frac{\lambda}{\mu_0}\frac{\partial\mu}{\partial\lambda}\right)$ is of the order of 0·00001, yet the absolute refractive-index is required to such a high precision that this uncertainty cannot be allowed. The difficulty is to know at what point the band of spectrum transmitted into the Rayleigh instrument can no longer be regarded as monochromatic radiation, so that it is the group velocity or refractive-index that is being measured. Wood's experiment shows that it must depend on the magnitude of $\partial\mu/\partial\lambda$, since in the anomalous region of sodium the relatively monochromatic Zeeman component of the helium line D_3 must be considered as a narrow band of continuous radiation. The writer hopes to carry out some experiments on this problem in the near future.

§3. DESCRIPTION AND EXPLANATION OF NEW METHOD

The fundamental disadvantage of the Rayleigh interference refractometer arises out of the fact that the primary slit of the collimator must be kept very narrow. As this slit is opened, the brightness of the fringe system increases rapidly, but after a certain point the visibility (as defined by Michelson) or clearness rapidly deteriorates until, when the slit subtends at the near nodal point of the collimator lens an angle equal to the angle between the fringes, all traces of a fringe pattern vanish. This is analogous to Fizeau's† method of finding the angular diameters of stars, which has been so successfully developed by Michelson.

W Since the angle between the fringes is given by λ/W, where W is the distance between corresponding points of the double slit, the obvious way of increasing the brightness would be to reduce W to its minimum value. In the standard commercial instrument the width of each double slit is 4 mm. and W is 12 mm. This latter value cannot be substantially reduced since a slight lack of alignment of the long gas tubes would effectively vary W. It is further desirable that the light should pass through the centre of a tube and not near the walls.

It will be remembered that in order to apply Fizeau's method to measure the angular diameter of stars, Michelson‡ employed two outer mirrors, with separation b, to receive the light from the distant star, and two inner mirrors, with separation d, to project the light beams into the telescope. This enabled him in effect to magnify the star, as far as interference fringes were concerned, in the ratio (b/d); for although the angle between the fringes remained at (λ/d), the fringes disappeared when the star subtended an angle $K(\lambda/b)$, K being a constant whose value is 1·22 for a uniformly glowing disc.

b, d

K

* *Phil. Mag.* **8**, 320 (1904).

† *Compt. Rend.* **66**, 934 (1868).

‡ *Astro. J.* **51**, 257 (1920).

The ordinary double-slit arrangement of the Rayleigh instrument, shown for reference in figure 1 (*a*), is modified as shown in figure 1 (*b*). A single slit S_b of width $2a$ is used and the beam is divided and displaced by some optical device so that the two beams have the same relative separation W as before. As far as the observing telescope is concerned, the angle between the fringes will still be (λ/W), but the primary slit Pa may now be opened until it subtends an angle $\frac{1}{2} \times 1{\cdot}43\lambda/a$ at the collimator objective before the fringes disappear. The arrangement in fact practically amounts to a Michelson star interferometer used in the reverse way, with the difference that the distance between the outer beams is fixed, and that we are using the arrangement for an entirely different purpose—to increase the brightness of the fringes.

While the same standard of visibility of the fringes is retained, the primary slit can now be opened to a width $0{\cdot}715W/a$ of its previous value, so that the brightness* of the fringes is increased as the square of this ratio. This, with the dimensions of W and a already quoted, should give an increase in brightness of 4·6 times if the losses at the displacing device be neglected.

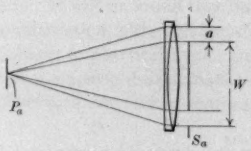

Fig. 1 (*a*).

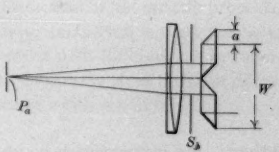

Fig. 1 (*b*).

The correctness of the above conclusions was first tested through the cooperation of Messrs A. Hilger Ltd. by using small totally-reflecting prisms as shown in figure 1 (*b*). It was found extremely difficult to work the tiny prisms to the necessarily very high standard required in interferometry. The optical retardation in each had to be identical, and the working of a long parallel strip prior to cutting in two proved impracticable owing to the springy nature of the very thin plate. Finally, it was found difficult to adjust two such small prisms and to hold them without distortion. Hence, even at the expense of a much greater loss of light, the well-known method of Albrecht's rhomb (see figure 2) was employed to separate the two beams. The first trial with an Albrecht rhomb was unsuccessful, and no

* If the image of the source were very carefully focussed by means of a wide-angled condenser lens system on an extremely *thin* slit, the gain in brightness would only be 2·145 times. Such a slit cannot be made. In practice, the usual spectroscope slit can be compared to a narrow opening between long parallel walls so that the oblique pencils from the condenser cannot enter. The light that reaches the objective enters the slit approximately as a parallel beam. When this slit is sufficiently narrow so that the whole objective is filled by the central maximum of this slit, a further reduction in slit-width to a half will reduce the intensity of the final image to a quarter since not only is the flux of light that enters the slit halved, but the light that enters is diffracted over twice the angle and consequently only half of it can enter the aperture. This is the principle of Vierordt's method of photometry, and explains the desirability of spectrographs having collimators of much longer focal length than telescopes or cameras, in which case the slit can relatively be much wider.

fringes could be obtained. By placing a narrow cross slit on the primary slit, the reason of the failure became evident—the point image from one half of the beam was slightly lower than that from the other, showing that the rhomb had a small pyramidal error of a few seconds of arc. When this was remedied, the fringes were easily found when the rhomb was placed symmetrically about the aperture and with its refracting edges parallel to the primary slit. It was verified that the primary slit could be opened two or three times wider than before without loss of visibility of the fringes.

The problem of the gain in intensity or brightness was more difficult, but on comparison of the brightness with various neutral-tint plates in the path of the modified instrument, the fringes had approximately the same brightness as in the original Rayleigh interferometer when the plates reduced the brightness to a quarter or a fifth. The exact value is relatively unimportant, the main point being to prove that the gain in brightness is more than proportional to the increased width of the primary slit.

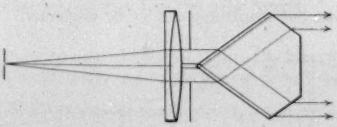

Fig. 2

Hitherto, the instrument behaves (apart from the increased brightness of the fringes) as a standard Rayleigh interferometer; the angle between the fringes and the distribution of light among the fringes remain exactly as before. The fringes at the focal plane of the telescope objective are so close together that a very short-focal-length cylindrical rod of glass must be used to magnify them sufficiently. This, not being achromatic, requires refocussing in different parts of the spectrum, so that in spite of the increased brightness of the fringes, the modified Rayleigh instrument cannot be successfully combined with a spectrograph.

The success of the first separation of the two beams from the centre naturally led to a trial of the reverse process at the telescope end. When the two beams are recombined by a second rhomb or analogous device, their separation W on entering the telescope objective is reduced to a third. The angle between the fringes increases from $(\lambda/3a)$ to $(0{\cdot}715\lambda/a)$ and the focal length of the eyepiece can be doubled while the same magnification as before is retained. As far as the telescope is concerned, what we now have is a two-plate transmission échelon, and the six or eight fringes obtained with the standard Rayleigh are reduced to the two or three orders seen with an échelon grating. It seems very difficult to derive an expression for the gain in brightness due to the reduction of the number of fringes visible, but practical experiment indicates that the gain on recombination of the beams is even greater than that obtained at the first stage. One contributory factor is that the reflection losses at the second rhomb are now much smaller, since the beams emerging from the first rhomb are, to a considerable extent, polarized.

The nett overall gain of brightness when two rhombs are used as described has not been carefully measured, but a conservative estimate would be about twenty times. The increase in brightness will be practically appreciated when it is realized that the white-light fringes with a 2-volt pocket-lamp source are brighter than the fringes of the standard Rayleigh with a Pointolite lamp source. While this is of value in making the apparatus more portable, the real advantage lies in the fact that it now becomes possible to carry out refractive-index measurements with the lines of helium, neon, etc., while previously the use of the Rayleigh with monochromatic light was confined to the stronger lines of mercury.

§4. THEORY OF THE SEPARATION AND INTENSITY OF THE FRINGES

The separation and intensity-distribution curves of the fringes bear little relation to those of the usual Rayleigh double-slit fringes, so that a theoretical analysis is desirable. It will be found that the fringe-displacement is not strictly proportional to the retardation, although the discrepancy is so small that it need not be taken into account in practical work. Suppose we have a path-difference p between the two beams entering the telescope. The resultant amplitude A_θ of the light diffracted in a direction θ due to two contiguous apertures of width a will be

p
A_θ
θ, a

$$2A_N \frac{\sin(\pi a\theta/\lambda)}{\pi a\theta/\lambda} \cdot \cos\left(\frac{\pi a\theta}{\lambda} + \beta\right) \qquad \text{.....(1),}$$

when 2β is the phase-difference, for a wave-length λ, due to the optical path-difference p, and A_N is the amplitude due to one slit in the normal direction. If X be written for $\pi a\theta/\lambda$ the expression becomes

β
A_N, X

$$2A_N \frac{\sin X}{X} \cdot \cos(X + \beta) \qquad \text{.....(1a).}$$

The distribution of amplitude in the fiduciary half of the field of view is given by the above expression when $\beta = 0$. This is proportional to $\sin\alpha/\alpha$, where $\alpha = 2X$ and corresponds to the pattern of a *single* slit of width $2a$.

α

If the central maximum is given for $\theta = 0$, the first minimum (zero) on either side corresponds to $\theta = \pm\lambda/2a$ and the secondary maxima occur at $\pm 1{\cdot}43\lambda/2a$. The separation of the fringes is then $0{\cdot}715\lambda/a$ and not λ/a as is found by considering the variation in the cosine term alone. For convenience, Schwerdt's factor has been determined more closely and it is found to be 1·43029.

For a fixed phase-difference 2β between the beams, the positions of the maxima are given by equating to zero the differential coefficient of the expression (1 *a*) with respect to X, whereupon we get the following equation:

$$\tan\beta = \frac{2X\cos 2X - \sin 2X}{2X\sin 2X - 2\sin^2 X} \qquad \text{.....(2),}$$

When β (half the phase-retardation) $= 0$ or π; $\tan\beta = 0$ and, from equation (2),

$2X \cos 2X = \sin 2X$. This is of the well-known form $\alpha = \tan \alpha$, giving maxima when $2X = 0$ and $2X = 1{\cdot}43029\pi$. The angle between the fiduciary fringes is then

$$\theta = 1{\cdot}43029\lambda/2a \qquad \text{.....(3)}$$

and corresponds also to a path-difference of one wave-length between the beams.

The true path retardation, given by $\lambda\beta\pi^{-1}$, has been calculated from equation (2) for various values of the position of the fringe-maximum expressed as fractions of $\frac{1}{2} \times 1{\cdot}43029\pi$ (i.e. of 128° 43′ 36″), and the results are given in the table, and in the graph of figure 3.

Table

Observed fractional fringe-displacement	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1
Actual retardation causing above (wave-lengths)	0	·0954	·1910	·2875	·3938	·4815	·5806	·6813	·7843	·8903	1
Correction	0	·0046	·009	·0125	·0162	·0185	·0194	·0186	·0157	·0097	0

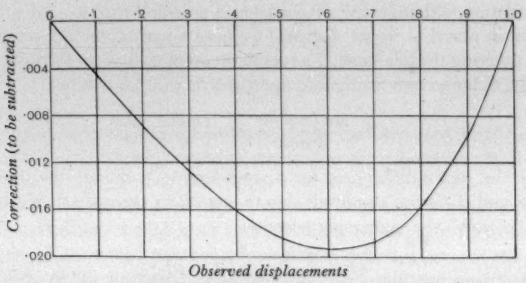

Fig. 3.

The observed fringe-displacement is, in general, slightly greater than the retardation (measured in wave-lengths) producing it, but the effect is so small as to be practically unimportant. The greatest deviation is for a retardation of approximately 0·6 of a fringe, when the discrepancy is less than one-fiftieth of the fringe width. Since these fringes approximate to a $\sin^2\theta$ intensity curve, they are rather broad and it is hardly likely that a fractional part can be determined to within this accuracy unless very special methods are employed.

It must be noted that this lack of linearity does not in any way affect visual determinations where coincidence is obtained between the two sets of fringes by tilting the compensator plates, since the correction term is zero when the total path-difference is any integral multiple of the wave-length employed. It could only be of importance in cases such as the method outlined in the following section, where the fractional part is determined by actual measurement and not by compensation methods.

§5. PROPOSED NEW METHOD OF DETERMINING THE DISPERSION CURVES OF GASES AND LIQUIDS

The very considerable gain in brightness of the fringes afforded by this method makes it possible to combine the instrument with a spectrograph, and by applying a variation of Benoit's fractional-part method, discussed below, the refractive indices of a substance for a large number of wave-lengths can be simultaneously determined.

For this purpose, the primary collimator slit and the Albrecht rhombs are mounted horizontally. The gas or liquid cells are mounted one above the other in one half of the beam and the compensator system is also rotated through 90° so that the two sets of fringes are now horizontal. These fringes are focussed on the slit of a spectrograph, and the arrangement is shown diagrammatically in figure 4.

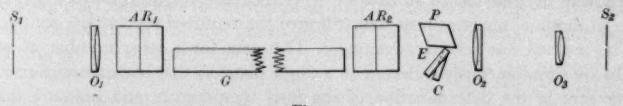

Fig. 4.

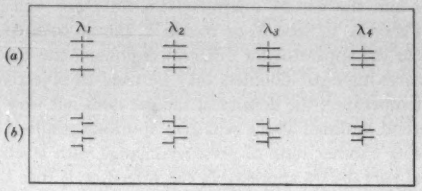

Fig. 5.

In addition to the Albrecht rhombs AR_1 and AR_2, a special astigmatic lens system O_3 is required. This lens, which must also be achromatic, has the plane of the vertical slit S_2 of the spectrograph as its focal plane, so that the horizontal fringes are in sharp focus. Simultaneously it should form, in the centre of S_2, an image of the vertical dividing edge E of the lifting prism P.

The appearance of the photographic plate for a wide slit S_2, with equal retardation in the gas tubes G and the compensator plates C parallel, is shown diagrammatically in figure 5 (*a*), the patterns in the two halves (divided by the broken line) for any wave-length being the same. As the wave-length becomes shorter, the distance between the orders becomes correspondingly shorter. Figure 5 (*b*) shows the appearance of the fringes, with a fixed path-difference between the two beams, due either to a difference of index in the tubes or to a relative tilt between the compensator plates. One half of the pattern remains as in figure 5 (*a*), the other half being displaced as shown. This displacement, measured in terms of the distance between the fringes in the other half, gives the fractional part of the retardation.

The proposed method of obtaining the dispersion curves for a gas is as follows. A photograph is taken, with a suitable source and with the two gas tubes exhausted. This should give a pattern similar to figure 5 (*a*), and affords a check of the correct adjustment of the apparatus. A small amount of gas is allowed to pass into one tube so that the pressure p at temperature t can be determined with fair accuracy. The tap to the tube is then closed and a photograph is taken on the same plate. This pressure is chosen to give an average displacement of one fringe. Suppose
f_1 that for a particular wave-length it is $(1 + f_1)$. A further photograph with a pressure $4p$, say, should give a displacement of $(4 + 4f_1)$ fringes, but the new fraction f_2 may be slightly different. The previous exposure now decides the integral number of wave-lengths n and the correct value for $4p$ will be $(n + f_2)$. A third exposure with a pressure of $16p$ should in turn be $(4n + 4f_2)$ and the exact fraction can be determined from this exposure. This is continued until a sufficient number of fringes have been displaced so that an experimental error of $\pm$ 0·05 or $\pm$ 0·1 of a fringe in the final measurement will still give the required percentage accuracy.

The method has several advantages. The data for a large number of wave-lengths are simultaneously collected on a single plate, so that one naturally exercises greater care in the determination of the final temperature and pressure than if independent determinations for each line have to be carried out. By suitable choice of quartz prisms and the use of quartz-sylvine-fluorspar objectives, the whole range from about 2000 Å. to 8000 Å. or 10,000 Å. can be covered. If the observer is willing to refocus the apparatus for different regions of the spectrum, the much less expensive quartz-fluorspar doublets may be used as objectives.

If at a given temperature the density of the gas does not vary linearly with the pressure, the method outlined above will give spurious results. Its accuracy can be checked by using another ratio of pressure-change with a second plate, and if the final fractional part differs appreciably the inference is that Boyle's law is not obeyed. Such cases can be investigated by the same method, provided the changes of pressure are made so small that no confusion as to the value of the integer of the fringe-displacement exists at any pressure in spite of the non-linearity of the curve. The data thus obtained would enable an accurate curve of the relationship between the density and the pressure for the substance in question to be drawn.

In applying the method to liquids, the simplest way is to have pairs of plates of quartz of equal thickness. Suppose the length of the liquid cells is 1 cm., convenient values of the thickness of the plates would be 0·99, 0·97, 0·91, 0·66 cm. These pairs, being cut from the same original plate, produce no fringe-displacement when placed one in each cell, so that effectively we have liquid cells of length 0·01, 0·03, 0·09, 0·27 and 1 cm. The exact values are unimportant provided they can be determined to within the requisite accuracy. A subsidiary determination with the Abbe or Pulfrich refractometer will enable the integer value for the fringe-displacement for the 0·01 cm. cell to be determined with certainty. The further procedure is the same as for gas determinations, but care must be taken with certain liquids that evaporation does not influence the results.

Messrs Adam Hilger Ltd. have, at the writer's suggestion, produced a cross-

slide photomeasuring micrometer. This instrument was primarily designed to measure the diameters of rings in Fabry-Perot-interferometer and reflection-échelon photographs. It consists of the usual long-screw photomeasuring micrometer L 13 and a short cross slide with a total traverse of only 2 cm. When required, the microscope can be rapidly transferred from one line to another and the arrangement is admirably suited for measuring the fringes described above. The long-screw scale need only be used when the identification of any line is a matter of doubt, as may be the case with, say, the iron spectrum.

§ 6. ACKNOWLEDGMENTS

In conclusion I wish to express my thanks to Mr T. T. Thomas, M.A., who has kindly undertaken the laborious task of calculating the table and prepared the graph of figure 3; and also to the technical staff of Messrs A. Hilger Ltd., who are the sole manufacturers of the instrument, for their cooperation in the experimental work.

DISCUSSION

Mr C. V. Jackson. The author has greatly under-rated the degree of accuracy which can be obtained with the Fabry and Perot interferometer. Thus, he states that it is possible to make measurements to 1 or 2 per cent of a wave in the case of the best fringes. Actually considerable experience, which I have gained in the course of interferometric determinations of wave-length, has shown me that even in the case of very poor fringes an accuracy of about 1 per cent can be reached, while for sharp fringes I have often found that my measurements are right to $\frac{1}{4}$ per cent or even better. In the measurement of the refractive index of air by means of the Fabry and Perot interferometer, the errors introduced by incorrect phase-corrections, or a slight error in the scale of the rings system, is of no consequence. For this reason there should be little difficulty in attaining an accuracy of about $\pm$ 0·0001 Å. by using this method. It is extremely doubtful if an accuracy as high as this could be attained by means of the method suggested by the author.

The author suggests that errors of about $\pm$ 0·0002 Å. may be introduced by the use of Meggers and Peters's tables, but I think this is very improbable on account of the good agreement between observed values in the region 2300–3500 and wave-lengths calculated from lines in the visible. If there were errors of the magnitude suggested by the author they would certainly upset this agreement, since all lines must have their wave-lengths reduced to vacuum for calculating from term values. I can see no advantage in the author's method over that used by Meggers and Peters.

Dr J. J. Fox. The author has done a service in calling attention to the comparative inadequacy of the data for the refractive index of air, when the progress in the accuracy of the determination of wave-lengths of spectral lines is considered. The most extensive determinations of the refractive index of air are due to Meggers

and Peters, carried out in 1918. The data obtained were doubtless sufficient to yield a more accurate dispersion formula than any other available, yet even it does not produce results better than those indicated in the paper. This suggests the desirability of securing some method or instrument capable of giving greater accuracy, especially when one wishes to determine refractive indices of gases both accurately and conveniently. The question of the desirability of reconsidering even the refractive index of air is brought out by the calculations of Opladen*, who derived the value $293{\cdot}03 \pm 0{\cdot}17$ for $(n-1) \times 10^6$ for λ 5461 from the best values available, while the value calculated from Meggers and Peters's formula is about 5 units lower in the seventh decimal place. The procedure described by the author has been found by us (at the Government Laboratory) to meet the requirements, inasmuch as it gives fringes of much greater brightness, permitting of the use of fainter spectral lines than hitherto and giving more comfort in visual observation.

Author's reply. Mr C. V. Jackson appears to have misunderstood my purpose. I tried to show that by employing lateral displacements and subsequent recombinations, the intensity of the fringe system in a divided-wave-front interferometer can be very considerably increased while, as is well known, such displacements of a beam will have no effect on the brightness of an optical image, as distinct from an interference fringe. Further, a type of interference system is described in which the fringe-displacement is not linearly proportional to the retardation or path-difference interposed. This, as far as I have been able to find, is the first instance in interferometry where such non-linearity has been found to exist.

The section on the refractive index of air, to which Mr Jackson takes such strong exception, is not strictly germane to the main substance of the paper, but I could not resist the opportunity of discussing this very important point for the reasons given below.

Apart from special purposes such as those of metrology, the real purpose of accurate wave-length determinations is to obtain the frequencies of these radiations, and the results so obtained can be used both to evaluate certain fundamental constants as well as to test the validity of theories of spectral emission. The primary standard is the wave-length of red cadmium radiation emitted under specified conditions and measured in dry air, the temperature and pressure of which are specified. The value adopted is a weighted mean of the independent results of Michelson and of Buisson, Fabry and Perot. The choice of secondary and tertiary standards has been made with great caution by the International Astronomical Union; values have not been accepted unless there was a sufficiently good agreement between the determinations made by independent observers.

Yet the real significance of all this, and of a great deal more work for which these standards have been used, is made to depend, in turn, on a single set of measurements of the refractive index of air which were carried out over fourteen

* *Z. Physik.* **42**, 160 (1927).

years ago, since which time there has been a considerable improvement in both the technique and the apparatus for precision wave-length measurements.

My estimate of the probable accuracy of Meggers and Peters's results (3 or 4 units in the seventh decimal place) is the value the authors quote as the probable error of a single determination. They do not specifically give an estimate of the probable accuracy of a value derived from their equations, but it can be inferred from the temperature and pressure tolerances mentioned that a probable accuracy of one unit should be attained. I feel that the considerable number of the readings taken does not so much appreciably increase the final accuracy of the value for any given wave-length, as determine in better detail the general shape of the dispersion curve.

I cannot accept Mr Jackson's assumption that the experimental agreement of tests of the combination principle is necessarily a proof of the accuracy of Meggers and Peters's results. Let us take an extreme case by way of illustration; suppose a line A at 10,000 Å. should be given by the difference between lines (C) at 2000 Å. and (B) at 2500 Å. The wave-number corrections for these values are usually taken as $A = -2{\cdot}738$, $B = -12{\cdot}053$ and $C = -16{\cdot}274$. If we assume that Meggers and Peters's results are uniformly 6 units in the seventh place too low, the following values: $A_1, -{\cdot}006$; $B_1, -{\cdot}024$; $C_1, -{\cdot}030$, have to be added to the above corrections. It will be noticed that $C_1 - B_1 = A_1$, so that the same agreement will be obtained in both cases.

Meggers and Peters's formula for the index of dry air under standard conditions gives for the red cadmium wave-length

$$(\mu_{15} - 1) \times 10^7 = 2758{\cdot}14.$$

The determination by Perard* for this wave-length, carried out at the Bureau Internationale des Poids et Mesures, gives the value 2764·13. I understand that as a result of a careful determination with étalons of far greater plate-separations than those used by Meggers and Peters, the Metrological Department of the National Physical Laboratory has arrived at a provisional value which is in very close agreement with the result of Perard.

This difference of six units in the seventh place introduces an uncertainty of $\pm{\cdot}0038$ Å. in the vacuum value of this wave-length. In brief, until this question is finally settled, wave-length determinations in air will have little real significance beyond the second decimal place in Ångstroms.

To return to Mr Jackson's comments, he estimates that there should be little difficulty in attaining an accuracy of $\pm 0{\cdot}0001$ Å. in the vacuum correction with the Fabry-Perot method. For the wave-length we are considering, this would require an accuracy of $\pm 1{\cdot}5$ units in the eighth decimal place for the refractive index, or that $(\mu - 1)$ must be obtained to within one part in 20,000. Taking the value of $\frac{1}{4}$ per cent as the possible error in the determination of the fractional part, which he quotes as practicable in the case of sharp fringes, the change in the order of

* A. Perard, *Phys. et Ra.* **6**, 217 (1925); see also *Trav. et Mem. Bur. Internat.* **18**, 34 (1929).

interference must be about 100 fringes to secure that the maximum possible error in the two fractional-part determinations (0·005) shall be in the correct proportion.

Since $(\mu - 1) = (n_{air} - n_{vac.})/n_{vac.} = 2764 \times 10^{-7}$, and $n_{vac.} \doteqdot 360{,}000$,

and the separation of the interferometer plates would have to be approximately 12 cm. With this separation, the fringes from even the red cadmium line will be very poor, so that the $\frac{1}{4}$ per cent accuracy in fractional-part determination would be utterly impossible. The 1 per cent accuracy suggested for poor fringes would require a plate-separation of 48 cm., for which distance no sign of a fringe would be seen. It is thus obvious that there is no hope of attaining this order of accuracy by means of the differential method employed.

I think the other opinions expressed by Mr Jackson are sufficiently answered in Dr Fox's communication. I particularly appreciate Dr Fox's comments, as he and his colleagues at the Government Laboratory are the only other investigators who have, as yet, had an opportunity of actually using the new instrument.

Postscript added during proof. Through the courtesy of Dr Lampe I have been able to examine the apparatus used and the data obtained for the refractive index of air, at the Physikalische Technische Reichanstalt. They employed a path-length of 4 metres, and their value for the index for this wave-length is again in very close agreement with that of Perard. This establishes beyond all doubt that the tables are in error and should be speedily revised.

ON THE REPRESENTATION AND CALCULATION OF THE RESULTS OF GRAVITY SURVEYS WITH TORSION BALANCES

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ABSTRACT. This paper deals with an alternative and, it is believed, superior method of indicating the functions of the Eötvös torsion balance, and of calculating those quantities, depending upon the distortion of the earth's gravitational field, which the balance measures. Remarks are also made on the representation of these quantities—the so-called *gravity-gradient* and *horizontal directive tendency*—and a convenient method is indicated of applying graphically the necessary corrections for the effects of earth's rotation and irregularity of the surface of the ground.

In most, if not all, of the many surveys of the departures from uniformity, or, more strictly, spherical symmetry, of the earth's gravitational field, carried out with torsion balances of the Eötvös type, it has been the practice to proceed directly to the calculation of the coefficients in the fundamental equation originally shown by Eötvös to be applicable to such instruments. Thereafter these coefficients have been used to derive certain other related quantities which are, in fact, those usually represented on the survey maps to show the state of the gravitational field. It is the purpose of this paper to propose a modification of the form of the fundamental equation such that the equation may contain explicitly from the outset the actual quantities which have ultimately to be calculated and used for representation. The result is that not only is the evaluation rendered more direct, but the significance of the terms in the equation, in relation to the measuring functions of the instrument, becomes much more obvious.

The Eötvös form of the fundamental equation* is

$$C_\alpha = \frac{K_1}{2}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + K_1 \frac{\partial^2 U}{\partial x \partial y}\cos 2\alpha + K_2\left(\frac{\partial^2 U}{\partial y \partial z}\cos\alpha - \frac{\partial^2 U}{\partial x \partial z}\sin\alpha\right) \quad \ldots\ldots(1).$$

Here C_α is the resultant horizontal force-moment acting on the suspended beam system, due to the non-uniformity (supposed small) of the gravitational field. This moment varies with the angle α which the vertical plane of symmetry of the beam system makes with the arbitrarily chosen vertical plane x, z. The constants K_1 and

C_α

α

K_1

* *Ann. der Phys.* **59**, 364 (1896). The derivation of the equation will also be found in the paper by Shaw and Lancaster-Jones, *Proc. Phys. Soc.* **35**, 152 (1923).

K_2 K_2 are instrumental. In the original Eötvös instrument K_1 was very nearly the moment of inertia of the whole beam system (which was confined nearly to one plane) about the axis of suspension; and K_2 differed very little from mhl, where
m, h, l m is the mass of the lower weight, h its mean distance below the beam, and l its mean distance from the axis of suspension. In later forms of the beam it has been necessary to calculate K_1 and K_2 for the more complicated mass distribution by means of the general formulae

$$K_1 = \int(\xi^2 - \eta^2)\, dm \quad \text{and} \quad K_2 = \int \xi z\, dm,$$

where ξ, η and z are the rectangular coordinates of the element of mass dm, the origin being on the axis of suspension, while ξ and η are horizontal and respectively in and perpendicular to the symmetrical vertical plane of the beam, and z is vertically downwards. It may be noted that in the Shaw and Lancaster-Jones gradiometer the mass-distribution of the beam is so chosen as to make K_1 equal to o, and that, in the Coulomb form of balance, symmetry with regard to a horizontal plane makes K_2 vanish.

The partial differential coefficients in the fundamental equation refer to the
U gravitational potential U. Frequently they are represented, for brevity, by the notation

U_Δ, U_{xy} $$U_\Delta = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}, \quad U_{xy} = \frac{\partial^2 U}{\partial x \partial y},$$

U_{xz}, U_{yz} $$U_{xz} = \frac{\partial^2 U}{\partial x \partial z} \quad \text{and} \quad U_{yz} = \frac{\partial^2 U}{\partial y \partial z},$$

and these four coefficients constitute the unknown quantities ordinarily derived in the first instance from the observations at any particular station, in a manner which will be described later.

These coefficients, although not giving a complete description of the field-variation (since the coefficient $\partial^2 U/\partial z^2$ is not determined by the apparatus), are related closely to two more fundamental quantities which have both magnitude and direction. Eötvös gave them the names *die horizontale Richtkraft* and *der Gradient der Schwerkraft*, and they are commonly known in English as the *horizontal*
H.D.T. *directive tendency* (or H.D.T.) and the *gravity-gradient* respectively. Although these names are open to criticism, it remains true that the quantities in question are of such special significance in describing the state of the gravitational field that they are in most cases chosen for representation upon the survey maps.

The gravity-gradient, strictly the maximum gradient in a horizontal direction
g, s of the vertical gravitational intensity, or $\partial g/\partial s$, where s is the direction of maximum variation, is a vector. It can, if desired, be regarded as having, in any two horizontal directions x and y, two rectangular components $\partial g/\partial x$ and $\partial g/\partial y$, fulfilling the relations

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial s}\cos\phi, \quad \frac{\partial g}{\partial y} = \frac{\partial g}{\partial s}\sin\phi,$$

ϕ and their implications, ϕ being the angle between s and x. Since $g = \partial U/\partial z$,

$\partial^2 U/\partial x\,\partial z$ and $\partial^2 U/\partial y\,\partial z$ are identified with $\partial g/\partial x$ and $\partial g/\partial y$ respectively; so that, if we denote the maximum gravity-gradient, $\partial g/\partial s$, by G, we have G

$$U_{xz} = \frac{\partial^2 U}{\partial x\,\partial z} = G\cos\phi \qquad \text{.....(2)}$$

and

$$U_{yz} = \frac{\partial^2 U}{\partial y\,\partial z} = G\sin\phi \qquad \text{.....(3).}$$

A knowledge of U_{xz} and U_{yz} thus implies a knowledge of G and ϕ, and conversely.

This deals with the transformation of two of the unknowns in the fundamental Eötvös equation. The remaining two, namely, U_Δ and U_{xy}, are related to the horizontal directive tendency, but in a rather different way. This name originated from a consideration of the behaviour of a balance beam of the Coulomb type, for example, a light horizontal rod with equal masses at the ends, in relation to the curvatures of the equipotential or "level" gravitational surface. Such a beam would tend to set itself so that its axis lay in the vertical plane corresponding to the least curvature of the level surface, curvature being reckoned positive downwards. The beam would also be in equilibrium, though now unstable, if lying in the plane of greatest curvature of the local equipotential surface. A maximum value of the controlling couple is reached when the axis of the beam bisects the angle between these two orthogonal planes. The value of this maximum couple is proportional to $g(c_1 - c_2)$, where c_1 and c_2 are the maximum and minimum curvatures of the level surface at the point of observation, and this product of the gravitational intensity and the curvature-difference is, in fact, identified with the so-called horizontal directive tendency. It is really a property of the field, not dependent on reference to any instrument, at any rate in a formal manner. For this reason the name in English is not a good one; but a more serious objection to it is that the gravity-gradient can equally be regarded as a horizontal directive tendency in respect of an apparatus having the full properties of the Eötvös instrument. For in that case the beam tends also to rotate horizontally into the direction of maximum gradient. The name in German does not suffer from this defect, but the quantity in question is not a force, as this name suggests. Actually the gravity-gradient and the horizontal directive tendency both have the same dimensions, namely, $1/\text{sec}^2$. It is perhaps too late in the day to suggest a new name, and the name "horizontal directive tendency" will therefore be used, somewhat reluctantly, in the rest of this paper. Several other names have been proposed, such as "curvature-value" or "curvature-difference," but they lack the desirable precision*. c_1, c_2

The quantity itself has magnitude and direction, but it possesses the latter in a limited sense. It is not a true vector, for its direction has no "sense." It is unaltered by rotation through π, but turning through $\frac{1}{2}\pi$ changes its sign. Its

* I have discussed this question with some mathematical colleagues. One of them has made the rather attractive suggestion that the difficulty might be solved by the invention of a composite word. He proposed *graviture* for the combination of gravity and curvature implied by a product of the type gc; then *graviture-difference*, or, more strictly, *principal graviture-difference*, would automatically signify the function $g(c_1 - c_2)$.

direction is taken to be that corresponding to stability, namely, the horizontal direction in which the vertical downward curvature of the equipotential surface is least. If we denote the scalar value of $g(c_1 - c_2)$ by R, and its direction, with respect to the x axis, by θ, it is related to U_Δ and U_{xy} by the equations

R
θ

$$2U_{xy} = 2\frac{\partial^2 U}{\partial x \partial y} = R \sin 2\theta \qquad \text{.....(4),}$$

$$-U_\Delta = \frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} = R \cos 2\theta \qquad \text{.....(5).}$$

These transformations complete what is required for our purpose.

Returning to the fundamental equation (1), we re-write it

$$C_\alpha = \frac{K_1}{2}\left[2\frac{\partial^2 U}{\partial x \partial y}\cos 2\alpha - \left(\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2}\right)\sin 2\alpha\right] + K_2\left[\frac{\partial^2 U}{\partial y \partial z}\cos\alpha - \frac{\partial^2 U}{\partial x \partial z}\sin\alpha\right],$$

and, by substitution from (2), (3), (4) and (5), we obtain

$$C_\alpha = \tfrac{1}{2}K_1 R(\sin 2\theta \cos 2\alpha - \cos 2\theta \sin 2\alpha) + K_2 G(\sin\phi\cos\alpha - \cos\phi\sin\alpha),$$

or,
$$C_\alpha = \tfrac{1}{2}K_1 R \sin 2(\theta - \alpha) + K_2 G \sin(\phi - \alpha) \qquad \text{.....(6).}$$

This is the form of the equation which is suggested for use in preference to that containing the U coefficients. It contains another, but related, set of four unknown quantities, R, θ and G, ϕ, which equally describe the field, and are already in the form required for map-making purposes. At the same time the new equation makes more evident the manner in which the total torsional control is made up of two distinct contributions, namely, that due to the horizontal directive tendency and that due to the gravity-gradient. If, for example, the azimuth, α, of the beam system happens to coincide with that of G, namely, ϕ, or is in the exactly opposite direction, the torsional control, C_α, is wholly due to R, i.e. to the difference between the principal curvatures of the level surface, in conjunction with gravity. If, on the other hand, the beam azimuth is inclined at any integral multiple of $\frac{1}{2}\pi$ to θ, the direction of R, the gravity-gradient G becomes wholly responsible for the torsion. The magnitudes of the two effects can accordingly be considered separately.

The effect of G resembles that of a uniform force-field, in that the force-moment is proportional to the sine of the angle between it and the beam-axis; and the instrumental constant—the "moment" of the beam from the present point of view—is K_2, which depends on the asymmetrical vertical distribution of mass, in the manner already indicated. Similarly, the other instrumental constant, $\frac{1}{2}K_1$, is a different "moment" which responds to the particular non-uniformity of field, represented by R; only in this case the rule for the calculation of the torsion is different, since the latter is now proportional to the sine of *twice* the angle between the beam-axis and R.

The new form of the equation also makes more evident the distinctive features

of certain particular designs of the torsion balance. If, for example, the moment K_2 is deliberately made negligible, as it would be for a Coulomb balance beam, we see that the second term of equation (6) vanishes, and the instrument becomes irresponsive to gravity-gradient. The equation for such beams reduces to

$$C_\alpha = \tfrac{1}{2}K_1 R \sin 2(\theta - \alpha) \qquad \text{.....(7)},$$

which forms the basis for the calculation of the magnitude and direction of the horizontal directive tendency only. Alternatively, should it be desired to measure the gravity-gradient vector only, the mass-distribution of the beam may be designed so as to render that other moment, $\frac{1}{2}K_1$, sufficiently small, and equation (6) becomes

$$C_\alpha = K_2 G \sin(\phi - \alpha) \qquad \text{.....(8)},$$

which is applicable to this special form of instrument. This is the principle applied in that elegant instrument, the gravity-gradiometer* of Shaw and Lancaster-Jones, which measures only the magnitude and direction of the gravity-gradient, giving negligible response to the horizontal directive tendency. It is evidently possible also to arrange the mass-distribution and degree of symmetry of the beam so as to fulfil the two conditions $K_1 = \int(\xi^2 - \eta^2)\,dm = 0$ and $K_2 = \int \xi z\,dm = 0$; but this would serve no useful purpose, as the instrument would then measure no feature of the non-uniformity of the gravitational field.

The method of derivation of the four unknowns R, θ, G and ϕ (instead of U_{xy}, U_{xz} and U_{yz}) differs somewhat from the normal practice, but it leads to no less simplicity in the calculations from the observations. If we introduce into equation (6) the customary substitutions for C_α, we obtain

$$n_\alpha - n = \tau^{-1}DK_1 R \sin 2(\theta - \alpha) + \tau^{-1}2DK_2 G \sin(\phi - \alpha) \qquad \text{.....(9)},$$

where n_α is the scale-reading when the beam has the azimuth α, and n is the (unknown) scale-reading corresponding to complete absence of torsion in the suspending wire. D is the ratio of the distance between the mirror and the scale to the width of one scale division, and τ is the torsion couple per unit twist of the suspending wire. It should be noted that in practice the torsion angle of the suspension is always very small, being seldom as much as one degree, so that the azimuth of the beam is practically the same as the rotation of the whole instrument from the standard direction. Writing $\tau^{-1}DK_1 = a$ and $2\tau^{-1}DK_2 = b$, both these being purely instrumental constants previously determinable, we obtain from equation (9)

$$n_\alpha - n = aR \sin 2(\theta - \alpha) + bG \sin(\phi - \alpha) \qquad \text{.....(10)}.$$

n_α, n

D

τ

a, b

Since n is unknown, as well as R, θ, G and ϕ, at least five equations are required, and these are obtained by taking observations (n_α) in a sufficient number of different azimuths (α). As in the employment of the usual form of the equation, the solution is simplified by using six† azimuths, equally spaced in a complete rotation of the instrument, the supernumerary equation in addition providing a check on the consistency of the observations. The following six equations are thus developed

* The *Mining Magazine*, May, 1929, contains a brief description of this instrument.

† *Note:* Actual settings are commonly reduced to three by working with double-beam instruments.

from (10), corresponding to the values 0°, 60°, 120°, 180°, 240°, and 300° respectively:

n_0 $$n_0 - n = aR \sin 2\theta \qquad + bG \sin \phi,$$

n_1 $$n_1 - n = aR \sin (2\theta - \tfrac{2}{3}\pi) + bG \sin (\phi - \tfrac{1}{3}\pi),$$

n_2 $$n_2 - n = aR \sin (2\theta - \tfrac{4}{3}\pi) + bG \sin (\phi - \tfrac{2}{3}\pi),$$

n_3 $$n_3 - n = aR \sin (2\theta - \tfrac{6}{3}\pi) + bG \sin (\phi - \tfrac{3}{3}\pi),$$

n_4 $$n_4 - n = aR \sin (2\theta - \tfrac{8}{3}\pi) + bG \sin (\phi - \tfrac{4}{3}\pi),$$

n_5 $$n_5 - n = aR \sin (2\theta - \tfrac{10}{3}\pi) + bG \sin (\phi - \tfrac{5}{3}\pi).$$

In the above the suffix of n indicates the multiple of 60° which is the corresponding azimuth; thus, n_4 corresponds to $4 \times 60° = 240°$.

It is unnecessary here to go through the steps of the evaluations of the five unknowns; it will suffice to indicate that by appropriate selection and addition or subtraction of equations, and by the use of well-known trigonometrical relations, the following results are obtained:

$$6n = n_0 + n_1 + n_2 + n_3 + n_4 + n_5^*, \qquad \text{.....(11a)},$$

which gives the natural zero of the instrument;

$$3n = n_0 + n_2 + n_4 = n_1 + n_3 + n_5 \qquad \text{.....(11b)},$$

which provides the check above mentioned;

$$\cot 2\theta = (n_2 + n_5 - n_1 - n_4)/\sqrt{3}.(n_0 + n_3 - 2n) \qquad \text{.....(11c)},$$

$$R = (n_0 + n_3 - 2n)/2a \sin 2\theta \qquad \text{.....(11d)};$$

$$\cot \phi = (n_4 + n_5 - n_1 - n_2)/\sqrt{3}.(n_0 - n_3) \qquad \text{.....(11e)},$$

$$G = (n_0 - n_3)/2b \sin \phi \qquad \text{.....(11f)}.$$

The two possible values of each of the angles 2θ and ϕ are found from (11c) and (11e), by reference to tables; and application in (11d) and (11f), respectively, enables the magnitudes of R and G to be calculated simply, besides removing the ambiguity of 2θ and ϕ. Thus R and G are determined both in magnitude and direction, and may be represented graphically on the survey map, as is customary, by straight lines in the appropriate directions and of lengths proportional to the magnitudes on a suitably chosen scale.

It may be noted that, for similar purposes, the usual method of evaluating first the coefficients U_Δ, U_{xy}, U_{xx} and U_{yz} requires the additional calculations corresponding to the relations

$$R = \sqrt{(U_\Delta{}^2 + 4U_{xy}{}^2)}, \qquad \tan 2\theta = -2U_{xy}/U_\Delta,$$
$$G = \sqrt{(U_{xx}{}^2 + U_{yz}{}^2)} \text{ and } \tan \phi = U_{yz}/U_{xx}.$$

The question may naturally be raised as to the advantage of obtaining the instrumental measures of R, θ, G and ϕ in the direct manner indicated, when, in fact, they are *resultant* effects attributable only partly to the actual gravitational

* This is only a special case of a general rule. If any number ν (necessarily integral) of azimuths are spaced with equal intervals in a complete rotation, it can be shown that

$$\nu n = n_0 + n_1 + n_2 + \ldots + n_{\nu-1}.$$

disturbance sought, namely, that due to buried structure. They contain components also for which the rotation of the earth and local topographical features are responsible. It might with some reason be argued that, as the elimination of the latter effects is necessary, it is better to have the resultant horizontal directive tendency and gravity-gradient in the form of x and y components, so as to reduce to scalar subtraction the corrections in question. The answer is that the elimination can be performed still more simply by graphical methods, ordinary vector diagrams being used for the gravity-gradient, and a modified construction for the horizontal directive tendency.

For example, if we denote by G_E the gravity-gradient due to the earth's rotation, its value is given very nearly by G_E

$$G_E = 8 \sin 2\lambda \times 10^{-9} \text{ sec.}^{-2}\text{*}$$
$$= 8E \sin 2\lambda, \qquad E$$

where λ is the latitude of the observation station. The direction of the vector G_E is λ in the northern hemisphere, towards the geographic north. It can be subtracted

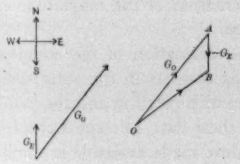

Fig. 1. Graphical method of obtaining difference between gravity-gradients.

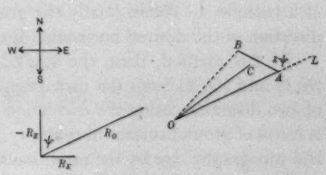

Fig. 2. Graphical method of obtaining difference between horizontal directive tendencies. The resultant, OC, bisects the angle AOB.

from the observed vector G_O by reversal and vector addition as indicated in G_O figure 1. Thus, if G_O and G_E are as shown, the vector diagram consists of OA, representing G_O, and AB, representing G_E reversed; and the remaining side of the triangle, OB, represents in magnitude and direction the gravity-gradient after the removal of the earth's rotation effect. Clearly other corrections may be made by repeated application of like rules, provided the correction data are available as vectors; and the final result is obtained from a single diagram in the form in which it is required, namely, a straight line of particular length and direction, representing completely the contribution of the underground structure to the local gravity-gradient.

The construction for the horizontal directive tendency is different, owing to the peculiar properties of this doubly-directed quantity. To subtract one from another one must first rotate it, not through 180°, but through 90°, and then perform the addition by a special set of rules. The angle between the two directions has to be doubled in the equivalent of a vector diagram, and the completing side

* 1 Eötvös unit, denoted by E., is equal to one thousand-millionth of a c.g.s. unit, or 10^{-9} sec.$^{-2}$, and is the small unit usually employed in torsion-balance work.

then represents the magnitude, but not the direction, of the resultant. The *direction* of the resultant *bisects* the internal angle between the initial and final sides. Thus, in order to eliminate from the observed horizontal directive tendency, R_O, the part R_E, due to the earth's rotation, given nearly by

R_O
R_E

$$R_E = 10E \cos^2 \lambda,$$

and directed east and west geographically, we represent R_O by OA, figure 2. We add $-R_E$, which has a north-south direction as already explained, by constructing the line AB so that the angle LAB equals twice the angle ψ between $-R_E$ and R_O, and making AB proportional to $-R_E$ on the scale adopted. Then, on the same scale, OB represents the magnitude of the resultant horizontal directive tendency, and its direction is along OC, where OC bisects the angle AOB. Thus, if OC is also made equal to OB, it represents in all respects the horizontal-directive-tendency element of gravitational distortion, other than that for which the earth's rotation is responsible. Here again other corrections, notably the topographical one, can be applied by continuing the use of similar rules, which now give a polygon instead of a triangle, to obtain finally the graphical representation of the magnitude and direction of the desired horizontal directive tendency.

ψ

In this method, then, the whole process of the derivation of the composite (R, θ) and (G, ϕ) from the instrumental observations consists in the construction of two diagrams, using the correction data for the earth's rotation and the various features of ground surface irregularity. The fact that these data, in respect of terrain and topography, are by the usual methods of procedure made available in the first instance as components (of the U_Δ, U_{xy}, U_{xz} and U_{yz} type) presents no difficulty. For in the gradient diagram these reversed components, $-U_{xz}$ and $-U_{yz}$, will simply constitute two successive steps at right angles to one another, instead of the one step which would obtain were their resultant already known. And the reversed horizontal-directive-tendency components, U_Δ and $-2U_{xy}$, will similarly form part of the horizontal-directive-tendency diagram, in place of the single line to which they are equivalent.

It seems worth while to mention a possible extension of the use of these diagrams, namely, to perform part of the integration necessarily involved in estimating the topographical effect. Starting from the station point as centre, each sector of the ground surface, lying between two close azimuths, can be regarded as contributing an element of gravity-gradient, $d(G)$, and an element of horizontal directive tendency, $d(R)$, each, if positive, in the direction of the mean azimuth of the narrow sector. If $d(G)$ is negative, it is directed oppositely to the sector azimuth; if $d(R)$ is negative, it is at right angles to the sector azimuth. The numerical values of the directed quantities $d(G)$ and $d(R)$ can be conveniently estimated by planimeter integration based upon "levelling" observations at selected radial distances, and the final integration for the whole range 2π of azimuths, can then be performed by the diagrammatic rules already described. The method has been worked out in detail, and has several interesting features, one of which is that the diagrams may consist partly of continuous curves such as arcs of circles. Drawings are

substituted largely for numerical calculations, and we still require only two diagrams altogether.

One hesitates, however, to propose definitely at present this development as an alternative superior to the elegant methods, particularly Schweydar's, which have been used widely and successfully in the field for estimating the topographical corrections. The basis of Schweydar's method* necessitates initial resolution into rectangular components, and integration with respect to azimuth *before* that with respect to radial distance. And his convincing use of the Fourier expansion rather suggests that his method deals as precisely as possible with the interpolations among the necessarily limited number of levelling observations. The only really satisfactory comparison of the two methods would be by results; and when sufficient field-practice has been obtained with the graphical method indicated, a fuller account of it may be offered for publication.

* W. Schweydar, "*Die topographische Korrection bei Schweremessungen mittels einer Torsionwage*," *Z. f. Geophys.* **3**, 17 (1927).

DISCUSSION

For discussion see p. 489.

SOME OBSERVATIONS WITH A GRAVITY-GRADIOMETER

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ABSTRACT. This paper gives an account of a series of observations with a Shaw and Lancaster-Jones gravity-gradiometer, during which it was discovered that very persistent, although small, electric charges could be developed on the mica ring forming part of the oscillation-damping system. The origin of these charges has not yet been ascertained precisely, but one of them has been located and measured approximately by suitable manipulation of the instrument. It is shown that such charges, which may persist for weeks, may not arouse suspicion, although in fact they lead to spurious results in the normal use of the instrument. Thus the apparent gravity-gradient determined by an instrument insidiously charged may have errors of magnitude much larger than the true gradient, and errors of direction of many degrees. It is shown that the effect can be effectively eliminated by introducing into the instrument a sufficient quantity of a suitable ionizing agent, such as meso-thorium. This precaution, which can be adopted readily in both old and new gradiometers, is proved to result in a notable improvement in the accuracy and reliability of the instrument.

§ 1. INTRODUCTION

The observations to be described were made with one of the two gravity-gradiometers used in the work of the Imperial Geophysical Experimental Survey which operated in Australia from 1928–30. It has since been acquired for the newly instituted courses of instruction in applied geophysics in the Imperial College, and it was during the preparation of the instrument, which arrived from Australia in a somewhat damaged condition, that the at first mysterious behaviour was noticed, and the explanation tracked down. When the instrument was purchased for the College it was not known that such great difficulties had been experienced with it in the Australian Survey that it had provided practically no satisfactory results. But evidence is found in the report* that makes it clear that the gradiometer now under consideration is in fact what is there called the "new gradiometer," i.e. the one added to the equipment during the latter part of the Survey; and it is quite apparent, from the remarks on page 321 of the report concerning the instrument, that its erratic behaviour then was due to the same cause which has now been discovered at the Imperial College, and not to imperfections of the torsion wires, as suggested in the report. The origin of the erratic behaviour is persistent electrification of the mica ring incorporated in the instrument for damping purposes, and the effect can be removed by ionizing the air within the instrument. As a result

* *The Principles and Practice of Geophysical Prospecting, being the Report of the Imperial Geophysical Experimental Survey.* (Cambridge University Press, 1931.)

this gradiometer has become docile and reliable instead of petulant and wayward, and now performs its function of measuring the magnitude and direction of the gradient of gravity in a most satisfying manner.

It is just possible, of course, that this particular instrument is peculiar in its liability to electrification; but it would not be safe to assume this, for mica is both an exceptional insulator and easily electrified. As will be seen, the effect can be readily eliminated and the performance and reliability of these elegant instruments thereby greatly improved. It is for this reason mainly, but also as showing the interesting manner in which the disturbing effect was brought to light, that an account of the observations is now presented.

§2. THE GRAVITY-GRADIOMETER

There has appeared as yet* no adequate description of this notable instrument of precision—the gravity-gradiometer. It was invented and designed by Dr H. Shaw and Mr Lancaster-Jones, and the first examples of it, including the one with which the following experiments were made, were manufactured by Messrs L. Oertling, Ltd. The inventors have already given a short account of it, but without instrumental details†. It is accordingly necessary to describe here those features which are related to the subject-matter of this paper. The instrument is a modification of the famous torsion balance of Eötvös‡, which measures small departures from uniformity in a gravitational field. In the Eötvös form the balance consists of a horizontal beam suspended at its mid-point by a suitable torsion wire and carrying at different levels at its ends relatively heavy masses, one being on the level of the beam itself and the other suspended from it by a wire. Such an arrangement can be made with an extraordinarily high sensitivity, and, with suitable precautions, and with observations of the state of twist of the torsion wire in a sufficient number of azimuths of the instrument as a whole, will measure with great precision the magnitude and direction of two quantities which describe partially the non-uniformity of the gravitational field at the place where the instrument is located. The first of these, commonly called the *horizontal directive tendency*, is in fact the difference between the principal curvatures of the local "level" surface multiplied by g, the vertical component of gravity. The second is the so-called *gravity-gradient*, i.e. $\partial g/\partial s$, where s is the horizontal direction of maximum change of g.

The modification due to Shaw and Lancaster-Jones consists in rendering the instrument irresponsive to the former of these two controls, for reasons given by those authors§; so that only the horizontal gravity-gradient is measured. This is secured by a suitable design of the distribution of the masses attached to the beam, and the method chosen has been to construct the beam in the form of a horizontal circular ring of aluminium with its centre directly below the point of suspension, and carrying three gold masses equidistant from the centre in azimuths at intervals

* It is understood that a full account of the instrument will be published shortly in the *Journal of Scientific Instruments*.

† *The Mining Magazine*, **40**, 272 (1929).

‡ *Ann. der Phys.* **59**, 370 (1896).

§ *Loc. cit.*

of 120°. Two of these are nearly in the plane of the ring and the other is supported above, at a height of about 50 cm., by a light aluminium tube. The arrangement is shown in figure 1, which represents also the ring of mica—the main feature for our consideration. This ring, of internal and external diameters about 4 in. and 6 in. respectively, is attached below the aluminium ring, and serves, by its movement when suspended in close proximity over a horizontal brass plate, to damp the oscillations of the beam about the vertical axes. The degree of damping—approximate aperiodicity is usually employed—is very sensitive to the interval between the mica ring and the brass plate below it, and provision has accordingly been made

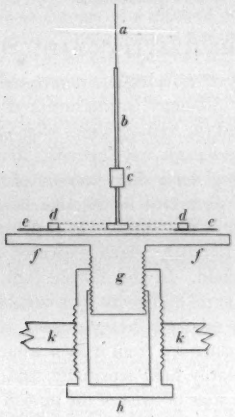

Fig. 1. Diagrammatic section of gradiometer beam system and control of damping plate.

a, lower part of suspending wire.
b, central rod of beam system.
c, mirror frame.
d, aluminium ring carrying gold weights (not in plane of figure and not shown).
e, mica ring.
f, brass damping plate.
g, upper element of differential screw for altering height of *f*, which is prevented from rotating by guides not shown.
h, graduated head of lower element of differential screw, engaging in fixed base of enclosure.
k, base of enclosure containing beam system. The enclosure is fixed to the upper part of a turntable (not shown) on which the whole instrument is mounted, and may be rotated bodily.

for altering very gradually the level of the plate. This is done by means of a differential screw operated by a screw-head outside the enclosure containing the suspended beam. One complete revolution of this screw head raises the plate 0·025 in. and, nominally, a manipulation of $\frac{1}{500}$ of this movement is possible. In practice it is found that approximate aperiodicity is secured by the air damping with about 0·025 in. between the mica ring and the plate, and, under these con-

ditions, the beam reaches equilibrium with sufficient accuracy in from 20 to 25 minutes after having the maximum amplitude of oscillation permitted in the apparatus. This limitation of amplitude is secured by means of adjustable stops, and it is usual in this way to allow an angular range of movement of only about 1 degree.

For purposes of transportation from one observing station to another the delicately suspended beam has to be clamped. This is done by the continued elevation of the damping plate, by means of the differential screw already mentioned, until the mica ring rests upon the plate and is eventually pinched between it and a brass ring fixed above. It should be noted, in relation to the sequel, that this necessary practical manipulation involves frequent contact, not free from friction and pressure, between the mica and the enclosing brass, during both the clamping and the releasing of the suspension of the beam.

It is unnecessary here to describe the means adopted for enclosing the suspended beam so as to prevent effectively the disturbances which might arise from air currents, or the optical system used to observe the angular position of the beam. It will suffice to mention that the divisions of the observation scale are such that about 40 of them correspond to 1 degree of rotation of the beam with respect to the torsion head of the suspension, and that readings can be made accurate to 0·1 division. An essential preliminary adjustment consists of orienting the torsion head carrying the upper end of the torsion wire so that the equilibrium reading, with the instrument located in a gravitational field nearly enough uniform, is approximately midway between those corresponding to the limiting stops. The weight of the beam complete is rather more than 80 gm., and the torsion wires are in the form of ribbons of a platinum alloy (similar to the well-known phosphor-bronze strip), having torsion coefficients (τ) round about 0·1 dyne-cm./radian. Two have been used in the present work—the one originally in the apparatus on arrival from Australia, with $\tau = 0{\cdot}159$ dyne-cm./radian, and a new one supplied by Dr Shaw, with $\tau = 0{\cdot}088$ dyne-cm./radian. τ

With a well-annealed torsion wire, once the torsion head has been finally adjusted and fixed, the natural zero of the instrument, i.e. the reading corresponding to a gravitational field precisely uniform, may be expected ideally to display constancy with respect to time. Apart from the source of disturbance with which we shall be dealing later, the behaviour of the torsion wires is in this respect highly satisfactory, at any rate under the conditions of use in a laboratory, as will be seen. This natural zero, however, can never be observed directly, because we have no place accessible where gravity is sufficiently uniform. For even in the absence of local disturbances, the northerly gradient of gravity, except near the equator and poles, is large enough to affect the instrument—surely a striking indication of its wonderful sensitivity. Generally, therefore, the reading alters with change of azimuth of the instrument as a whole, and practice consists in observing the readings (after equilibrium is attained) with the instrument as a whole in a sufficient number of different azimuths, a graduated turntable being incorporated for the purpose.

With the gradiometer a minimum of three readings in three azimuths are

required completely to determine the gravity-gradient, and, incidentally, the natural zero of the instrument. In practice one or two more readings are taken in order to check the constancy of the zero. The author has found it convenient to throw the fundamental equation into the form:

$$n - n_\alpha = qG \sin (\alpha - \phi),$$

α where α is the azimuth of the symmetrical vertical plane of the beam with respect
G to an arbitrary direction (such as the magnetic meridian), G is the resultant
ϕ horizontal gravity-gradient $\partial g/\partial s$, and ϕ its azimuth referred to the standard above
n, n_α indicated. The unknown natural zero reading is n, and n_α is the observed reading
q with the beam in the azimuth α. The constant q is a purely instrumental one, involving, in a manner into which we need not enter here, the distribution of the mass of the beam, the torsion coefficient, and the dimensions of the optical system.

To provide the three equations necessary to determine the unknown magnitudes n, G and ϕ, it is customary to choose values of α equal to 0°, 120° and 240° respectively. The author has found it more convenient to use four values, namely, 0°, 90°, 180° and 270°, the supernumerary one providing a convenient check on zero-constancy.

The four equations obtained are:

$$\alpha = 0, \quad n - n_0 = -qG \sin \phi,$$

$$\alpha = \pi/2, \quad n - n_1 = \quad qG \cos \phi,$$

$$\alpha = 2\pi/2, \quad n - n_2 = \quad qG \sin \phi,$$

$$\alpha = 3\pi/2, \quad n - n_3 = -qG \cos \phi,$$

where n_0, n_1, n_2 and n_3 are respectively the readings observed with the instrument in azimuths 0, $\pi/2$, $2\pi/2$ and $3\pi/2$. By adding and subtracting these equations appropriately we obtain

$$n_2 - n_0 = 2qG \sin \phi,$$

$$n_1 - n_3 = 2qG \cos \phi,$$

and

$$n_0 + n_2 = 2n = n_1 + n_3.$$

The last equation provides the zero-constancy check, and the first two give the gravity-gradient in magnitude and direction. For

$$\tan \phi = (n_2 - n_0)/(n_1 - n_3),$$

and

$$G = (n_2 - n_0)/2q \sin \phi,$$

the ambiguity of ϕ being removed by reference to the last equation.

An actual example from observations at a particular location in the College laboratory will illustrate the application of these equations, and the kind of accuracy which may be hoped for under good conditions.

Date—January 5, 1932.

Constant of instrument $= 1/2q = 10{\cdot}9 \times 10^{-9}$ sec.$^{-2}$
$= 10{\cdot}9$ E.*

Time	Azimuth of instrument	Scale reading
10.30 a.m.	0°	$n_0 = 113{\cdot}9$
10.55	90°	$n_1 = 114{\cdot}5$
11.20	180°	$n_2 = 120{\cdot}1$
11.45	270°	$n_3 = 119{\cdot}4$
12.10 p.m.	0°	$113{\cdot}9$

The last reading is an additional check for zero-constancy, which is here excellent.

The natural zero is $n = \frac{1}{2}(n_0 + n_2) = 117{\cdot}0$,

or $n = \frac{1}{2}(n_1 + n_3) = 116{\cdot}95$,

and the closeness of these figures again shows satisfactory behaviour.

From the above equations:

$$\tan\phi = (n_2 - n_0)/(n_1 - n_3) = -1{\cdot}265.$$

Hence $\phi = 128°{\cdot}3$ or $231°{\cdot}7$.

But since $n_2 - n_0$ (equal to 6·2) is positive, the equation

$$G = (n_2 - n_0)/2q \sin\phi$$

shows that $\sin\phi$ is positive. Hence $\phi = 128°{\cdot}3$ and

$$G = 6{\cdot}2/2q \times 0{\cdot}785 = 7{\cdot}9/2q = 86 \text{ E.}$$

Thus the gradient measured is 86 Eötvös units in a direction making an angle of 128° with the standard direction. It will be noted that if the readings could be always made precise and reproducible to 0·1 division, an accuracy of between 1 and 2 Eötvös units could be expected.

§ 3. ERRORS DUE TO ELECTRIFICATION

The above outline of the functions and use of the gravity-gradiometer enables us now to consider the special observations which form the subject of this paper. A more or less chronological order will be adopted in the description. When the instrument was first set up after the necessary repairs, it was found usually to give self-consistent results in the various stations in which it was used, but not always. More particularly immediately after the transfer from one station to another, in which the clamping and unclamping of the beam was involved, there were variations from consistency amounting to as much as 20 E. in the magnitude and 16° in the direction of the gravity-gradient determined. There were also unexplained and apparently capricious changes of the natural zero of as much as 7 scale-divisions

* "E." denotes 1 Eötvös unit, which is 10^{-9} c.g.s. unit of force-intensity gradient, i.e. 10^{-9} dyne/gm. cm., or 10^{-9} sec.$^{-2}$ E.

during one night, even when the beam was hanging free over the whole period. The cause of this rather erratic behaviour was at first assumed to be poor quality in the torsion wire, which had been similarly suspected by the Australian observers, who seem to have experienced even more marked inconsistency of the kind described. Accordingly it was decided to insert a new wire, and one tested by Dr Shaw was obtained, having a smaller torsional coefficient than the previous one.

The insertion of a new suspension, and the balancing of the beam, are matters of some delicacy. It is important, of course, that the plane of the mica ring should be parallel to the damping plate in its horizontal adjustment, and that there should be between them no hairs or dust-particles likely to interfere with free movement in the relative positions of close proximity (about 0·025 in. apart) in which they have to be used. For this reason the ring was rubbed with chamois leather, and marked electrical attraction was for a short while thereafter noticed between the ring and the damping plate. It was expected that this electrical effect would soon disappear, as, in fact, it did, so far as directly visible attractions were concerned. The erection of the apparatus was completed, apparently in a satisfactory manner, and electrification ceased to be suspected. One disturbing feature remained, however, in the final adjustment. This was the continued slow creep of the zero, which lasted several days and amounted, in all, to more than 20 degrees, before equilibrium was apparently reached. Again, this effect was—as it now appears, wrongly—attributed to the torsion wire, as an elastic after-effect due to the application of the load suspended upon it. Thereafter the behaviour of the instrument seemed to be normal, or at any rate self-consistent in repeated readings, as table 1 shows.

Table 1. Readings taken on February 29, 1932.

Time	Azimuth of instrument	Scale-reading
10.15 a.m.	0°	98·4
10.40	90°	94·2
11.20	180°	112·9
11.50	270°	117·4
12.20 p.m.	0°	98·6
12.50	270°	117·5
2.15	180°	112·8
2.45	90°	94·4
3.10	0°	98·9

Normally the readings should be constant in any particular azimuth. It will be observed that this is satisfactorily so, and that the reading in the 0° azimuth changed by only 0·5 division in about 5 hours. But the results of these observations are not even approximately in accordance with previous determinations in the same location. They give, in fact, a value of the gravity-gradient differing by nearly 100 per cent in magnitude and 20° in direction from the best earlier results.

This was the dilemma when, almost by chance, an observation was made which eventually pointed the way to the solution. The suspension had been secured with a view to testing whether the zero-creep reappeared upon release, and the damping

plate was later lowered to a position not quite so far from the mica ring as previously. It was found that the zero had changed by about 9 divisions, but was not obviously creeping. This led to systematic tests being made of the effect of other changes in the position of the damping plate, relative to the mica ring, with the surprising result that the scale-reading, and therefore the equilibrium orientation of the beam, was related in a periodic manner to the space interval between the mica ring and the plate. Moreover, the period was that corresponding to one complete rotation of the screw which controls the movement of the plate. The process of observation

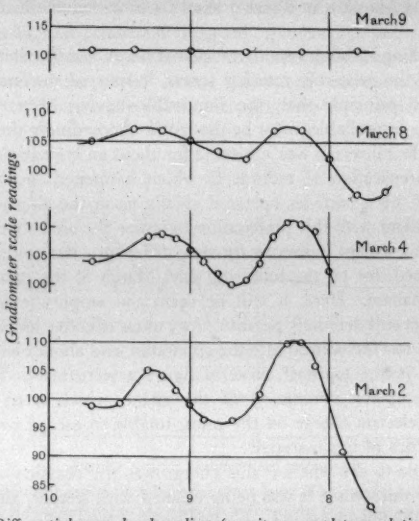

Fig. 2. Comparison of electrical effects in relation to the lapse of time, and to the introduction of ionizing material into the gradiometer.

was necessarily laborious, for a time interval of at least 15 minutes had to be allowed after each new adjustment for equilibrium to be attained. Yet the relation indicated was quite definite, and readings could be closely repeated, especially if taken after screw-rotations always in the same sense.

The results are shown graphically in the lowest curve in figure 2. The abscissae are the screw-head readings, which decrease as the damping plate rises, and a change of reading of unit corresponds to one complete revolution of the screw-head. The form of the curve obviously suggests the operation on the beam of a horizontal force-moment which increases as the damping plate approaches the mica ring, and also fluctuates considerably with the rotation of the driving-screw. Electrification was again suspected; a charge localized on a comparatively small

region of the mica, accompanied by a periodic relative tilt of the damping plate owing to imperfect travel of the actuating screw, would account qualitatively for the observations. And this is, indeed, the explanation. But at the time it was difficult to believe that a charge could remain insulated so persistently, even though the weather at the time was very dry; for the mica ring had been already suspended freely for several days—from February 29 to March 2, without any contact which might have produced new electrification. Moreover, it was found that the effects could be repeated without much change on subsequent days. Thus, for example, the curve labelled March 4 in figure 2 shows a little reduction of amplitude, but not much.

Still, after seeking in vain for another explanation, such as gravitational asymmetry in some part of the relatively rotating screw, it became necessary to adopt the Sherlock Holmes principle that, the impossible having been eliminated, what remains, however improbable, must be the truth. Accordingly the instrument was opened up, and the mica ring was subjected for about an hour to the radiation from a fairly active preparation of radium D which happened to be available. The ionizing effect of the α-particles operated chiefly upon the air above the mica, as it was not convenient with this preparation to ionize the air between the mica and the damping plate, except indirectly by slow diffusion. But evidently some effect had been produced, for on the following day, March 8, the curve so labelled in figure 2 was obtained. Here, it will be seen, the amplitude is definitely less, although the effect still definitely persists. As a more effective ionizer about $\frac{1}{3}$ mgm. of meso-thorium was introduced into the apparatus, and about one hour afterwards the observations in the topmost curve of figure 2 were taken. This shows very strikingly the complete elimination of the effect, which must accordingly be attributed to an electric charge on the mica, unable to escape owing to the high insulating properties of this material.

It is impossible to say whether this charge was the remains of that originally developed on the mica while it was being cleaned with leather, although probably it was so; for in the meantime the mica had been at least once in contact with the damping plate, and this might have disturbed the earlier electrical distribution. But the fact remains that a small charge (an estimate of its amount is made later) persisted nearly unchanged in amount and location on the mica for more than a week. For while the constancy of amplitude in the earlier curves implies little variation in the amount of the charge, the constancy of the positions of the maxima and minima equally implies its stationary position.

It is interesting to examine a little more closely the mode of operation of this electrical effect, now happily eliminated, and to consider its possible bearing upon normal gravity determinations made with the instrument. The complete removal of the electrification made known precisely for the first time the reading, 111·0, corresponding to the equilibrium of the suspended system under the sole action of the torsion in the wire and the differential gravity forces. Comparison of this with the readings indicated in any of the lower figure 2 curves shows that the effect of electrification was apparently two-fold, first, a deflection (in the sense of

reduced reading) which became notably greater as the damping plate approached the mica, and, secondly, a fluctuating deflection, correlated to the azimuth of the damping plate screw-head. This latter deflection appeared to be superimposed upon the first as a pseudo-zero-line, and evidently increases in amplitude as the distance between the mica ring and the damping plate is reduced. An attempt has been made to indicate the dissection of the curves in figure 3. The x-axis itself is taken to represent the undisturbed zero reading; the differences between this and the other readings are the ordinates, which thus become positive; and the abscissae are the azimuths of the screw-head itself (instead of the readings on its drum) reckoned from its angular position corresponding to contact between the damping plate and the mica. These abscissae are thus also proportional to the distance between the mica and the plate. Actual deflections are represented by the full curve, and the dotted line is a guess at the changing zero about which the fluctuations seem to occur.

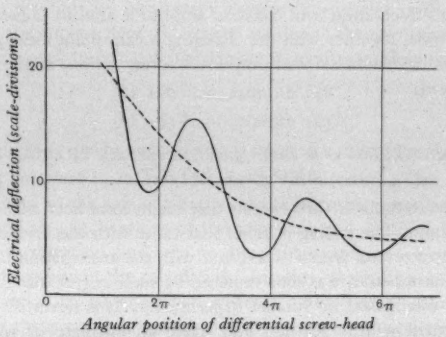

Fig. 3. Electrical effect of rotation of differential screw-head. The angular position represented of the screw-head is proportional to the distance between the mica ring and the damping plate, 2π being equivalent to 0·025 in.

A satisfactory explanation of the electrical effect can be made on this basis. In order to produce an angular deflection at all the charge must be acted upon by a force which has a horizontal component, for a precisely vertical force would produce no moment about the vertical axis of the system. Moreover, since the charge on the mica was always very close to the relatively extensive brass damping plate (the maximum separation was less than 0·1 in.) it is reasonable to suppose that practically all the lines of force terminated normally upon this plate. This implies that the plate itself was never truly horizontal. The evidence provided in figure 3 that, on the average, the force increased notably with diminution of the plate distance, also suggests strongly that the charge was very much localized, in comparison with the separation. The explanation of the effects accordingly offered is that a charge on the mica, practically constant in amount and confined to a comparatively small area, was attracted by the damping plate with a force which

always had a horizontal component, owing to this plate being inclined to the horizontal. This inclination varied from its mean value owing to imperfect action of the differential screw, the variation being repeated for each complete rotation of the latter. Consequently, the effective part of the plate (which does not itself rotate bodily)—i.e. that immediately underneath the charge—executed periodic changes of inclination, and thereby varied periodically the magnitude of the horizontal force-component. The total effect would, of course, be a mixed one, combining the results of inclination and proximity. If, for example, the screw had been perfect in its action, so that the damping plate maintained a constant inclination, we should have expected simply a gradual growth of the deflection as the plate rose, owing to the increase in the magnitude of the electrical attraction—some such curve, in fact, as the dotted line in figure 3. On the other hand, if the charge had been distributed over so great an area that the magnitude of the attraction was independent of plate distance, the periodic error of the screw would have given rise to deflections fluctuating with constant amplitude about a constant mean. The changing amplitude, together with the changing mean, point clearly to a localized charge on the mica, combined with imperfect screw action, as the cause of what has been observed.

§4. QUANTITATIVE ESTIMATE OF ELECTRIFICATION AND PLATE-TILT

It is easy now to think of further tests that might have been made to render the matter more certain. The feeling of relief that came with the disappearance of the electrification after several weeks' work, and with the anticipation that the gradiometer would behave better, was soon replaced by some regret that the same charge could not be re-established for further experiments. It is nevertheless possible to carry the argument a little further, and make an estimate of what was in all probability a not very different charge, and of its effect on gradiometer observations normally carried out. The observations recorded in table 1 were made at a time just previous to the discovery of the electrical effect, and the mica was undoubtedly then charged also. In the interval the mica ring had rested (it was not clamped) for 50 minutes upon the damping plate. This may, of course, have changed the charge both in magnitude and location, but the little evidence of comparative readings which is available makes it fairly certain that the charge before the interval was in the same position and substantially the same in amount as it was afterwards. At any rate the charge was what had survived after six days of free suspension, and was not enough to make the behaviour of the gradiometer appear other than normal.

We can use these observations in combination with corresponding ones obtained when the mica had been completely discharged, to form an estimate of the mean tilt of the damping plate and the magnitude of the charge. In these observations, of course, there is no question of variation of plate-distance, which is constant at the normal damping position. Any electrical effect is due solely to variation of

plate-inclination with change of azimuth of the instrument as a whole. The comparative readings are given in table 2, in which are recorded also the limits of free swing, for reasons which will appear later.

Table 2

Azimuth of instrument	Gradiometer readings		Limits of free swing
	(*a*) Previous to discharge	(*b*) After discharge	
0°	98·4	111·0	76–135
90°	94·2	112·2	77–136
180°	112·9	122·0	73–132
270°	117·4	120·6	72–131
0°	98·6	110·9	

Both sets of readings are satisfactory as regards repetition in the same azimuth, but their wide and inconsistent differences are due to the electrification in the first case. If they are used to calculate the apparent gravity-gradient, we obtain:

(*a*) 166 E. at azimuth 148°.

(*b*) 83 E. at azimuth 127°.

The latter, of course, is the correct result, and, as is evident, the former differs from it by 100 per cent in magnitude and 21° in direction. This large difference must be attributed to the axis of rotation of the instrument not being vertical, for otherwise the relative inclinations of the mica ring and damping plate would not change, and electrification could do no more than produce a constant deflection, which would disappear in the calculation. In other words, spurious results would be avoided, even with an electrified system, if the axis of rotation were truly vertical. But it turns out that the axis would have to be vertical with impracticable precision.

The levels provided on the instrument are such that one division on the bubble scale corresponds to about 20 seconds of arc, but they are difficult to adjust so as to use fully this sensitivity, which, indeed, is quite unnecessary in normal practice. They were not, in fact, read very carefully while the instrument was in the adjustment with which we are at present concerned. But it happens that some incidental and not very accurate readings, namely, those corresponding to the limits of free swing shown in table 2, provide a relatively sensitive method of estimating the extent of the departure of the rotation axis from the vertical. If this axis were truly vertical, rotation of the instrument would bring about no relative alteration of the positions of its various parts, including the suspension, and contact of the latter with the stops would occur always at the same readings. The fact that these limiting readings did not remain constant accordingly implies that the axis of rotation was not quite vertical; and the change of the limits with azimuth, used in combination with the geometry of the instrument, shows that the error of adjustment was approximately 30 seconds of arc. Thus, during a complete rotation, the

damping-plate tilt altered through a maximum range of just about 1 minute—a change of angle entirely negligible in the normal gravitational use of the apparatus, but responsible for a large part of the spurious electrical effect.

It turns out that besides this periodic electrical disturbance associated with azimuth there was a constant effect due to the plate having a small permanent tilt upon which the variable part during rotation was superimposed. If we take the differences between corresponding readings in the (*b*) and (*a*) columns of table 2, we isolate from the true gravitational control the whole of that due to plate tilt and electrification. These differences, which correspond to electrical deflections in the sense chosen for positive azimuth, are given in the second column of table 3.

Table 3

Azimuth (α)	Electrical deflection (scale-divisions)	
	Observed	Calculated
0°	12·6	12·5
90°	18·0	18·1
180°	9·1	9·0
270°	3·2	3·3
360°	12·6	12·5

If our explanation of the effect of plate tilt is correct we should expect these observations to be representable by an equation of the form

$$d = a + b \sin (\alpha + \epsilon),$$

d, *a*, *a*, *b* with d as the electrical deflection, and α as the azimuth of the instrument; a, b

ϵ and ϵ being constants. Although the test is not very stringent, there being only one observation in excess of the number of constants, the equation

$$d = 10{\cdot}7 + 7{\cdot}6 \sin (\alpha + 13^\circ{\cdot}3)$$

does in fact fit all the points with a precision equal to the accuracy of reading, namely, 0·1 division. We may therefore take the curve shown in figure 4 as re-

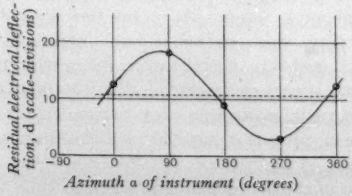

Fig. 4. Electrical effect of change of azimuth of instrument. Calculated curve $d = 10{\cdot}7 + 7{\cdot}6 \sin (\alpha + 13^\circ{\cdot}3)$; observed points denoted by circles, and zero of sine curve by dotted line.

presenting by its ordinates the course of variation of total plate tilt with azimuth. This may be dissected into the constant part, 10·7, and the variable part which has an amplitude of 7·6. The former corresponds to the permanent part of the

plate tilt and the latter to its change with azimuth. We have already seen that the variable part had an amplitude of 30″, and we can now conclude that permanent part was 30 × 10·7/7·6 or 40″ approximately—again an angle of no importance in gravitational practice with the instrument*.

This estimate of the magnitude of the plate tilt provides means of making a rough estimate of the amount of electrical charge which was operative, if certain assumptions are made as to its distribution. The distance between it and the plate was 0·025 in. or 0·064 cm., and it is difficult to believe that the charge was confined to a region on the mica small compared with this, although the evidence points to its not being large. With this reservation, and assuming nevertheless that the charge was effectively at a point on the mica about midway between the inner and outer edges of the ring, we can calculate its amount, for we know the change of scale-reading, and, therefore, the torsional force-moment, corresponding to a given plate tilt. Thus for any azimuth we equate the horizontal component of the force on the electric charge (this force itself being always normal to the plate) multiplied by its distance from the axis of suspension, to the residual torsion in the wire, as indicated by the difference between the scale-readings under charged and uncharged conditions. The result obtained is that the charge was about 0·07 e.s.u. This is certainly an under-estimate, for the assumption of any distribution of the charge wider than a point leads to a higher value. For example, if we suppose it to be spread uniformly over 1 cm.^{2} the amount becomes three times as great. Equally certainly this is an over-estimate, for the observations show a marked dependence of attractive force upon distance, even at small separations of the order of 1 mm., between the mica and brass surfaces. It is safe, therefore, to take it that the charge was of the order of 0·1 e.s.u.

§5. PRACTICAL CONCLUSIONS

This charge was much too small to make itself evident except in such an extremely delicate system as the gradiometer. Indeed, its marked disturbing effect is a tribute to the sensitivity of this nevertheless portable and robustly constructed instrument. The important practical point is, however, that electrical charges, small even in comparison with that above calculated, must be eliminated whenever formed. That they can be produced by the contact between the mica and brass, necessary during transportation of the instrument, has been shown by subsequent observations.

It is only rarely that the effect becomes obvious, but when it is remarked that on more than one occasion the release of the suspended system has been followed by a period of erratic and unstable behaviour lasting two hours, even with the meso-thorium within the instrument, it will be realized that less violent and obvious effects are likely to be much more frequent. In fact, a very small charge might be

* It is perhaps worthy of note that a new adjustment of the levelling, made subsequently with greater care, resulted in the limit readings becoming practically independent of the azimuth. This showed that the axis of rotation had been rendered much more truly vertical.

much more insidious than a large one, for it would probably fail to arouse suspicion and consequently lead to spurious gravitational results.

Safety in this respect lies in making provision either for the prevention of the development of all charges of electricity, however small, or for their rapid dissipation. The substitution of some suitable metal for the mica would effectively fulfil the former alternative, because the whole apparatus would then be of conducting material, and its inner parts could sustain no electrification. Or the mica might be coated with metal, for example by spluttering; but this would probably not be satisfactory or permanently effective, owing to the liability of the metal to be rubbed off by contact with the brass clamps. The author, however, favours the second alternative, namely, the use of sufficient ionizing material suitably disposed to dissipate quickly any charges formed. The mica has several features in its favour—its flatness and lightness, for example—and ionization has been proved a reliable means of getting rid of the electrification, whether this is produced during the original setting up of the apparatus or by the contacts inevitable in normal use.

Since the introduction of the meso-thorium the instrument under discussion has been a joy to use, and its performance has more than realized expectations. It seems almost certain that, when it was first set up with the new suspension, the considerable initial electric charge on the mica disappeared much more slowly than one would have anticipated, its gradual reduction of control giving rise to the prolonged creep of the equilibrium reading. Even after apparently normal equilibrium had been reached there remained a relatively small but insidious charge, which would have taken weeks to diminish by leakage to a really negligible amount. The insertion of the meso-thorium greatly accelerated this process, and its presence has continued thereafter as an effective safeguard. And, except on those rare occasions already mentioned when the temporary erratic behaviour was too self-evident to be misleading, the observations with the instrument have been extraordinarily self-consistent in spite of frequent use of the clamping device. Deliberate attempts have been made to facilitate electrification by encouraging friction between the mica and its clamps. But in practically every case its effect, if it was ever produced, had disappeared before the normal minimum time after release for taking a reading—about 25 minutes.

As illustrating the very marked improvement in the performance of the instrument in the absence of electrification, the results shown in table 4 and obtained in

Table 4

Date	G (Eötvös units)	ϕ (degrees)
March 10	83	127·3
,, 14	85	127·7
,, 15 (1)	82	126·3
,, 15 (2)	83	126·7
April 1 (1)	81	127·0
,, 1 (2)	82	127·0
Mean	83	127·0

the same place on different occasions, for the magnitude (G) and the azimuth (ϕ) of the gravity-gradient, are worth noting.

The extreme variation of the magnitude is only 4 E. and of the direction only 1·4 degrees. These may be compared with the errors of 83 E. and 21° for which electrification was on one occasion responsible. Moreover, the equilibrium reading of the instrument in a fixed azimuth has not been observed to vary by as much as 1 division for a whole month, although the clamping device has been used frequently during this period. When it is remembered that constancy in this respect is only demanded for rather less than two hours—the maximum time normally occupied at a station—the present reliability of the instrument becomes apparent. It seems reasonable, after the experience described with this one instrument, to doubt whether all the field observations so far made with gradiometers have been sufficiently free from the spurious effects of electrification. In some cases, probably only a few, quite large errors may have been involved, but there has been the danger that many of the results have been at least slightly tainted. The uncertainty in this matter arises from the way in which the electrical effect may remain hidden, allowing results which are really vector mixtures of the true and the spurious to pose as genuine gravity-gradients. A too variable natural zero should, perhaps, have aroused suspicion, as, indeed, it sometimes has done; but the elastic properties of the torsion wires have had to bear the blame. The torsion wire has now been found not guilty, at least of the chief indictment; and the real culprit has been detected and abolished, or, rather, deported. It is hoped and believed that this example will be followed, not only in newly manufactured gradiometers, but in those also which have already served in the field. For the means of eradication is both simple and inexpensive, and its application will without doubt enhance still more towards perfection the accuracy and reliability of the already wonderful performance of these instruments.

DISCUSSION ON THE PRECEDING TWO PAPERS.

Dr H. Shaw. The author has obtained some very simple and convenient formulae for the expression of these gravitational effects. His method of completely separating the two portions of the horizontal couple will be most helpful to the student in enabling him to form a better mental picture of exactly what is happening in the instrument. With regard to the graphical method put forward in the paper I am inclined to be a little hesitant, for although its simplicity is obvious in the laboratory, its application in the field would I fear be somewhat more difficult. However I should like to see it tested under field conditions, against the methods at present in operation, and if it proves quicker and more convenient, there is little doubt that it will be applied extensively.

With regard to the second paper, I would say that the metal disc suggested by Prof. Rankine was tried before mica was adopted, and was discarded on account of the difficulty of securing a ring which is at the same time flat, light and rigid. Ordinary mica too was soon found to be useless, and even ruby mica, apparently quite free from intrusions, was often found to possess impurities which in the earth's magnetic

field gave deflections of from 10 E. to 40 E. It will be seen therefore that it has proved to be a matter of some difficulty to obtain mica which is up to the standard required for these instruments.

With reference to the electrical troubles which form the subject of this paper, it is very curious that prior to Prof. Rankine's experiments we had not encountered anything at all comparable to the disturbing effects he describes, and there can be little doubt that the trouble is due, at any rate in part, to a too energetic use of the chamois leather. I was reminded this morning that Prof. Boys warned us of this (and other) troubles in 1921, when the Eötvös torsion balance was first exhibited and described at a geophysical discussion of the Royal Astronomical Society.

I would like to ask the author whether the long zero-creep referred to on pp. 479, 480 was upward or downward, and whether it may be attributed wholly or only in part to the gradual leakage of the electrical charge. With regard to the possibility of errors in the field observations so far made with gradiometers, I would say that the constancy of the natural zero is perhaps one of the observer's main safeguards. It happens however that this point has long been recognized as an important indicator of the efficient operation of all instruments of the Eötvös torsion balance type, and so far as our own field operations are concerned it has always been kept under close observation, so that I have no fears. It may be of interest to note that the first gradiometer, which has now been in operation for over four years, is still in the field, where it continues to give the most gratifying results.

Users of this instrument are much indebted to the author, firstly for discovering and tracking down this source of danger, secondly for determining its nature, and lastly (what is perhaps even more important) for showing how it may be completely eliminated.

Mr E. Lancaster-Jones. The treatment outlined in the first paper will greatly assist students who appreciate geometrical or graphical illustrations and dislike analysis. The ordinary analytical treatment, using axial components of the gravity magnitudes, has however the merit that the component effects are simple algebraical numbers, and therefore can easily be summed by unskilled operators or by mechanism. At the same time there is a lot to be said for plotting the terrain and topographical-correction quantities, as a check on numerical calculations.

As regards the second paper, all users of the gradiometer will be greatly indebted to Prof. Rankine for this investigation. It has revealed a hitherto unsuspected source of trouble, and indicated the cure. It is highly improbable, fortunately, that previous field-work results obtained with the gradiometer have been subject to unsuspected errors by reason of electrical effects of this type. Readings have always been repeated and zero-fluctuations carefully scrutinized. Had electrical effects been present, they must surely have revealed themselves by reason of the instability of the zero and non-repetition of readings in a given azimuth. In practice, the instrument is supported on footplates bedded usually in soft ground, and a certain amount of "settlement" is inevitable; this causes the level of the instrument to vary sufficiently to accentuate the electrical effects very considerably.

Also, in most of the practical surveys so far undertaken with the gradiometer the results have been checked by using other instruments at intervals, and very few discrepancies have been revealed, except in the Australian work. The ionization treatment advocated by Prof. Rankine should certainly be adopted, but it may be as well to consider methods of clamping the beam at its metal parts, leaving the mica disc "in the air."

Prof. W. WILSON. The author's vectorial methods are very valuable, even though they may not be the most suitable for purposes of computation in the field. I would suggest that the horizontal directive tendency is a tensor of rank 2.

Dr D. OWEN. It seems almost incredible that in an ordinary damp atmosphere electric charges should persist on the surface of mica for a period of a fortnight. I should like to suggest that charges may occur in the interior of the mica sheet owing to shearing produced in the course of the adjustment. The degree of non-homogeneity in mica seems to warrant the possibility of such an effect.

Dr J. E. R. CONSTABLE. I was very interested in the author's account of the trouble which he experienced owing to charges on the mica disc, as some time ago, while working with a Geiger counter, I experienced considerable trouble due to the same cause. A Geiger point counter fitted with a mica window rapidly ceased to function when exposed to β and γ radiation. Touching the surface of the mica with an earthing wire always restored the activity at once. In view of the fact that I was using up to 2000 volts between the case of the counter and its inner electrode it would seem that potential-differences of this order can be generated by the charges on the mica. These charges only appeared when the counter had the potential-difference applied to it, so that Prof. Rankine's method of removing the charges by mesot-horium radiations should be quite satisfactory provided there are no electric fields in the neighbourhood. To secure the highest efficiency it might therefore be advisable to enclose the meso-thorium in an earthed metal container.

Mr J. H. AWBERY said that the quasi-vectorial diagram in figure 2 of the first paper reminded him of diagrams used in the theory of finite rotations, the plane of figure 2 corresponding to the spherical surface dealt with in the theory.

Mr J. GUILD asked whether a paper disc could be substituted for the mica disc which had proved so troublesome.

Mr T. SMITH. As regards the first paper: as a rule I much prefer working with rectangular to polar coordinates, but comparison of equations (1) and (6) of the paper on gravity surveys makes it quite apparent that the latter are much to be preferred here. In field work the instrument is set by means of an angular scale, and the solution given in equations (11) is the simplest and most direct that can be expected. The use of graphical methods is a distinct question. Personal preferences may carry some weight here. As an accuracy of about 1 per cent is sufficient, graphical methods are certainly admissable and will probably be more quickly carried out than the resolution required by the alternative method. The construction for R may be compared with that for the combination of two astigmatic lenses.

As regards the second paper: the paper on the gradiometer is a most interesting account of the way in which trouble with an instrument was tracked to its source. Dr Shaw has told us that Prof. Rankine was misusing the instrument in rubbing the mica ring with the chamois leather, but this misuse, in combination with somewhat imperfect action of the screw, has been the means in Prof. Rankine's hands of entirely altering the value of the instrument. I can only hope that the author will always be as fortunate in his mishandling, more particularly if it frequently results in the presentation of such an interesting paper to the Physical Society. Like all who have listened to his explanations I would like to express my appreciation of the very lucid way in which both papers have been presented to us.

Author's reply. With regard to the first of these two papers I welcome the expression of views by two such experienced workers with torsion balances as Dr Shaw and Mr Lancaster-Jones. I agree that the ultimate test of the relative merits of the customary analytical method, as contrasted with the graphical method now proposed, is in respect of rapidity and convenience in the field work. My students and I are relatively inexperienced in this aspect of the work, but we hope gradually to accumulate data to render the comparison possible.

I am not sure that I understand the terms used by Prof. Wilson, but, whatever the horizontal directive tendency is called, I have recognized that the graphical rules for its addition suggested other similar directed quantities of higher orders. So far I have resisted the temptation to investigate these mathematically, because their practical application was not obvious. It is interesting to note that the graphical rules for addition of the *R*-values are similar to those applicable in the theory of finite rotations, as indicated by Mr Awbery, and in the theory of the combination of astigmatic lenses, as Mr T. Smith has mentioned.

On Mr T. Smith's other remarks I wish to say how gratified I am to have for my mode of transformation of the equations the approval of so penetrating and reliable a mathematician. This is a real encouragement to go on further in the direction indicated, and I shall do so.

The remarks of Dr Shaw and Mr Lancaster-Jones on my second paper are also valuable as showing the infrequent incidence of marked electrical effect in the many surveys already carried out with gradiometers. But I do not think it is safe to assume, as Dr Shaw apparently does, that strong electrification of the mica is always due to too vigorous rubbing during cleaning. Considerable charging occasionally occurs as a consequence of the normal clamping and releasing of the beam, and one cannot be sure that small effects do not occur quite frequently. In reply to Dr Shaw's question, I would say that the zero-creep was in the direction corresponding to increasing reading, i.e. a rotation in a counter-clockwise sense as viewed from above the mica ring. I am convinced that the zero-creep observed was almost if not wholly due to leakage away of the original electric charge, for, after provision had been made for rapid discharge, the zero was very constant, in spite of repeated alternations of loading and unloading of the suspension. I agree with Dr Shaw and Mr Lancaster-Jones that constancy of repeated readings would be a

satisfactory test of the absence of electrification, but constancy over many hours would be required, because of the slow rate of leakage. As I have indicated in the paper, repetition of readings was apparently satisfactory over the normal observation-period, even when the mica was strongly charged. On the other hand, the "settlement" to which Mr Lancaster-Jones has referred, by varying the axis of rotation in field use, would provide a sure criterion.

In reply to Dr Owen I should remind him that actually the observations were made in very dry weather. Apart from this, however, I do not think the charges could be internal, for, if so, neither the positive nor the negative could leak away, and their combined effect would be nil. Also, the quick disappearance of the charges in ionized air suggests external location on the mica, because diffusion into the mica sheet would presumably be very slow.

Dr Constable's experience with the Geiger counter confirms the view that one has to be very careful of electrical effects on insulators. My own recent observations have, in fact, made me suspicious of all insulated suspensions, or suspensions composed wholly or partly of insulating material.

The inventors of the gradiometer would be better able than I to answer Mr Guild's question; but Dr Shaw's remarks would seem to imply that paper could not be made, or ensured to remain, flat enough to serve for damping purposes. Mr Lancaster-Jones's suggestion to clamp the system by its metal parts and leave the mica always free is ingenious, but I think ionization is safer.

Mr T. Smith emphasizes an interesting point, the implications of which are somewhat curious. It is true that it was the imperfection of the differential screw which led to the charging being discovered and eliminated. I doubt very much whether the spurious effect could have been distinguished otherwise from normal gravity effects, for the manipulation of the damping plate formed the crucial test. Does this imply that sometimes imperfect workmanship is to be not only tolerated but approved?

THE FALL OF POTENTIAL IN A CHARGED INSULATED CABLE

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ABSTRACT. The expansion theorem of Heaviside is applied to the solution of the problem in which a cable, having been charged until it reaches its steady state, is insulated at the sending end; and the potential is required at any point and at any time after insulation. It is found that at the sending end there is an initial steep fall of potential which is due to a part of the charge being drained away from this end, in order to equalize the potential throughout the line when the exciting source has been removed. After the potential has become uniformly distributed, it falls with time according to a simple exponential law.

§ 1. INTRODUCTION

V_0
V_t, t

When a condenser with a good solid dielectric between its plates is charged to a potential V_0 and then left insulated, it is easily shown that the potential V_t at any time t after insulation is given by

$$V_t = V_0 e^{-Gt/C},$$

G, C

where C is the capacity of the condenser and G is the leakance or the reciprocal of the dielectric resistance. Use has been made of this formula to deduce the resistance of the dielectric when the capacity of the condenser is known. The method is a very old one, and is believed to be due to Latimer Clark (*c.* 1867) who applied it to some of the early submarine cables. Fleeming Jenkin[1] described a method of measurement in which an "inferred zero" was used, and we find the method advocated for cable-testing by various writers since that time.

A long cable is, however, not exactly equivalent to a condenser. We have to consider the fact that the capacity and leakance are distributed throughout its length. The aim of the present investigation is to find an expression for the potential at any point along the conductor of a cable which has been charged and then insulated, the far end being free.

§ 2. HEAVISIDE'S EXPANSION THEOREM

This very useful theorem was stated without proof in Heaviside's *Electromagnetic Theory*[2]. Recently Vallarta[3] has reconstructed the argument from Heaviside's scattered writings. In many cases in which a steady force is discontinuously applied or removed, the expansion theorem leads quickly to a solution which could only be obtained otherwise by Fourier integrals.

The method of working may be illustrated by considering a circuit of impedance Z which has a steady e.m.f. V_0 suddenly impressed upon it. The current i is given by Z, i

$$i = V_0/f(p) \qquad \text{.....(1),}$$

where $f(p)$ is a function known as the "operational-impedance." This is derived from the ordinary impedance Z to alternating current by the substitution of the Heaviside operator p (or d/dt) for the imaginary operator $j\omega$. The expansion theorem then states that $f(p)$

$$i = V_0 \left\{ \frac{1}{f(0)} + \sum_p \frac{e^{pt}}{pf'(p)} \right\} \qquad \text{.....(2),}$$

where the value or values of p are obtained by equating $f(p)$ to zero. The steady term $f(0)$, obtained by putting $p = 0$ in $f(p)$, is the resistance to direct current remaining when all transients have died away.

§ 3. APPLICATION TO A CABLE WITH UNIFORMLY DISTRIBUTED CONSTANTS

The line is assumed to be of length l and to possess resistance, capacity and leakance R, C and G per unit length. The inductance is assumed to be negligible. l R, C, G

With the far end free, a potential V_0 is applied at the sending or head end of the line, and after the steady state has been attained the head end is insulated. The potential $V_{x,t}$ at any point distant x from the head end at any time t is required. $V_{x,t}, x, t$

The steady-state current into the line is given by the well-known expression

$$i_0 = V_0 \sqrt{(G/R)} . \tanh l\sqrt{(RG)} \qquad \text{.....(3).}$$

Now the action of freeing the cable is equivalent to impressing an equal and opposite current on the line at the head end, the other conditions remaining the same. The potential at any point x will then be expressed by

$$V_{x,t} = V_{x,0} - V_x' \qquad \text{.....(4),}$$

where $-V_x'$ is the effect due to the current $-i_0$ at the head end. $V_{x,0}$ is the potential at the point x at the moment of freeing the cable.

From transmission theory[4] we have

$$V_{x,0} = V_0 \cosh x\sqrt{(RG)} - i_0\sqrt{(R/G)} \sinh x\sqrt{(RG)} \qquad \text{.....(5),}$$

but
$$i_0\sqrt{(R/G)} = V_0 \tanh l\sqrt{(RG)}.$$

Therefore
$$V_{x,0} = V_0 \frac{\cosh (l-x)\sqrt{(RG)}}{\cosh l\sqrt{(RG)}} \qquad \text{.....(6).}$$

Again[5]
$$V_x' = V_0 \frac{\cosh P(l-x)}{\cosh Pl} \qquad \text{.....(7),}$$

but
$$V_0 = i_0 Z_0 \coth Pl.$$

Therefore
$$V_x' = i_0 Z_0 \frac{\cosh P(l-x)}{\sinh Pl} \qquad \text{.....(8),}$$

where
$$Z_0^2 = R/(G + pC) \qquad \text{.....(9),} \quad Z_0$$

and
$$P^2 = R(G + pC) \qquad \text{.....(10).} \quad P$$

Substituting for i_0 from (3),

$$V_x' = \frac{V_0\sqrt{(G/R)}\tanh l\sqrt{(RG)}}{f(p)} \qquad \text{.....(11),}$$

where

$$f(p) = \frac{P}{R}\cdot\frac{\sinh Pl}{\cosh P(l-x)} \qquad \text{.....(12).}$$

Putting $p = 0$ in (12) we have

$$f(0) = \sqrt{(G/R)}\cdot\frac{\sinh l\sqrt{(RG)}}{\cosh (l-x)\sqrt{(RG)}} \qquad \text{.....(13).}$$

Converting (12) to circular functions,

$$f(p) = -\frac{jP}{R}\cdot\frac{\sin jPl}{\cos jP(l-x)} \qquad \text{.....(14).}$$

Equating to zero,

$$\sin jPl = 0,$$

$$P^2l^2 = -m^2\pi^2 \qquad \text{.....(15),}$$

where m is zero or any integer.

Substituting from (10),

$$p = -(m^2\pi^2 + GRl^2)/CRl^2 \qquad \text{.....(16).}$$

Differentiating (14) with respect to P and substituting from (15),

$$\frac{d}{dP}f(p) = \frac{Pl}{R\cos(1-x/l)\,m\pi}\left[\frac{\sin m\pi}{m\pi} + \cos m\pi\right],$$

but $2PdP = CRdp$ from (10). Therefore

$$f'(p) = \tfrac{1}{2}Cl\sec(1-x/l)\,m\pi\,[\cos m\pi + \sin m\pi/m\pi] \qquad \text{.....(17),}$$

$$\frac{1}{pf'(p)} = -\frac{2Rl\cos(1-x/l)\,m\pi}{(m^2\pi^2 + GRl^2)(\cos m\pi + \sin m\pi/m\pi)} \qquad \text{.....(18).}$$

Hence, combining (11), (13), (16) and (18) in the expansion formula, we have

$$V_x' = V_0\sqrt{\frac{G}{R}}\cdot\tanh l\sqrt{(RG)}\left[\sqrt{\frac{R}{G}}\cdot\frac{\cosh(l-x)\sqrt{(RG)}}{\sinh l\sqrt{(RG)}}\right.$$
$$\left. - 2Rl\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}\cdot\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}\cdot t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)}\right]$$
$$= V_0\left[\frac{\cosh(l-x)\sqrt{(RG)}}{\cosh l\sqrt{(RG)}} - 2l\sqrt{(RG)}\cdot\tanh l\sqrt{(RG)}\right.$$
$$\left.\times\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}\cdot\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}\cdot t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)}\right] \qquad \text{...(19).}$$

Finally from (4) and (6) we have

$$V_{x,t} = 2V_0 l\sqrt{(RG)}\tanh l\sqrt{(RG)}\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)}$$
$$\qquad\qquad \text{.....(20).}$$

This is the required expression for the potential at any point at any time after

freeing. To obtain the potential at the head end, x is made equal to zero, and we find

$$V_{0,t} = 2V_0 l\sqrt{(RG)}.\tanh l\sqrt{(RG)} \sum_{m=0,1,2\ldots} \frac{\exp\left(-\dfrac{m^2\pi^2 + GRl^2}{CRl^2}\,t\right)}{(m^2\pi^2 + GRl^2)\,(1 + \tan m\pi/m\pi)} \quad \ldots\ldots(21).$$

§ 4. NUMERICAL EXAMPLE

In figure 1 is shown the potential at various times after freeing at any point in a cable 1000 miles long, having the following constants per mile:

$$R = 10\Omega, \quad G = 10^{-9}\ \text{mho}, \quad C = \tfrac{1}{3}\mu\text{F}.$$

It is seen that the potential at the head end begins to fall immediately, while at a point further down the line the potential remains steady for an appreciable time. In this particular case the potential distribution is practically uniform after two seconds.

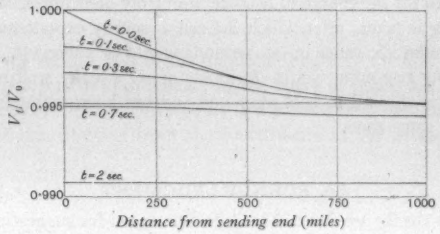

Fig. 1.

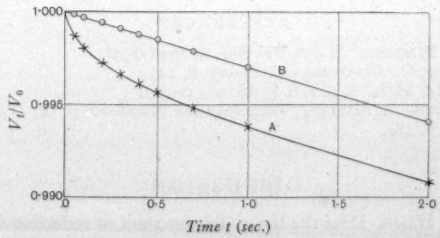

Fig. 2.

Curve A in figure 2 shows the fall of potential with time at the head end of this same cable, calculated from equation (21). Curve B in the same figure was obtained by putting R equal to 0 in (20) or (21) so that

$$V_t = V_0\, e^{-Gt/C} \quad \ldots\ldots(22),$$

which, as was pointed out in § 1, means that we have neglected the distributive effect and treated the cable as an ordinary condenser. This curve is shown for comparison with the more accurate expression (21).

§ 5. CONCLUSION

The expression obtained for the fall of potential at the head end of the cable shows that there is an initial sharp decline which is not predicted by the approximate formula (22). This effect is due to the redistribution of the charge along the cable, tending to uniformity throughout, when once the source has been removed.

In measuring the insulation resistance of a long cable by the loss-of-charge method, the higher potential value is commonly taken to be that of the charging battery, and the lower value is observed at time t after insulation. In these circumstances the apparent resistance will be too low, owing to a part of the potential fall being due to causes other than leakage through the dielectric. It is not suggested that the cumbersome expression (21) be used to evaluate the resistance, but time should be allowed for the charge to become uniformly distributed before the higher potential reading is taken, after which the fall is simply exponential. In the case considered above an allowance of two seconds would be sufficient.

The foregoing reasoning would obviously apply equally well to the case of a long bar which is allowed to cool after one end has been maintained at a steady temperature for some time.

§ 6. ACKNOWLEDGMENT

The author's thanks are due to Mr G. E. Stevenson for suggesting the problem, and for his interest in its solution.

REFERENCES

(1) Jenkin, Fleeming. *J. Soc. Tel. Eng.* **2**, 169 (1873).
(2) Heaviside, O. *Electromagnetic Theory*, **2**, 127.
(3) Vallarta, M. S. *J. A. I. E. E.* **45**, 383 (1926).
(4) For example, E. Mallett, *Telegraphy and Telephony*, p. 34. (London, 1929.)
(5) *Loc. cit.* p. 184.

DISCUSSION

Capt. C. W. Hume. Does the theory take account of reflection at the receiving end?

J. H. Awbery. Mr McCleery states that the solution would answer also the question of the temperature-distribution in a long bar which is allowed to cool after one end has been maintained at a steady temperature for some time. Would he explain for what conditions this solution would apply? Is it when the bar is heat-insulated, or when its surface loses heat at a rate proportional to the excess above

air temperature? This question is equivalent to asking what are the boundary conditions in the electrical case actually solved—is the outside of the dielectric taken to be at the potential of the earth, and is this the same as the potential at the distant end of the cable?

I should like to ask also whether the expansion theorem as used here gives the same form of solution as would Fourier's method (though possibly with less labour), or does it give a different, and perhaps more rapidly convergent, series?

AUTHOR'S reply. In reply to Capt. Hume: reflection at the receiving end is taken care of by the tanh term in the expression for the current into the line.

Mr Awbery asks if the bar in the heat analogue is insulated. If this were the case the bar could not cool and the problem would not arise. Actually the lateral heat-loss from the bar, proportional to the excess above air temperature, corresponds to the leakage of electricity through the dielectric, proportional to the potential excess of the conductor above earth. The potential at the distant end of the cable is not zero unless the line is infinitely long; the theory assumes that the far end is free, i.e. insulated, and its potential at time $t = 0$ is given by

$$V_l = V_0 \operatorname{sech} l\sqrt{(RG)}.$$

In reply to the second part of the question: Fourier's method would require a large number of terms to approximate to the curve of potentials. In the present method the series is rapidly convergent; it was found for the numerical example quoted that the contribution from the fourth term of the series was quite negligible.

THE PROPAGATION ALONG THE EARTH OF RADIO WAVES ON A WAVE-LENGTH OF 1·6 METRES

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ABSTRACT. A previous paper described an investigation of the attenuation of ultra-short radio waves when transmitted directly along the earth's surface. The present paper reports the progress made in the continuation of this research, the particular wave-length to which attention has recently been given being 1·6 metres. A brief description is given of the simple, but efficient, transmitting and receiving apparatus which has been employed for the experiments on this wave-length.

Measurements of the field-intensity at different distances from the transmitter have been carried out for various heights of the apparatus above the ground level. When both transmitter and receiver are used very close to the ground, the attenuation curve obtained is similar to that encountered at longer wave-lengths. When, however, the apparatus is elevated by an amount comparable with, or greater than, the wave-length, the field-intensity-distance curves have maximum and minimum values, the positions of which depend upon the actual heights employed. These maxima and minima are due to interference between waves transmitted directly from the transmitter to the receiver, and those which arrive at the receiver after reflection from the earth's surface.

Theoretical curves having the same characteristics have been calculated from a consideration of the reflection coefficient of the earth's surface, account being taken of the electrical properties of the earth. By a comparison of such theoretical curves with the experimental results, the effective conductivity of the earth appears to be about 95×10^8 e.s.u. (resistivity 95 ohm-cm.) at the very high frequency of 190 megacycles per second employed. This is higher than the values of 5×10^8 to 30×10^8 e.s.u. previously obtained at frequencies of 30 to 60 megacycles per second, and these in turn were higher than the values obtained in earlier work, at a frequency of 1 megacycle per second. Owing to this considerable increase in the value of the conductivity as the frequency is raised, the experimental method does not enable the dielectric constant of the earth to be ascertained with any great accuracy, although a value of 10 gives suitable agreement between the theoretical and experimental results in the present case. At the same time this consideration indicates that, from the point of view of practical communications, the value of the dielectric constant of the earth is not a controlling factor in determining the propagation of waves over the earth's surface on either long or short wave-lengths, except in situations where the conductivity of the ground is abnormally low.

§ 1. SUMMARY OF PREVIOUS WORK

In a previous paper[1] the authors described an investigation of the attenuation of radio waves, of wave-length between 5 and 10 metres, when transmitted directly along the earth's surface. A comparison was made between the experimental results and those calculated from a simple wave-attenuation theory, and thereby values were found for the electrical conductivity of the earth for alternating-current

conditions at the frequencies of 30 to 60 megacycles per second employed. The present paper describes an attempt to extend this work in the direction of higher frequencies (i.e. shorter wave-lengths) with a view to increasing our knowledge both of the propagation of waves along the earth's surface and of the electrical constants of the earth. In particular it was desired, if possible, to obtain a more accurate knowledge of the effective dielectric constant of the earth, by operating at such a high frequency that this quantity might be expected to become as important as the conductivity in determining the mode of propagation of waves along the earth's surface.

In the region of frequencies now in question, some investigations have been made by M. J. O. Strutt[2] at a wave-length of 1·42 m. (210 Mc./sec.), and he has obtained a value of conductivity in excess of 10×10^8 e.s.u. for soil which at lower frequencies possessed a value of 1×10^8 e.s.u.

§2. DESCRIPTION OF APPARATUS AND METHODS EMPLOYED

(*a*) *Transmitter*. For the source of oscillations in these experiments, a low-power transmitter was constructed using a short-path-type receiving-valve connected to a single-tuned circuit with capacitative retroaction. The circuit arrangement adopted is shown in figure 1 *a*. The oscillatory circuit consisted of an inductance of a single turn, $1\frac{1}{2}$ in. in diameter, of $\frac{1}{8}$ in. copper tube, and a small variable air condenser having a maximum capacity of about 20 $\mu\mu$F. A similar condenser was employed for retroaction, while suitable choke coils, having a self-resonance at approximately the working wave-length, were used in the four direct-current supply leads to the valve electrodes. The valve, with its cap removed to reduce inter-electrode capacity, was mounted in an inverted position, and the oscillatory circuit was mounted as closely as possible to the leads from the valve electrodes. The tuning and retroaction condensers were controlled through extended handles from the front panel, while a filament resistance and grid-bias battery were mounted on the base-board. The direct-current supplies for the filament and anode circuit were obtained from batteries contained within a screened box upon which the set stood. The relative magnitude of the oscillatory current was measured by coupling to the inductance a single-turn loop connected to a non-contact-heater thermo-junction unit, the output side of which was connected to a pointer microammeter of suitable size for mounting on the set. This current-measuring combination was calibrated periodically with direct current, but as far as possible in each set of field-strength measurements the output from the transmitter was kept constant; a steady deflection on the microammeter was maintained by varying the voltage supplied to the valve anode circuit. Every effort was made to keep the dimensions of the apparatus as small as practicable in comparison with the working wave-length. In this connection the chief difficulty is encountered with the high-tension battery supply. In order to maintain a constant-current supply of the order of 20 mA. at 120 V. to the valve anode circuit, it was necessary to employ a large-size dry battery which, together with a 4-volt accumulator for the filament supply, was contained in a metal-lined box of

approximate dimensions 50 × 25 × 20 cm. The maximum dimension is thus seen to be less than one-third of a wave-length (160 cm.), and as, further, the choke coils employed were found to be very efficient in confining the oscillatory current to the tuned circuit, it is considered reasonable to assume that the loop inductance acts as a localized source of radiation for the purpose of this investigation.

(*b*) *Receiver*. The first stage of the receiver comprised a detector-oscillator unit which was identical with the transmitter just described. This stage was used in an oscillating condition for the heterodyne reception of the signals from the transmitter. After rectification the beat note of audible frequency was passed through a single amplifying stage to an output transformer connected to a pair of telephones. The audio-frequency potential-difference across these telephones was measured by means of a valve voltmeter of normal design and construction. A circuit diagram of the receiver is shown in figure 1 *b*.

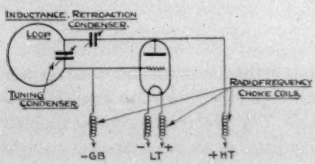

Fig. 1 (*a*).

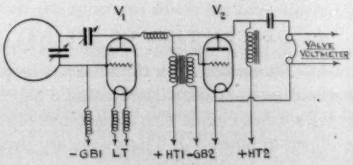

Fig. 1 (*b*).

An overall calibration of this receiving apparatus was carried out by measuring the output voltage for various values of the transmitter current to which the field-intensity at a fixed distance would be proportional. By this means it was ascertained that over a reasonably adequate working-range the output voltage was proportional to the field-intensity to an accuracy (of the order of 10 per cent) sufficient for the investigation.

(*c*) *Experimental procedure*. For the initial experiments the procedure was similar to that adopted in the case of the somewhat longer wave-lengths*. With the transmitter set in operation on a suitable site, measurements of the received field-intensity were made with the receiver at various distances from the transmitter while constant operating conditions were maintained throughout. The site employed for the measurements carried out at the National Physical Laboratory, Teddington, was the same as that used in the measurements on wave-lengths of from 5 to 10 m. as described in the former paper[1]. Owing to the shorter wave-length of 1·6 m. employed in the present case, however, it was necessary to take greater precautions to secure freedom from interference-effects from surrounding objects. The site chosen was flat and clear of all obstacles over an area of radius at least 100 m. from the transmitter, and as the majority of the measurements to be described were made at distances of less than 50 m., these conditions were considered to be satisfactory. On a wave-length of 1·6 m. great care has to be taken to ensure that the observers,

* See page 594 of reference (1), p. 508.

whose height averaged 1·8 m., did not produce spurious effects on the radiation. The first observer who controlled the transmitter was always at a reasonable distance from the apparatus, and during actual measurements he observed the deflection of the ammeter at the transmitter by the aid of a telescope. At the receiving end the second observer was always placed in a position below the set, which in some cases involved lying flat on the ground. In any series of measurements the positions of both observers were maintained constant in relation to the apparatus.

In order to study the propagation of waves directly along the earth's surface, it is necessary that the heights above the ground of the radiating and receiving points should be small compared with the wave-length. A practical limit was set in this direction by the dimensions of the apparatus. With the transmitter standing on its battery box placed on the ground, the height of the small radiating loop above the earth's surface was 50 cm. The height of the receiving loop similarly placed was 53 cm. For other measurements the heights of the transmitter and receiver were increased up to about 3 m. from the ground by placing the apparatus on portable wooden tables or stools.

§ 3. DISCUSSION OF RESULTS OBTAINED

(*a*) *Experimental results.* When the heights of the transmitting and receiving loops were about 0·50 m. or less than one third of the wave-length, it was found that the field-intensity decreased steadily as the distance was increased, so that the

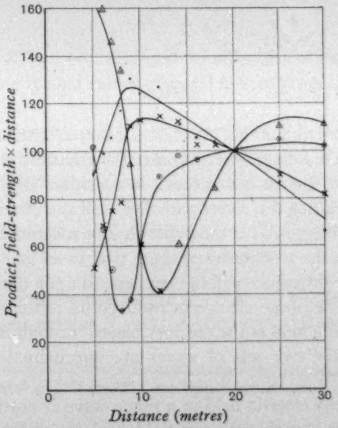

Fig. 2. Experimental results. Wave-length, 1·6 m.; height of receiver, 1·4 m.; height of transmitter, 0·5 m. (dots), 1·2 m. (crosses), 2·1 m. (circles), 2·8 m. (triangles).

attenuation curve was of the type normally experienced on longer wave-lengths. When, however, the transmitter and receiver were raised to a height comparable

with or greater than the wave-length, the field-intensity passed through successive maximum and minimum values as the distance between transmitter and receiver was increased. Figure 2, for example, shows a set of experimental results plotted for a fixed height of receiver and for four different heights of the transmitter. These graphs show, in the usual manner, the relation between the distance and the product of field-intensity and distance. It is seen that this product passes through maximum and minimum values, the position of which depends upon the height of the transmitter. As the transmitter is raised the maximum moves out from its first position at a distance of about 9 m. to its last position at a distance of 27 m. This maximum is followed by a trough or minimum which is at a distance of less than 5 m. for the first two heights and then occurs at distances of about 8 and 12 m. for transmitter heights of 2·1 and 2·8 m. These curves represent, in fact, a type of interference pattern in the radiated field, the positions of the fringes depending upon the height of the transmitter above the ground.

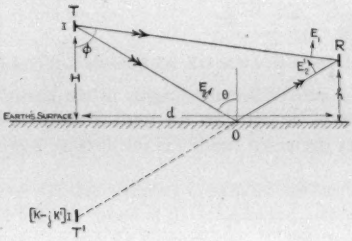

Fig. 3.

The mode of production of this interference pattern can easily be understood by the aid of figure 3. This diagram represents the transmitter T and receiver R, at heights H and h above the earth's surface. It is evident that two sets of waves will reach R from T; one along the direct path TR, and the other along the path TOR, after reflection from the ground at the point O. The magnitude of the resulting field at R will depend upon the amplitudes of these two waves and also upon their phase-difference. The phase-difference will in turn depend upon the actual path-difference of the waves and on the phase-change introduced on reflection at the ground. The intensity of the resulting field at the receiver passes through maximum or minimum values according as the two sets of waves are approximately in the same or in opposite phase.

The direction of the electric field in a plane wave is normal to the direction of propagation of the wave, so that the two fields arriving at R may be represented by the two vectors E_1, E_2'. If an aerial were used for reception at R the difference in direction between E_1 and E_2' would have to be taken into account when the field-intensity at R is calculated theoretically. A small loop at R, however, has the same reception characteristics for all directions in the plane of the coil. Similar conditions

apply at the transmitter T, so that the intensity of radiation from T along the two paths TR and TO are the same. The ray TOR, however, undergoes reflection at O, the coefficient of reflection being determined by the boundary conditions at O in accordance with Fresnel's equations. These equations are strictly valid only if the distances of transmitter and receiver from the reflecting surface are such that the wave incident at the surface is plane and also that the re-radiation from the surface becomes plane before it reaches the receiver. These two conditions were fulfilled in the majority of the experiments by keeping the transmitter and receiver at minimum distances of about one wave-length above the ground. Even in the case in which the heights of transmitter and receiver above the ground were only 0·5 m. (less than one-third of the wave-length), Fresnel's equations appeared to apply to an approximate extent; but it is proposed to examine this point in more detail later.

(*b*) *Theoretical curves.* Suppose that the complex coefficient of reflection of the ray TO incident at the angle θ at O, figure 3, is represented by $(K - jK')$ in which j is equal to $\sqrt{-1}$. It can be shown from Fresnel's equations that

θ, K, K'

j

$$K - jK' = \frac{(k^2 + 4\sigma^2/f^2)\cos^2\theta - (c^2 + d^2) - 2j\cos\theta\,(kd + 2c\sigma/f)}{(k^2 + 4\sigma^2/f^2)\cos^2\theta + (c^2 + d^2) + 2\cos\theta\,(kc - 2d\sigma/f)},$$

where σ is the conductivity and k is the dielectric constant of the ground, f is the frequency of the wave incident on the earth's surface at the angle θ, and the values of c and d are given by

σ, k, f

c, d

$$c^2 - d^2 = k - \sin^2\theta,$$

$$cd = -\sigma/f.$$

The values of K and K' can be calculated for all angles of incidence, θ, and a selection of graphs for some typical values of σ and k were given in a paper published by Wilmotte [3].

The problem of calculating the field-intensity at R is simplified if we consider the radiation along the path TR to come from a source at T of strength, say, I, and the radiation along TOR to come from a similar source at T' of strength $(K - jK')\,I$, where T' is the image of T in the earth's surface. The field at R due to I is proportional to I/TR, and that due to the source $(K - jK')\,I$ is proportional to

I

$$(K - jK')\,I/T'R,$$

and these two fields differ in phase by $(2\pi/\lambda)\,(T'R - TR)$. It can be shown that the product of the resultant field at R and the horizontal distance between T and R is proportional to

$$\left[\sin^2\phi + K^2\sin^2\theta + K'^2\sin^2\theta + 2K\sin\theta.\sin\phi.\cos\frac{2\pi}{\lambda}(T'R - TR) - 2K'\sin\theta.\sin\phi.\sin\frac{2\pi}{\lambda}.(T'R - TR)\right]^{\frac{1}{2}},$$

where ϕ is the angle the direction TR makes with the vertical.

A set of theoretical curves calculated from this formula for fixed heights of transmitter and receiver and for various typical values of k and σ/f is given in figure 4.

These curves show, as before, the relation between the product of field-intensity times distance, and for distances up to 30 m. from the transmitter. By inspection of these curves it can be seen immediately that the dip in the curve corresponding to a minimum value of the product of signal-strength and distance becomes more marked as the value of σ/f is raised from 5 to 100. An increase in the value of k from 0 to 10 is also effective in reducing the depth of the trough although to a less marked extent. An additional effect of increasing the value of k is that it moves the position of the minimum towards the transmitter.

(*c*) *Comparison of theory and experiment.* By superimposing the three most suitable curves with the corresponding experimental results we obtain figure 5 for a transmitter-height of 2·1 m. From this comparison it is seen that the most appropriate value of σ/f is about 50, with k equal to 10. It is considered that such values serve to locate the position of the trough on both coordinates to within the probable experimental accuracy. Similar comparisons have been carried out for two other heights of the transmitter, and the results are reproduced in figures 6 and 7. In all the cases so far investigated it appears that the theoretical curves calculated for $\sigma/f = 50$ and $k = 10$ fit the experimental results to within the accuracy (about 10 per cent) that is at present obtainable in working on such short wave-lengths. It must be pointed out that with the large value of σ/f which appears in this result, the method does not enable the value of k to be determined with any great accuracy, since within the limits 0 and 10 the value of k does not greatly affect the actual location of the curves.

In this respect one of the objects of carrying out this investigation has failed, for if σ is given the value of 10^8, which is found for wave-lengths of the order of 300 metres (frequency 1000 kc./sec.), the value of σ/f at a wave-length of 1·6 m. (frequency 190 Mc./sec.) would be about 0·53 and it is likely that k would have been the controlling factor in determining the coefficient of reflection of the waves. This conclusion indicates that, from a practical point of view, the value of the dielectric constant k is not a controlling factor in determining the propagation of radio waves over the earth's surface on either long or short wave-lengths, except in situations where the conductivity of the ground is abnormally low.

(*d*) *Value of conductivity of the earth.* From the results of the investigation described in the previous paper[1], it was seen that the value of the effective conductivity of the earth determined for wave-lengths between 5 and 10 m. (frequencies 60 to 30 Mc./sec.) was within the limits 5×10^8 and 32×10^8 e.s.u., whereas the values previously obtained on wave-lengths between 350 and 750 m. (frequencies between 860 and 400 kc./sec.) lay between $0{\cdot}2 \times 10^8$ and 4×10^8 e.s.u. The results of the experiments described in the present paper indicate that on the wave-length of 1·6 m. (frequency 190 Mc./sec.) the conductivity of the earth in the same locality is about 95×10^8 e.s.u.* This affords some confirmation of the fact that there appears to be a definite and substantial increase in the effective value of the earth's conductivity under alternating-current conditions as the frequency is increased above one

* The corresponding resistivity is 95 ohm-cm.

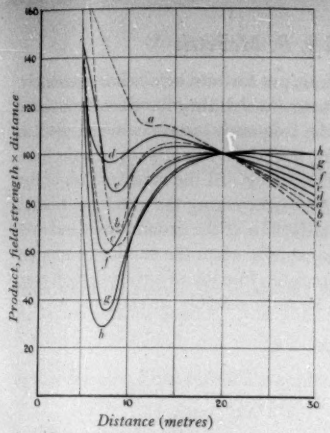

Fig. 4. Theoretical results.

a: $\kappa=0$ $\sigma/f=5$ | b: $\kappa=0$ $\sigma/f=10$ | c: $\kappa=0$ $\sigma/f=20$

d: $\kappa=10$ $\sigma/f=5$ | e: $\kappa=10$ $\sigma/f=10$ | f: $\kappa=10$ $\sigma/f=20$ | g: $\kappa=10$ $\sigma/f=50$ | h: $\kappa=10$ $\sigma/f=100$

Height of transmitter = 2·1 m.

Height of receiver = 1·4 m.

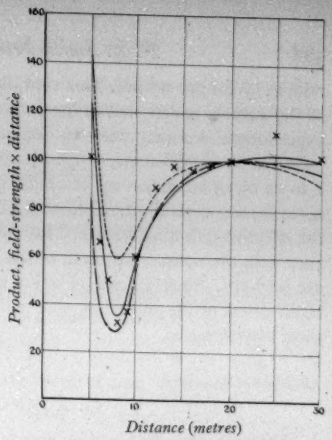

Fig. 5. Comparison of theoretical and experimental results.

Height of transmitter = 2·1 m.
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.

Calculated curves: ---- $\kappa=10$: $\sigma/f=20$
—— $\kappa=10$: $\sigma/f=50$
— — $\kappa=10$: $\sigma/f=100$

Experimental results: ×

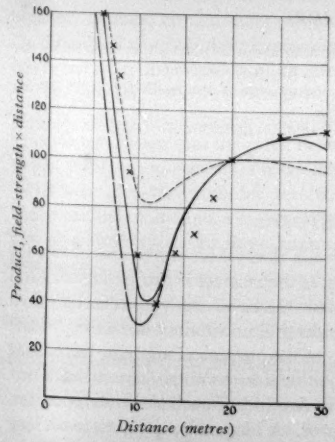

Fig. 6. Comparison of theoretical and experimental results.

Height of transmitter = 2·8 m.
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.

Calculated curves: ---- $\kappa=10$: $\sigma/f=20$
—— $\kappa=10$: $\sigma/f=50$
— — $\kappa=10$: $\sigma/f=100$

Experimental results: ×

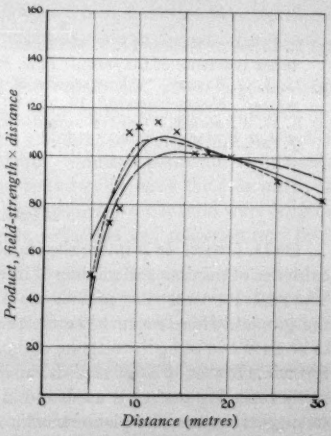

Fig. 7. Comparison of theoretical and experimental results.

Height of transmitter = 1·2 m.
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.

Calculated curves: ---- $\kappa=10$: $\sigma/f=20$
—— $\kappa=10$: $\sigma/f=50$
— — $\kappa=10$: $\sigma/f=100$

Experimental results: ×

million cycles per second. This view, however, is put forward somewhat tentatively as the authors realize that in the field experiments which they have conducted the experimental accuracy tends to decrease as the frequency rises. It is proposed to continue the experiments, as opportunity offers, over the range of wave-lengths from 1 to 10 m. in an endeavour to obtain greater accuracy. In the continuation of the investigation careful attention would be paid to ascertaining the exact position of the effective reflecting surface. This may not always be at the ground level and may vary with the moisture-content of the ground, chiefly when the surface conditions are very dry. Since a depth of soil of an appreciable fraction of a wave-length may be involved in the reflection, this point can be most suitably investigated at very short wave-lengths.

§ 4. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board, and acknowledgment is due to the Department of Scientific and Industrial Research for granting permission for publication. The authors desire to thank Messrs A. C. Haxton and H. M. Bristow for much assistance both in the experimental measurements and in connection with the various theoretical calculations involved.

REFERENCES

(1) R. L. Smith-Rose and J. S. McPetrie. "The attenuation of ultra-short radio waves due to the resistance of the earth." *Proc. Phys. Soc.* **43**, 592–610 (1931).
(2) M. J. O. Strutt. "Measurement of the conductivity of the earth for short electric waves." *Naturwissenschaft*, **17**, 727–8 (1929).
(3) R. M. Wilmotte. "General formulae for the radiation distribution of antenna systems." *J. Inst. E.E.* **68**, 1174–90 (1930).

DISCUSSION

Dr D. Owen. It is interesting to observe in these short-wave experiments the existence of maxima and minima of intensity as the receiver is carried further back. The measurements thus provide an important means of studying reflection from the ground. There must, however, be considerable uncertainty as to the point O to be chosen in using the diagram of figure 3 for the purpose of applying the Fresnel formula. It should be of material assistance therefore to obtain data when a perfectly reflecting surface is used, and it seems feasible, at such short wave-lengths, to suggest the use of a plane metallic reflector. A long strip, say 5 to 10 m. long by 2 m. wide, of the finest copper sheet might be mounted on a light frame for this purpose. If suitably inclined it would also serve to divert the reflected beam completely from the receiver, so permitting the intensity of the direct beam to be separately ascertained.

In regard to the numerical values, I should like to ask to what extent the variations between the limits quoted depend on wave-length, and how far on the nature and degree of wetness of the soil.

Mr F. D. Smith. The equation in § 3 (*b*) reduces to

$$K - jK' = \frac{(2\sigma/f)\cos^2\theta - 1 + 2j\,(\sigma/f)^{\frac{1}{2}}\cos\theta}{(2\sigma/f)\cos^2\theta + 1 + 2\,(\sigma/f)^{\frac{1}{2}}\cos\theta}$$

when

$$\sin^2\theta \ll k \ll \sigma/f,$$

differing in the sign of the imaginary term from the corresponding equation (29) of R. M. Wilmotte's paper*. Now the experimental results indicate values

$$k = 10, \quad \sigma/f = 50,$$

so that this simplified formula is applicable in the present case. In the extreme case

$$(2\sigma/f)\cos^2\theta \gg 1, \quad K \fallingdotseq 1 \quad \text{and} \quad K' \fallingdotseq 0,$$

and in the other extreme case

$$(2\sigma/f)\cos^2\theta \ll 1, \quad K \fallingdotseq -1 \quad \text{and} \quad K' \fallingdotseq 0.$$

The reflection is practically perfect except when $(2\sigma/f)\cos^2\theta$ does not differ greatly from unity, that is, except when $\cos^2\theta$ does not differ greatly from 0·01. When the reflection is practically perfect, the interference pattern is practically independent of σ/f and no estimate of the numerical values of σ/f can be made. On the other hand, when $\cos\theta \fallingdotseq 0{\cdot}1$ the most favourable condition for forming an estimate of σ/f obtains. In these circumstances would it not have been better to have equalized the theoretical curves and the experimental results at a shorter distance of, say, 5 m., where the reflection is practically perfect?

The general agreement between theory and experiment, which is particularly good in view of the experimental difficulties encountered at 1·6 m., indicates the valuable conclusion that the earth behaves as a highly conducting sheet practically impenetrable to radio waves of wave-length 1·6 m. Similar experiments with shorter waves may give very complex results. The boundary between the two media is somewhat indefinite and a region of gradual transition over several wave-lengths may sometimes exist. In these circumstances, refraction and reflection may both take place.

Authors' reply. In reply to Dr Owen: the uncertainty in our knowledge of the exact point of reflection from the ground was realized by us, and a brief reference to the further experiments to be carried out in this connexion is made at the end of § 3 (*d*) of the paper. We had also appreciated the desirability of using metallic reflecting sheets laid over the ground surface and we have actually commenced experiments on these lines. In the first place, for reasons of economy, we are using small-mesh galvanized iron wire netting and have acquired a piece of dimensions 16 m. × 4 m., which should be adequate as a reflecting surface for a wave-length of

* *Loc. cit.*

1·6 m. With the aid of this netting we shall endeavour to determine the smallest size of sheet actually required, and we shall then replace this with a sheet of solid copper or copper gauze. We also visualize the possibility of extending the experiments to reflection from a sheet of water, in particular with a view to measuring the electrical properties of sea-water at high radio frequencies. The variation in the values of the conductivity of the soil with wave-frequency given in the paper are quite definite and are much larger than the variation experienced with the moisture-content of the soil under normal field conditions in this country. A laboratory investigation of the electrical properties of soil at various frequencies and over a wide range of moisture-contents is also in progress, however, and the results of this should provide more exact knowledge on these points.

In reply to Mr F. D. Smith; the error in sign in the formulae in Wilmotte's paper to which he refers was noticed some time ago, and the corrected form was actually used by us in our previous paper published last year. It appears to us that in reducing this formula by making various approximations for extreme cases, Mr Smith is elucidating the form of the graphs for K and K' (given in figures 4 and 5 of R. M. Wilmotte's paper*) for these cases. Incidentally the fact that for the case when $(2\sigma/f)\cos^2\theta \ll 1$, K is equal to -1 and not $+1$ gives rise to a certain difficulty, since the direct and reflected waves are 180° out of phase with each other, and the total field-strength at the surface would appear to be zero, a deduction which is not borne out by experiment. The explanation of this anomaly is that, as explained in the paper, the equations from which the formula for the reflection coefficient was derived do not hold for the case in which the receiver (or transmitter) is very close to the surface. With regard to the choice of distance at which the equalizing of the curves and results was carried out, 20 m. was selected because it was considered that at this distance the overall accuracy of the experiments was greatest.

We agree with Mr Smith's remarks on the boundary conditions which may lead to complex results at shorter wave-lengths, and our future investigations will be directed towards obtaining more knowledge on this point. It may be stated, however, that part of the object of working in the wave-length-range 1 to 10 m. was to investigate this complexity, due to the supposed increasing importance of k at very high frequencies. As our results show, however, the value of σ has increased so much at these very short wave-lengths that the value of $2\sigma/fk$ is still large and thus k appears to play only a minor part in the properties of soil at high frequencies.

* *Loc. cit.*

NOTES ON SURFACE-TENSION MEASUREMENT

By ALLAN FERGUSON, M.A., D.Sc., F.Inst.P. and S. J. KENNEDY, B.Sc.

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ABSTRACT. The first part of this paper deals with a method for the accurate determination of the surface tension of liquids available in volumes of not more than one or two cubic millimetres. The method described does not involve any knowledge of the density of the liquid.

The second part of the paper describes a series of measurements of the variation with concentration of the surface tension of aqueous solutions of *p*-toluidine. Here, also, the method employed is independent of a knowledge of the density of the solution.

§ 1. THE MEASUREMENT OF SURFACE TENSION USING A SMALL QUANTITY OF LIQUID

Some time ago, one of us (A. F.)* pointed out that it was possible to obtain accurate values for air-liquid or interfacial tensions using as little as one or two cubic millimetres of liquid. The method then described consisted in drawing a short thread of liquid into a vertical capillary tube 1 mm. or less in bore, attaching the capillary to a manometer and to a simple arrangement for varying the pressure, and increasing the pressure to a value $g\rho_1 h_1$ such that the thread of liquid in the capillary is forced down until the surface of the liquid at the lower end of the capillary is *plane*. If ρ_2 is the density of the liquid under test, and h_2 the length of the thread of liquid, we than have

g, ρ_1, h_1

ρ_2, h_2

$$\gamma = \tfrac{1}{2} gR\,(\rho_1 h_1 + \rho_2 h_2) \qquad \text{.....(1)},$$

exactly, where R is the radius of curvature of the upper meniscus at its vertex, and γ the surface tension of the liquid. Assuming a zero contact angle and substituting for R the value

R

γ

$$R = r\,(1 + r^2/6a^2) \qquad \text{.....(2)},$$

where r is the internal radius of the tube and as usual $a^2 \equiv \gamma/g\rho_2$, we find

r, a

$$\gamma = \tfrac{1}{2} gr\,(\rho_1 h_1 + \rho_2 h_2) + \tfrac{1}{6} g\rho_2 r^2 \qquad \text{.....(3)},$$

as the working equation from which to determine the surface tension.

Obviously, the equation as it stands necessitates a knowledge of ρ_2, the density of the liquid under test, and this determination may be a matter of some difficulty if only a few cubic millimetres of liquid are available. Two courses are open; we may, without appreciable loss of accuracy, shorten the length of the thread of liquid until $\rho_2 h_2$ is negligible in comparison with $\rho_1 h_1$, or we may make a number of

* *Proc. Phys. Soc.* **36**, 37 (1923).

determinations, varying the value of h_2 and, writing the equation for γ in the form

$$h_1 = -\frac{\rho_2}{\rho_1}\left(h_2+\frac{r}{3}\right)+\frac{2\gamma}{gr\rho_2} \qquad \ldots\ldots(4),$$

we see that if we plot h_1 against $(h_2 + \frac{1}{3}r)$ we can obtain γ without the necessity for a direct determination of ρ_2.

In this paper we consider another method of obviating this difficulty—that of using the capillary tube in a horizontal position. This not only relieves us from the necessity of determining the density of the liquid under test, but simplifies a little the experimental arrangements. Figure 1 shows the disposition of the apparatus*.

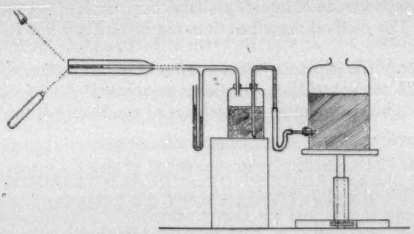


Fig. 1.

By means of the simple arrangement shown on the right-hand side of the drawing, the liquid thread is forced along the capillary until the meniscus at the open end of the tube, at first concave, flattens out until it is exactly plane. Further increase of pressure makes the meniscus convex. A number of experimenters who have used the method previously described seem to have found difficulty in hitting off the plane-position. We therefore show in the plate, *b*–*f*, a series of photographs of the end of the capillary: *b* and *c* show concave menisci, *e* and *f* convex menisci, and *d* as nearly as possible the plane position. The corresponding pressures, as read on an aniline manometer, are shown in table 1.

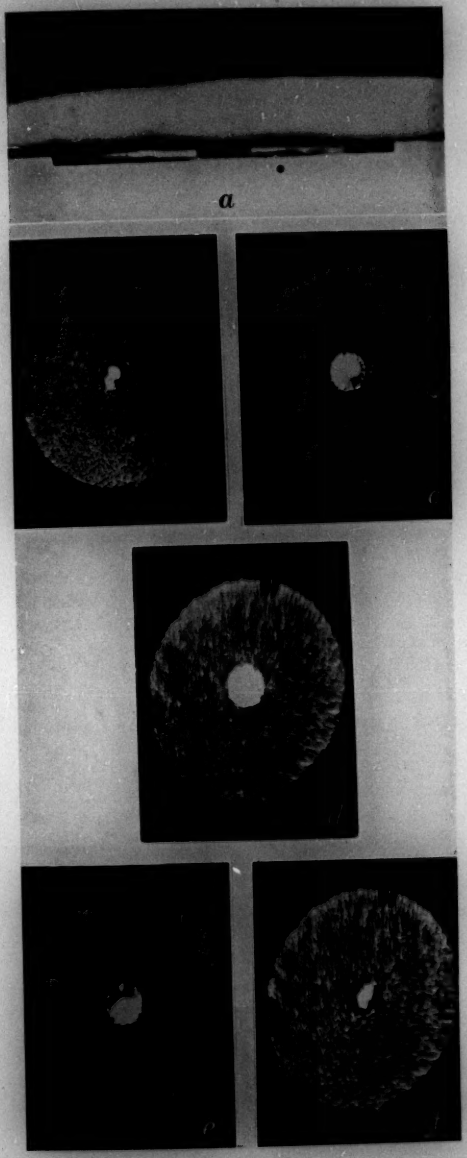
Table 1.

Photograph	*b*	*c*	*d*	*e*	*f*
Pressure (cm. aniline)	2·689	2·836	2·887	2·955	3·108

It must not be imagined that these steps of pressure in any way represent the delicacy of the setting, which is very sensitive and clear-cut. The steps are made at intervals as large as 0·5 mm. or thereabouts simply to show more clearly on the photographs the march of the observed phenomena.

Obviously, the bore of the tube used must be small enough to ensure that there is no serious gravitational distortion of the meniscus. The problem of the distortion

* It is obvious in the figure that the drawing is not all made to the same scale. The horizontal capillary on the left is much enlarged for convenience. Moreover, the filament lamp and low-power microscope for viewing the image of the filament in the meniscus are, of course, situated in a horizontal plane perpendicular to the plane of the rest of the drawing.



[To face page 512

of the meniscus in a horizontal capillary tube is not a promising subject for mathematical attack, and, fortunately for our purpose, it can be discussed on a purely experimental basis. We have seen that if a column of liquid of length h_2 requires a head h_1 of another liquid to force it to the "plane" position, then

$$2\gamma/gr = \rho_1 h_1 + \rho_2 (h_2 + \tfrac{1}{3}r) \qquad \text{.....(5).}$$

If we have a horizontal tube and find that a manometer liquid of density ρ_1 and giving a head h_1' forces the liquid under test into the plane position, we may put

$$2\gamma/gr = \rho_1 h_1' \qquad \text{.....(6),}$$

if the tube is sufficiently narrow to enable us to neglect gravitational forces in comparison with those due to surface tension.

The procedure adopted was to make a test first with the tube vertical, then with the tube horizontal, and to compare the values of $\rho_1 h_1 + \rho_2 (h_2 + \frac{1}{3}r)$ and of $\rho_1 h_1'$. If this test is carried out with tubes of different bore, it is not difficult to find an outside limit for the bore of a tube which may be safely employed with liquids whose surface tensions and densities are of the order usually encountered. Thus using aniline as the test liquid and manometer liquid, and a tube of just less than a millimetre bore, all that is necessary is to compare the values of h_1' and $h_1 + h_2 + \frac{1}{3}r$. The figures in table 2 were obtained.

Table 2.

Radius of capillary, 0·045 cm. Temperature, 16·5° C.
Test liquid, aniline. Manometer liquid, aniline.

h_1' (cm.)	1·936	1·938	1·937	1·933	1·933	
$h_1 + h_2 + \frac{1}{3}r$ (cm.)	1·938	1·938	1·938	1·935	1·934	1·935

Mean value of $h_1 + h_2 + \frac{1}{3}r$ = 1·936 cm.
Mean value of h_1' = 1·934 cm.

If the test liquid differs in density from the manometer liquid, then the comparison must be made as mentioned above. Such a comparison has been made using water as the test liquid, aniline as the manometer liquid, and tubes of three different radii. The results corrected to read in centimetres of aniline are shown in table 3.

Table 3.

Internal diameter of tube (cm.)	h_1' (cm.)	$h_1 + h_2 + \frac{1}{3}r$ (cm.)
0·126	2·410	2·407
	2·379	2·389
	2·422	2·413
	2·363	2·362
0·100	3·064	3·054
	3·145	3·139
	3·051	3·058
	3·023	3·031
0·092	3·213	3·225
	3·227	3·218
	3·213	3·202
	3·176	3·167

It seems, then, that with tubes of less than 1 mm. bore, we are quite safe in employing a horizontal tube and in using the simple formula

$$\gamma = \tfrac{1}{2}rh\rho g \qquad \text{.....(7),}$$

where h and ρ refer, of course, to the manometer liquid. An experiment was carried out to determine the surface tension of water, and the simple formula just given was used. The results are given in table 4.

Table 4.

Tube 1. Radius 0·036 cm. Temperature 15° C.

h (cm.)	4·070	4·060	4·069	4·069	4·064	4·068

Mean value of h = 4·067 cm.

γ = 73·47 dyne/cm.

Tube 2. Radius 0·046 cm. Temperature 15° C.

h (cm.)	3·188	3·184	3·189	3·181	3·182	3·191

Mean value of h = 3·186 cm.

γ = 73·55 dyne/cm.

Tube 3. Radius 0·063 cm. Temperature 15° C.

h (cm.)	2·320	2·323	2·317	2·322	2·320	2·321

Mean value of h = 2·320 cm.

γ = 73·38 dyne/cm.

Surface tension of water γ = 73·47 dyne/cm. at 15° C.

Finally, an attempt was made to determine a number of interfacial tensions. Suppose we have two liquids α and β in contact in a horizontal capillary tube. If the pressure inside the tube is increased to an excess value gph sufficient to make plane the surface of the liquid β at the open end of the capillary tube, a simple argument shows that, if we assume zero contact angles throughout,

$$\gamma_{\alpha\beta} = -\gamma_{\alpha} + \tfrac{1}{2}rh\rho g \qquad \text{.....(8).}$$

The results of experiments made on six different liquids are shown in table 5.

Table 5.

Liquid	γ_{α} (dyne/cm.)	$\gamma_{\alpha\beta}$ (dyne/cm.)	Temperature (° C.)
Ether	17·41	10·56	16·0
Carbon tetrachloride	27·05	44·98	16·5
Benzene	29·14	33·67	15·0
Chloroform	27·50	30·02	15·0
Toluene	28·84	37·73	16·0
Ethyl bromide	24·52	31·16	17·0

The interfacial tensions are all determined against water, and the values are in very fair agreement with those determined by different methods. The values for γ_{α} cited in table 5 were determined by this method.

To sum up: whilst the method, in common with most capillary tube methods, assumed a knowledge of the contact angle, in most cases assumed to be zero, as at present described it may fairly be called accurate and has many advantages over the ordinary capillary-rise method. (1) The technique is simpler and cleaning operations are more easily carried out. (2) No knowledge of the density of the liquid under test is required. (3) The amount of liquid used may be cut down to one or two cubic millimetres, without any loss of precision. (4) For tubes of less than a millimetre bore, the simple formula may be applied. Hence it follows that the manometer *may be made direct-reading, as its indications need only be multiplied by a factor which, for a given capillary, is constant.* There is, of course, no restriction to the U-tube type of manometer. It is conceivable that a robust and yet sensitive direct-reading instrument may be constructed employing a manometer of aneroid type.

§ 2. ON THE SURFACE-TENSION/CONCENTRATION CURVES FOR AQUEOUS SOLUTIONS OF *p*-TOLUIDINE

Some time ago, a method for surface-tension measurements was described by one of us (A. F.) in a paper published in collaboration with Mr P. E. Dowson*. The method consisted in immersing a capillary vertically to a depth h_2 in the liquid of density ρ_2 under test, and measuring on a separate manometer the pressure $g\rho_1 h_1$ required to force the liquid meniscus to the bottom of the capillary and to hold it there.

The method has obvious advantages over the capillary-rise method inasmuch as: (1) No calibration of the tube is necessary. (2) Pressure measurements are made on a separate manometer and may be magnified at will. (3) The cleaning of the apparatus is facilitated. (4) There is very little adsorption at the glass-liquid surface of the tube, as compared with the amount likely to take place in measurements made by the ordinary capillary-rise method; this is a matter of some importance in measurements made on solutions.

The method described in the first part of this paper, the capillary-rise method, and the method used in the measurements now to be described, are all closely allied. In each instance a *pressure excess* and a *curvature* are measured; in each instance a *plane* surface of reference is provided.

In the capillary-rise method the liquid under test acts as its own manometer, and the pressure required is determined by measuring the vertical distance between the meniscus vertex and the plane surface of the liquid in the outer container. In the method described in the first part of this paper the *total* pressure excess is measured on a separate manometer, and the plane surface of reference is obtained as described. In the present method the pressure excess required is, small corrections apart, measured as the *difference* between the pressure read on the manometer and the pressure due to the immersion of the meniscus to a depth h_2 below the *plane* surface of the liquid under test.

* *Manchester Memoirs*, **65**, No. 5 (1921); *Trans. Far. Soc.* **17**, 384 (1921).

A simple argument shows that, for a liquid of zero contact angle, the surface tension is given by

$$\gamma = \tfrac{1}{2}gr\,(\rho_1 h_1 - \rho_2 h_2) + \tfrac{1}{6}g\rho_2 r^2 \qquad \text{.....(9).}$$

If we vary h_2 and measure the corresponding values of h_1 we see that, rearranging equation (9) above, we may put

$$h_1 = \frac{\rho_2}{\rho_1}\left(h_2 - \frac{r}{3}\right) + \frac{2\gamma}{g\rho_1 r} \qquad \text{.....(10).}$$

Hence, if we plot h_1 as ordinate against $(h_2 - \frac{1}{3}r)$ as abscissa, the intercept of the line so obtained gives the surface tension of the liquid and obviates any necessity for the determination of its density.

But we may avoid the density-determination in another manner. If, as a particular case, we make h_2 equal to $\frac{1}{3}r$, we have

$$\gamma = \tfrac{1}{2}g\rho_1 h_1 r \qquad \text{.....(11)}$$

as a very simple formula for the evaluation of γ, necessitating no knowledge of ρ_2 and yet taking into account the small correcting term of the first order. The uppermost photograph in the plate shows that it is quite easy, with a tube having a bore of a millimetre or less, to immerse the tube to the required depth and to observe accurately the equilibrium position of the meniscus at the lower end of the tube. The adjustment is easily made, either by raising the vessel containing the liquid under test until, as shown by observation of the image of the bottom of the tube reflected in the liquid surface, the end of the tube is just touching that surface, and then racking up the vessel through the additional third of the radius; or with the assistance of a needle point fixed to the capillary at a known vertical distance from the end of the tube, and far enough away from the tube to ensure its touching a portion of the liquid surface that is sensibly plane. In either case, it must be remembered
a that if a thick-walled capillary tube of external cross-section a is employed, when it has been racked down into the liquid through a distance equal to $\frac{1}{3}r$, the immersion may not be equal to this on account of the liquid displaced by the tube. A correcting
A factor depending on a/A, where A is the cross-sectional area of the vessel containing the liquid, is necessary, and this correction must be made or precautions must be taken to ensure that it is negligible.

Observations of the variation of the surface tension of aqueous solutions of *p*-toluidine were made some years ago by Edwards*, using the method as described by Ferguson and Dowson†. Later, Gans and Harkins‡ covered the same ground using a drop-weight method, and obtained a curve between surface tensions and concentration, differing considerably from the curve obtained by Edwards. The matter was investigated further by R. C. Brown§, who determined the concentration/surface-tension relations by three or four different methods and found that the values obtained from Ferguson's method did not disagree with the results of the

* *J. Chem. Soc.* **127**, 744 (1925).
† *Trans. Far. Soc.* **17**, 384 (1921).
‡ *J. Am. Chem. Soc.* **52**, 2289 (1930).
§ *Phil. Mag.* **11**, 686 (1931).

experiments of Gans and Harkins. It seemed, therefore, that a useful test of the technique just described would be found in a series of measurements of the surface tension of aqueous solutions of *p*-toluidine.

Para-toluidine of research quality was obtained and was recrystallized from aqueous alcohol. The crystals so obtained were dried in a desiccator for several days.

Aqueous solutions of known concentration—always expressed as x gm. in 100 cm.^{3} of water—were made up, and the surface tensions were measured in the manner indicated*.

The radius of the tube was measured by means of a travelling microscope made by Watson and reading to 0·01 mm., and the pressures, measured by means of a simple U-tube manometer containing aniline, were read on the same instrument. The usual precautions in respect of cleanliness were taken. The results are shown in table 6.

Table 6. Authors' results for *p*-toluidine at 16° C.

Concentration (gm./100 cm.^{3})	Pressure (cm. aniline)	γ (dyne/cm.)
0·0	3·167	73·10
0·0233	3·082	71·11
0·0467	3·082	71·11
0·0935	3·030	69·94
0·100	3·031	69·96
0·131	2·953	68·16
0·150	2·933	67·70
0·187	2·846	65·69
0·200	2·811	64·88
0·225	2·727	62·95
0·250	2·679	62·30
0·262	2·644	61·00
0·300	2·549	58·83
0·333	2·510	57·99
0·375	2·462	56·82
0·450	2·310	53·32
0·500	2·257	52·01
0·600	2·140	49·36

Table 7. Gans and Harkins's results for *p*-toluidine at 20° C.

Concentration (gm./100 cm.^{3})	γ (dyne/cm.)
0·0	72·75
0·0649	71·63
0·0767	71·38
0·1257	69·82
0·2126	64·57
0·2944	60·38
0·3616	57·73
0·5286	52·35
0·6659	49·10

* In every case, tap water was employed for the manufacture of the solutions.

The agreement with the observations of Gans and Harkins, which were made at a temperature of 20° C., is best shown by means of the curve of figure 2. Here the full line represents the results of our own experiment—the actual figures obtained are shown thus ⊙—and the small crosses show the numbers given by Gans and Harkins. To make the comparison strict, since our own figures were obtained at 16° C., the full line should be moved slightly downwards—how much it is difficult to say, as the temperature-coefficient is unknown—but it is not likely that the shift will be greater than 0·4 or 0·5 of a unit of surface tension. The dotted line shows the results obtained by Edwards.

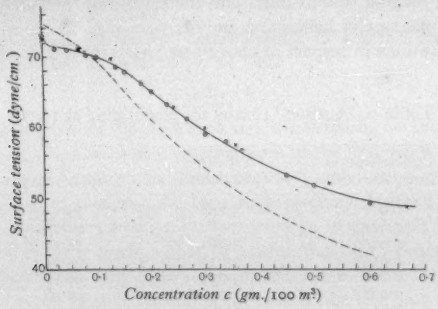


Fig. 2.

We have, then, three sets of values for the variation of surface tension with concentration for *p*-toluidine solutions—one set obtained by the drop-weight method, one obtained by the method originally described by Ferguson and Dowson, and the third set obtained by the variation of this method just described. The values in the first and third of these sets are in reasonable agreement, and it seems probable therefore that the results obtained by Edwards are due to some difference in the quality of the *p*-toluidine which he employed.

One or two side-issues of minor interest may be noted, arising from experiments made with a view to obtaining some knowledge of the variation of the surface tension of *p*-toluidine with time.

Two solutions were prepared, and kept in separate beakers. One beaker was kept covered, the other was left exposed to the atmosphere. The surface tensions were determined at approximately daily intervals. After a slight rise the surface tension of the liquid in the covered beaker remained constant over a period of 10 days.

In spite of steady evaporation (which was duly measured) from the open beaker and a very obvious fouling of the surface, the surface tension of this specimen rose fairly regularly from an original value of 51·1 dyne/cm. to a value of 71·0 dyne/cm. at the end of 33 days. Suspecting a possible change in the toluidine we applied the usual test* and found that the solution did not give the slightest response. We place

* Diazotization and addition of ferric chloride solution.

the fact on record; what chemical change had taken place we do not presume to say.

We noticed also that the solution was remarkably sensitive to fog and smoke, and might, indeed, be used as a nephometer. A day of heavy fog caused a sudden drop in the surface tension from 65·5 dyne/cm. to 63·7 dyne/cm., followed the day after by a recovery to 64·9 dyne/cm. Similarly an invasion of smoke from a chimney caused a drop of 6 dyne/cm. followed by an equal recovery.

To sum up: in the second part of this paper we describe a series of determinations of the surface tension of *p*-toluidine by a variant of a method previously described. The technique is very simple, and results may be obtained and worked out in a few minutes. The method possesses the advantages of the pressure method described in the paper by Ferguson and Dowson and, as with the method described in the first part of this paper, it may be made direct-reading inasmuch as the readings of the manometer have merely to be multiplied by a factor which for a given capillary is constant, in order to obtain the surface tension in dynes per centimetre.

And, it must be noted, this simple process includes the first-order meniscus corrections.

§ 3. ACKNOWLEDGMENT

These experiments were carried out in the laboratories of the East London College and our thanks are due to Prof. H. R. Robinson for the facilities which he placed at our disposal.

DISCUSSION

Mr R. C. Brown. The authors have evolved a method of measuring surface tensions of small quantities of liquid which is an undoubted advance on the older method of holding the capillary tube vertically, particularly as the necessity of knowing the density of the liquid is eliminated.

I was particularly interested in the results for para-toluidine solutions obtained by the second method described in the paper. As they remark, the curve which the authors obtain is in fair agreement with that of Gans and Harkins and those which I myself determined some time ago; but the agreement is only fair, and it seems to me that this must always be so. Para-toluidine is very capillary-active and is strongly adsorbed on to the liquid/air interface, and it is the surface concentration which determines the surface tension of the solution. The surface concentration is, of course, a function of the bulk concentration, but it seems reasonable that two separate observers should put exactly the same numbers of grams of solid into 100 cm^3 of solution and yet obtain somewhat different numbers of molecules per cm^2 of the surface film, which we can suppose is 1 molecule thick. The curve itself, when inverted, very closely resembles those obtained when the surface "pressure" of a monomolecular film is plotted against its area, and I suppose this is what we

have been doing in an indirect (and therefore less accurate) way, since the surface pressure of a film is the difference between its surface tension and that of pure water.

It is known that a film of camphor on a water surface will evaporate into the air. Might not the surface film of para-toluidine do the same thing? Thus the molecules of solid in an exposed surface of a solution would continually escape and be replaced by others coming from the bulk until, in time, the solution would become almost pure water. This would explain the failure of a 33-days-old solution to respond to a chemical test for para-toluidine.

Mr T. Smith. The authors have removed one measurement by rotating the capillary tube through a right angle from the vertical to the horizontal position, but have been led to adopt conditions in which the meniscus is distorted, and are limited to tubes of very small bore so that it may be negligible. A procedure which promises some advantages is to keep the tube vertical and use the plane surface criterion, but measure the pressures for the two vertical positions obtained by rotating the tube through two right angles about a horizontal axis. The equation for the surface tension then takes the very simple form

$$\gamma = \tfrac{1}{4} g r \rho_1 (h_1 + h_1') = \tfrac{1}{4} r (h_1 + h_1').$$

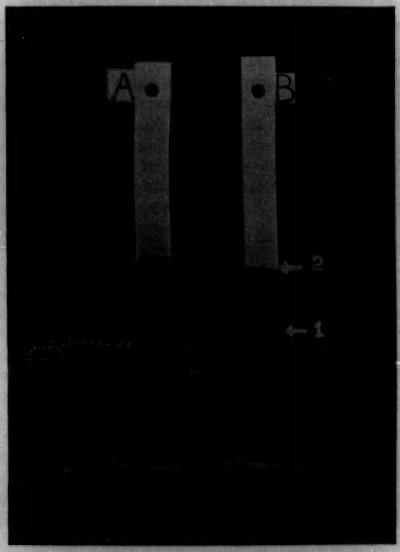
Authors' reply. We desire to thank Mr Brown for his very helpful remarks, particularly for those concerning the possible evaporation of *p*-toluidine from the surface film. The suggestion is a most useful one, and its possibilities should be explored.

Mr T. Smith's suggested method gives the required elimination, and very neatly. It must be remembered, however, that one of the difficulties encountered in the practice of the vertical-tube method with volatile liquids is due to the very rapid evaporation which takes place, causing a shortening of the liquid column. We have found it advisable to take the pressure reading immediately after measuring the length of the liquid column, and we fear that the time involved in the reversal and resetting may introduce an element of error, at least where volatile liquids are concerned. It is one of the advantages of the horizontal-tube method that this source of error does not enter into the problem. Another, but quite minor, advantage is the ease with which the method may be made direct-reading.

DEMONSTRATION

"Capillary adsorption due to surface tension." *Demonstration given on May* 20, 1932, *by* D. OWEN, B.A., D.Sc., F.Inst.P.

A solution of fuchsin in water, or ordinary red ink, has a surface tension slightly lower than that of pure water. When a vertical strip of filter paper is dipped into such a solution a red band advances up the paper, but immediately in front of it, constituting a vanguard of narrow depth, there is a colourless wet fringe. The fuchsin molecules thus show a preference for the walls of the capillaries, whilst the water molecules seek the axes and so advance further.



An aqueous solution of a salt, on the other hand, has a surface tension exceeding that of pure water. It was therefore anticipated that if the experiment were made with red ink or fuchsin solution to which some salt had been added, the fuchsin would be forced more strongly to the walls, whilst the salt molecules would tend to the centre of the capillaries. The coloured region would in this event be shortened, and the colourless region increased in length.

On trying the experiment these conclusions are found to be strikingly realized. In place of a narrow colourless band of about a millimetre the colourless band now extends over a length approaching that of the coloured region itself. This is shown in the photograph reproduced in the illustration, where *A*, representing the rise with red ink, shows an inconspicuous colourless fringe at the top; whilst *B*, representing the rise when the mixture of red ink and salt is used, shows the red band terminating at 1, and the colourless band extending from 1 to 2. The experiment works well with a solution of common salt of moderate strength.

REVIEWS OF BOOKS

Chaucer on the Astrolabe (with the original illustrations). Second and abbreviated edition, revised by R. T. GUNTHER. Pp. 96, 68 illustrations. (Oxford: Printed for the author, 5 Folly Bridge, Oxford.) 7*s*. 6*d*. net.

Dr Gunther, to whose painstaking researches on mediaeval instruments we owe so much, has put us deeper in his debt by issuing an edition of Chaucer's treatise at a price which puts the volume, even in these hard times, within the reach of most of us who are interested in such matters.

How comes it that even now Chaucer's merits as a man of science are not so fully recognized as they might be? Of the generous width and accuracy of his knowledge no one can doubt who has carefully read the treatise—compilation though it may be. Chaucer himself disarms criticism on this point when he says to his son, "I ne usurpe nat to have founde this werk of my labour or of myn engyn. I nam but a lewd compilatour of the labour of olde Astrologers, and have hit translated in myn English only for thy doctrine; and with this swerd shal I sleen envye." But there is yet more evidence of Chaucer's learning in the exact and careful references, especially to matters astronomical, which are scattered throughout his works, references which show him as a painstaking enquirer, well abreast of the science of his day. But despite this, and despite the fact that, till the twentieth century, Chaucer's was the standard English treatise on the use of this fascinating instrument, his name does not loom largely in the story of the development of physical science. His fame as poet overshadowed these lesser activities, and made it all the less likely that the historian of science should look to Chaucer for any serious contribution to his tale.

The treatise on the astrolabe is by no means difficult of access to-day. Standard one-volume editions of Chaucer's works usually include it. Skeat produced a scholarly and fully annotated edition as one of the publications of the Early English Text Society, and Dr Gunther issued an annotated edition in 1929 as volume 5 of *Early Science in Oxford*. Now we have the handy volume under review, which may be regarded as a second edition of the one just mentioned.

"Never the time and the place and the loved one all together"—so, at least, may we lament when we turn over the pages of the cheap editions. Those in the one-volume Chaucers are frankly pared to the bone. Illustrations, annotations, bibliographical details—all are gone, and the reader is left with the bare text. Skeat delighted us with a wealth of varied lore, but sorely cut down the number of plates.

Now Dr Gunther presents us with a text written in modernized form, and accompanied by all the illustrations which serve to give liveliness to Chaucer's explanations of the problems to be solved. But here he stops; we find practically no bibliographical details, and none of those discussions on side-issues which add so much to the interest for those of us who dearly love a digression.

Is it too much to ask that, when a third edition appears, Dr Gunther should write a score or so of pages by way of introduction to the work? Such an introduction would add little to the cost of the book, much to its already great value. A. F.

Les Principes de la Mécanique Quantique, by P. A. M. DIRAC, translated by Al. PROCA and J. ULLMO. Pp. viii + 314. (Paris: Les Presses Universitaires de France.) 95 fr.

When the tourist wishes to refer to a certain well-known work he calls it Baedeker, and not Baedeker's Guide-book, and in the same way one who wishes to refer to the volume which guides him round the subtler parts of the quantum theory calls it simply Dirac. Dirac has already been translated into German, and we now welcome the French translation. The translators, Messrs Proca and Ullmo, have themselves made good contributions to the subject and are eminently competent to carry out their task, and they have succeeded in reproducing to a great extent in the translation the lucid style of exposition of the original. There is a small addition by Mr Proca at the end, an account of the theory of Poisson brackets, which will prove useful especially to two classes which must now include nearly all physicists—the young who came straight to the new theory without having to labour at the classical, and the old who knew all about the brackets once, but have forgotten them in the struggle to understand the new developments. C. G. D.

An Outline of Wave Mechanics, by N. F. MOTT. Pp. 155. (London: Cambridge University Press.) 8*s*. 6*d*.

The general plan of this book is less ambitious than that of many recent books on the subject. There is no effort to discuss questions of philosophy and metaphysics, and no advanced mathematics are used. In particular, Hamiltonian and Jacobian methods in dynamics are not once referred to in the whole of the book.

The word "outline" is included in the title and is an excellent description of the scope of the work. The subject-matter includes a treatment of the problems of the hydrogen molecule and the helium atom, including the explanation of the existence of non-combining terms, whilst there is also an account of the success of wave mechanics in explaining radioactive disintegration, and the concluding chapter of the book deals with electron spin.

A word which would equally well describe the contents is "survey." We pass rapidly over the main roads of this tract of country, in the company of a guide who is familiar with it; he points out the salient features of the landscape, and after enjoying his escort, most travellers will feel that they have noticed regions which they wish to explore in greater detail at some future time, either alone, or in the company of a pioneer who has already examined them. Nevertheless, we cannot yet say that the perfect book on wave mechanics has been written. When it appears, it will most certainly have many features in common with this one, but, of course, it will contain examples for the student-reader to cut his teeth on. We may also suppose that it will not contain the infelicities of expression which are noticeable with unpleasing frequency in this book.

"Suppose first that there *were* no interaction,...then each electron *will* move...."

"Let us consider what *will* happen if such an atom *were* placed in a magnetic field...."

"The pair may have *either* of the *three* values...."

Sentences such as these jar the nerves, although they do not in any way lessen the scientific value of the work in which they occur; the next, however, might well cause a certain puzzlement: "An electron shut up in a box could not have any energy, but could only exist in stationary states of energy E_1, E_2...." It means that an electron in a box could not have any arbitrary amount of energy, but could exist only in stationary states having energy E_1, E_2.

Having only one other criticism, and that an equally trivial one, the reviewer may as well dispose of it at this stage. It relates to the innovation by which a determinant is symbolized by placing two vertical lines on each side of the array of its elements. This use of two vertical lines on each side of an array of quantities has already been appropriated for

two different purposes, and it is a great pity to use it in a third way, when the ordinary determinant notation has for so long been well established.

Turning now from the differences between this book and the ideal, to consider the similarities, we find in the first place that the historical development of the subject is almost entirely disregarded. The author has considered wave mechanics as a whole, and has set out its ideas and methods in an order determined only by the logic of the processes, and not by the accidental chain of circumstances which led to their formulation. To take an example, we may refer to the treatment of the principle of indeterminacy. Instead of being invited to consider hypothetical experiments with γ-ray microscopes, the reader is led to expect the principle simply from an examination of the law of propagation of a wave packet, combined with the principles previously developed. The fact that the theory provides no wave packet capable of describing a beam of electrons with less than a certain dispersion in their generalized coordinates, together with the fact that it gives results which accord with experience, is taken to show that an electron beam with less than this minimum dispersion could not exist in nature. It follows that for a single electron there must be a similar dispersion in its possible coordinate-configurations; this is simply one way of stating the principle in question.

In a somewhat similar way the very foundations of wave mechanics are so developed that each step follows almost as a logical necessity from what precedes. One of the most elegant examples of this treatment is the section which leads up to the determination of the boundary conditions satisfied by a de Broglie wave.

The book will appeal not only to those who find the text-books of de Broglie and Temple useful, but also to that class of physicists who, though not primarily mathematical physicists, wish to understand the workings as well as the results of the new methods. Such a work was sorely needed, and one must be thankful to the author for filling the gap so well.

J. H. A.

Partial Differential Equations of Mathematical Physics, by H. BATEMAN, M.A., Ph.D. Pp. xxii + 522. (London: Cambridge University Press.) 42*s*.

This is a large volume of a little over 500 pages devoted to the consideration of a large number of problems occurring in physics. The title indicates that the problems are those in association with which a differential equation presents itself for solution. Such equations are of a very varied character and a choice has to be made in the compilation of even a large volume. The author chooses those which fall into a particular form which covers a surprisingly useful and wide range.

In text-books on physics it occurs frequently, on account of limitations of space and of the impossibility of turning a physics text-book into a mathematical one, that equations are solved incompletely and sometimes by methods that are mathematically unsound. This book is a work of reference in which these deficiencies are made good. One has only to glance through the book to realize how wide is the field of mathematics which the physicist must explore. The reader is usually anxious to hurry on from one region to another without too much attention to the connecting paths, so that an author has no easy task before him when he sets out to make a mathematical chart, which, while sufficiently complete, is not so detailed that it becomes irritating. In this respect we believe the author to be successful.

The problems are grouped in chapters under summary headings and an index of subjects and authors makes reference easy. The second chapter, for example, is entitled, "Applications of the Integral Theorems of Gauss and Stokes," and problems in hydrodynamics, elasticity and heat with many others are associated under this heading.

The general method is to assume as a starting-point results which are directly to be obtained from works on physics and to proceed to the solution, overlapping being thus avoided: for example in a problem on the bending of a beam, the expressions for

shearing-force and bending-moment form the starting-points and we then have a purely mathematical discussion of equations resulting in particular problems. This is exactly what the physicist, at any rate, requires.

This problem also illustrates a case where boundary conditions have to be considered. The author discusses various types of boundary conditions in the first chapter—a very useful discussion for physicists in general, and for those in particular who are interested in the new developments, where boundary conditions are of great importance.

The author does not consider the differential equations of the new quantum theory, but the new theory, in some of its aspects, makes so much use of classical methods that the student of recent advances in atomic physics will find useful material here.

Chapter IX, on paraboloidal coordinates, introduces Hermite's polynomial, and examples of orthogonal functions are scattered about the book. A number of books have been written in order to assist the physicist in the mathematics appropriate to his subject, especially by continental writers. This work in English is welcomed particularly on account of its originality and usefulness, but also on account of the unity introduced into a subject which might easily have become a tedious collection of isolated problems.

H. T. F.

Solutions Superficielles Fluides à deux Dimensions et Stratifications Monomoléculaires, by ANDRÉ MARCELIN. Pp. 163 with 86 figures including 8 coloured plates. (Paris: Les Presses Universitaires de France.) 80 fr.

The term "surface solution" has come to be the recognized physical name for molecules of a substance in the surface of a liquid, which, by their presence, alter the surface tension of the liquid. The study of such two-dimensional solutions has in some ways proved more fundamental than the study of bulk solutions. For instance, the "pressure" of a layer of molecules can be directly measured by the force on 1 cm. of a floating barrier separating the layer from a free surface of the solvent, whereas osmotic pressure-determinations are apt to be clumsy and their results difficult to interpret.

After a short introductory chapter M. Marcelin gives us the history of the surface film, beginning with divers observations on the calming of the sea by oil which suggested that possibly a film whose thickness was comparable with that of a single molecule would produce the desired effect. The author then describes the experiments of Miss Pockels, Lord Rayleigh, Devaux and himself, and gives the values of so-called molecular diameters calculated therefrom. This is followed by an outline of Langmuir's orientation theory.

A large section of the book is devoted to the refined experiments of Langmuir, Adam and the author himself, which led to the establishment of the perfect-gas relation connecting the surface pressure of a film with its area and temperature, provided the film is unsaturated. The absorption equation of Gibbs is treated briefly both from the classical and modern points of view.

No less interesting than the foregoing is the account of the experiments of Guyot on the lowering of the Volta effect between an electrolyte and a metal by the presence of a surface film. The results of this investigation give us values for dipole moments and the average angle which the axis of the molecule makes with the plane of the film.

Finally M. Marcelin gives accounts of optical investigations of the black spots of soap films and the thickness of paratoluidine crystals formed both from solution and by sublimation. Several very good colour-reproductions are given.

The whole volume is full of interest and should bring home to the reader the fact that in the study of thin films we have a powerful means of obtaining insight into molecular behaviour.

Every important experiment that is mentioned is clearly described, but should the reader desire further detail he will find the extensive bibliography of great value.

R. C. B.

The Adsorption of Gases by Solids. A General Discussion held by the Faraday Society, January 1932. Pp. 448. (London: Gurney and Jackson.) 15*s*.

The contents of this volume, including summaries of work already published and also new experimental and theoretical papers, cover a wide field. Much discussion centred about the reality of two types of adsorption, one of which is small and in evidence at low temperatures and involves low binding energy; while the second type is larger and in evidence at higher temperatures, involves large heats of adsorption, and proceeds with measurable velocity. The second type, called "activated adsorption," or "chemisorption," is considered by some contributors as due to the removal of adsorbed films of foreign material with increasing temperature. New experimental methods include the great increase which is effected in the electron-emissivity of tungsten by small adsorbed amounts of electropositive metals, the study of electron-diffraction from surfaces on which there is adsorbed gas, and the transformation of ortho- into para-hydrogen on surfaces. On the whole, the experimental side is less prominent than the theoretical, and in the latter section there are papers of considerable interest in which the effects of van der Waals's forces, now known from quantum mechanics to be inversely as the seventh power of the distance, and quantum valency forces arising from the interaction of electrons, are considered. The effects of electrical forces due to dipoles are discussed also, although in one communication it is claimed that these are usually negligible. The interaction between an inert-gas atom and a metal, the latter considered as containing a perfectly polarizable electric continuum, and between a gas atom and an ionic crystal, have been worked out. An interesting discussion as to a possible explanation of activated adsorption starts from the supposed dissociation of the adsorbed gas molecule when the cohesion between its constituent atoms and the adsorbent is greater than the dissociation energy. The approach of atoms to the metal surface gives rise to irregularities with new conduction-electron levels in the metal atoms, and this may explain activated adsorption at higher temperatures. There is much experimental evidence that adsorbed particles may be in a mobile state and capable of moving about over the surface as a kind of two-dimensional liquid. Their penetration into microscopic cracks in the solid, caused by unbalanced surface forces, will explain absorption into the interior of the solid, which may also be activated. Many other aspects of the subject are dealt with, and the volume provides a useful survey of recent advances in the subject, many of direct interest to physicists. J. R. P.

Manual of Meteorology, Volume 4. *Meteorological Calculus: Pressure and Wind*, by Sir Napier Shaw, LL.D., Sc.D., F.R.S., with the assistance of Elaine Austin, M.A. Pp. xx + 359. (London: Cambridge University Press.) 30*s*.

The issue of this volume completes a manual which a former number of these *Proceedings* anticipated "would probably be for some time the standard work on meteorology." In the dedication of it to the memory of W. H. Dines, F.R.S., it is called a "volume of reminiscences," but half of it was issued for official use in the Meteorological Office in 1918 and to the public a year later. The 160 pages of this earlier issue have now been expanded to 195 pages by the addition of new matter such as the section on "peculiarities of surface isobars." Two new chapters on the laws of atmospheric motion and on the general equations of motion of a parcel of air serve as an introduction to the chapters of the previous issue, and the volume closes with two further new chapters, one on hypotheses and realities as to low- and high-pressure areas and the other entitled "retrospective and prospective." In the preface Sir Napier Shaw takes credit to himself for the "discursiveness" which has been charged against volume 3 and claims that volume 4 has the same characteristic. With regard to the last two chapters most readers will admit his claim, but they will feel some doubt as to the earlier ones. The last chapters constitute a summary of

the authors' views, which depart considerably from those generally held. They regard the horizontal motion of the wind as the independent variable and the pressure as the dependent. Velocity and entropy are regarded as the two quantities which determine all atmospheric phenomena. Apart from these innovations the two chapters present the views of the Norwegian meteorologists, but the presentation is not always as clear as it might be and terms are introduced which are not in common use, e.g. "syncopated by half a period" for delayed by half a period (p. 331), "underworld" for the lower air (p. 335), "if the earth went dry" printed as a quotation but having no reference to prohibition (p. 333). An index of 11 pages is provided and the volume ends with 10 pages entitled, "The Drama of the Atmosphere," and a list of the "dramatis personae"—that is, the contents of the whole manual.

The printing and general appearance of the volume are excellent. C. H. L.

Wireless Receivers: The Principles of their Design, by C. W. OATLEY, M.A., M.Sc. Pp. vii + 103. (London: Methuen & Co., Ltd.) 2*s*. 6*d*.

It would be impossible to do full justice to this title in the space of one of Messrs Methuen's "Monographs on Physical Subjects," to which series this little book belongs. Mr Oatley has, however, made good use of the space at his disposal to produce what should prove to be a useful introduction to a popular subject.

The book is divided into chapters dealing with the various "stages" into which a wireless receiver may, in general, be divided—namely, the aerial and tuner system, h.f. amplifier, detector, l.f. amplifier, and power stage; the fundamental functions of each stage and the way in which incorrect design may affect operation being explained in precise mathematical language. A bibliography is appended, which serves to amplify the text considerably and especially to draw the reader's attention to the many recent advances in the design of highly selective broadcast receivers; comprising some fifty items, this cannot be considered as unduly bulky.

Two special classes of receiver do not receive mention in the text. One of these, the superheterodyne, claims a special section of the bibliography, and the other, the superregenerative, although likely to become of importance with increasing interest in ultra-high-frequency telephony, is of such limited application at the moment that omission of its description is scarcely surprising.

The average student of physics has but a vague knowledge of wireless principles and of thermionic valve circuits; this little book can be recommended as offering an excellent means of placing such knowledge on a sounder, quantitative basis. R. A. F.

Magnetism and Electricity (second edition), by E. NIGHTINGALE, M.Sc. Pp. xvi + 294. (London: G. Bell and Sons, Ltd.) 4*s*. 6*d*.

The author, who has already produced a very valuable elementary book on sound, has now issued the second edition of this *Electricity and Magnetism*, which is of School Certificate standard.

Great attention is devoted to the historical development of the subject, with a wealth of illustration in the way of photographs and small but clear diagrams, which almost average one per page. The mathematical treatment is not advanced, but numbers of examples are included at the ends of the chapters. Emphasis is also laid on the practical applications of the principles described. Frictional electricity is treated, probably with good reason, on conventional lines, and the electron is only mentioned towards the end of the book. Credit is fairly given to Cavendish for the discovery of Ohm's law by means of a condenser-discharge through his own body. Other interesting matters include electric cardiographs, electric ovens, and maps showing the cable systems of the world and the "main grid" transmission lines of England. The book is indeed of remarkably good value and printing and production are all that could be desired. J. E. C.

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THE CONCEPT OF CAUSALITY

BY PROF. MAX PLANCK

The seventeenth Guthrie Lecture, delivered on June 17, 1932.

I AM deeply sensible of the honour which you have conferred upon me in having allowed me to speak at the present meeting, which is devoted to the memory of Frederick Guthrie, one of the founders of the Physical Society of London. My pleasure is the greater in accepting your kind invitation, as I too rank amongst the physicists who have followed the steps of this celebrated investigator, inasmuch as his scientific career led him from University College, London, to German Universities, to Heidelberg and to Marburg.

It is true that a physicist of to-day, particularly if he is only a theorist, cannot dream of mastering the whole range of subject-matter which had to be understood as a matter of course by a physicist in the middle of last century. For Guthrie was not only a physicist, he was also a professional chemist; and it is just in the boundary province of these two disciplines that lies his most important work. His investigations on capillarity and heat-conduction have made him well known throughout the world, and his name will for all time be connected with the concepts of critical solution-temperature, of the eutectic alloy and of kryohydrate.

To-day the investigator who wishes to further the advance of his science substantially is obliged, by the rich abundance of accumulated material, to specialize narrowly the subject-matter of his researches. But on the other hand this involves the danger that he will find it more and more difficult to awaken any interest in his work among his scientific fellow-workers, engaged in different problems.

In order nevertheless to approach my task to-day with some hope of not boring you, I have chosen a subject which, though of rather an abstract nature, is just for that reason of fundamental significance for all domains of physics, and indeed for the whole of science—the question of the concept of causality. It is a question which could scarcely have been put forward a generation ago, when the classical theory was in its prime. But to-day, when the supporting pillars of the theory are in danger of tottering, this question stands in the centre of our interest.

Whoever seeks to clarify the opposing opinions in the present quarrel about the significance and the validity of the law of causality in modern physics, must naturally begin by stating that primarily everything depends on an agreement as to the sense in

which the word "causality" is used in physical science. At the outset we agree that in speaking of a causal link between two successive events we mean a certain connection, subject to law, between the two events, of which the earlier event is called the cause, the later one the effect. But the question is, in what does this particular kind of connection consist? Is there any infallible sign proving a certain event occurring in nature to be causally conditioned by another?

The numerous investigations hitherto undertaken concerning this question show that the surest approach to a clear answer is obtained by connecting the question with the possibility of making correct predictions of the future. Indeed, for proving that any two events are causally connected, there is no more unobjectionable means than that which consists in showing that from the occurrence of one event the occurrence of the other event can always be concluded in advance. That was already known to the farmer, who demonstrated *ad oculos* to the incredulous peasants the causal connection between artificial manure and fertility of soil. The peasants refused to believe that the lush growth of clover on the farmer's field was due to artificial manure, and sought for other reasons. So the farmer had certain narrow furrows ploughed on his field; then he shaped them into letters and manured them profusely, so that after the shooting up of the seed the following sentence was legible in distinct clover-writing: "This strip of land has been manured with gypsum."

As a starting-point for all further considerations I will therefore use the following simple proposition, applicable also beyond the domain of physics: *An event is causally conditioned if it can be predicted with certainty*. Thereby of course I only wish to say that the possibility of making a correct prediction for the future forms an infallible criterion for the existence of a causal connection, not by any means that the two mean one and the same thing. I need only recall the well-known example, that in the daytime we are quite able with certainty to predict the advent of night, yet day is not the cause of night.

But on the other hand we often assume the existence of a causal connection in cases where there is no possibility at all of a correct prediction. Think of the weather forecasts! The unreliability of weather prophets has become proverbial; and yet there is no trained meteorologist who does not look upon the occurrences in the atmosphere as causally determined. Thus we see that if we wish to reach the essence of the concept of causality, we must sift matters more minutely.

In the case of weather forecasts we may easily suppose that their unreliability is only conditioned by the size and the complicated nature of the object under consideration—the atmosphere. If we take only a small quantity of it, say a litre of air, we shall far more probably be able to make correct predictions as to its behaviour under external influences, such as compression, heat, moisture and the like. We know certain physical laws which enable us to predict more or less positively the results of the corresponding measurements, such as increase of pressure, increase of temperature, or condensation.

On closer observation, however, we arrive at a very remarkable conclusion. Even if we choose ever so simple conditions and use ever such delicate measuring instruments, we shall never succeed in calculating in advance the results of our

measurement with absolute accuracy; that is to say, not so accurately that it will agree with the measured number to the last decimal place. There always remains some residuum of inaccuracy, in contrast with purely mathematical calculations such as those of $\sqrt{2}$ and π which can be stated exactly to any number of decimal places. And what applies to mechanical and thermal phenomena is valid for all regions of physics, including electrical and optical phenomena.

Our experiences therefore compel us to recognize the following statement as a given and established fact: *In not a single instance is it possible to predict a physical event exactly.*

On placing this fact side by side with the proposition, which served as our starting-point, that an event is causally conditioned if it can be predicted with certainty, we are confronted by a vexatious but unavoidable dilemma. Either we stick to the wording of the original proposition, so that there is not a single instance in nature in which a causal connection can be asserted; or we make room for the assumption of a strict causality, when we are compelled to subject our original proposition to a certain modification.

There are nowadays a number of physicists and philosophers who have decided in favour of the first alternative; I will call them the indeterminists. According to them there is absolutely no real causality in nature—no strict law. It is only an illusion given us by the appearance of certain rules which are never absolutely valid, although often they are very approximately so. On principle the indeterminist seeks a root of a statistical kind for every physical law, for gravitation, and for electrical attraction. For him they are all laws of probability, only relating to mean values from numerous homogeneous observations, and possessing only approximate validity for single observations, always admitting, therefore, of exceptions.

A well-known example of such a statistical law is the dependence of gas-pressure on the density of gas and on temperature, in accordance with the kinetic theory of gases.

This theory is directly confirmed by investigating the fluctuations with time which occur in the pressure acting on a very small part of the sides of the vessel. It is well known that fluctuations of this kind, caused by the irregular impacts of the molecules, are quite universally observed, whenever high-speed molecules are in touch with bodies susceptible of motion. They are shown in the so-called Brownian molecular movement and also in the behaviour of an extremely sensitive balance, which never comes to complete rest but incessantly performs little irregular oscillations around its position of equilibrium.

And as in this case of the gas laws, the indeterminist wishes to attribute also all other physical laws ultimately to chance. For him nature is governed by statistics alone. His aim is to base physics on the calculus of probability.

But in fact the science of physics has hitherto developed on the opposite basis. It has chosen the second of the two alternatives: that is to say, in order to be able quite strictly to maintain the law of causality, it has slightly modified the starting-point, which was that an event was causally conditioned if it could be safely predicted. The modification consists in using the word "event" in a slightly altered

sense. It is not to one single actual measurement, always containing casual and unessential elements, that the theoretical physicist gives the name of event. He reserves this name to an imagined process, going on in another world: we will call it the physicist's world-picture, which is substituted for the actual one given by our senses and by measuring instruments acting as a kind of refined sense. The physicist's world-picture is a mental construction, arbitrary to a certain extent; an idealization, created for the purpose of escaping from the uncertainty which inheres in every individual measurement, and of becoming able to establish sharply defined conceptual relations. In physics therefore all measurable quantities—lengths, intervals of time, masses, charges and the rest—have a double meaning, according as to whether we consider them as given directly by measurement, or as transferred into the physical world-picture. In the first meaning such quantities can only be defined inaccurately and can therefore never be represented by precise numbers. But in the physicist's world-picture they stand for definite mathematical symbols, which can be operated with according to strict rules. If in physics we make use of a trigonometric equation for calculating the height of a tower, then in speaking of the height we mean quite a definite thing, a well-defined quantity. Yet the actual measurement of the height does not furnish a definite quantity. Consequently the so-called true height of the tower is a different thing from the measured height. Exactly the same argument applies to the frequency of vibration of a pendulum or to the brightness of an incandescent lamp. Likewise every universal constant, for instance the charge on an electron, is a different thing from the actually measured charge.

The clear and logical distinction between the magnitudes and quantities of the world of sense and the similarly named magnitudes and quantities of the physicist's world-picture is absolutely indispensable for the clarifying of conceptions. Without this distinction a discussion about these questions is futile.

Therefore it is wrong to state (as some do) that the world-picture of physics contains or ought to contain only directly observable quantities. On the contrary, directly observable quantities do not appear at all in the world-picture. It contains nothing but symbols. Besides, there are always in the world-picture elements which for the world of the senses have only a very indirect significance or none at all, such as ether-waves, partial vibrations, coordinate systems and the like. Such elements at first act as ballast, but they are put up with in view of the decided advantage afforded by the introduction of the mental world-picture. This advantage is, that the world-picture enables us to carry through a strict determinism.

To be sure, the world-picture always remains an auxiliary conception: what we are eventually concerned with is of course the events in the world of the senses and their approximately correct forecasting, which in classical theory is effectuated in the following manner. First an object of the world of the senses, say a system of material bodies in any measured state, is symbolized, that is to say transferred into the world-picture. In this way a definite physical system in a definite initial state is obtained. In like manner the influences which are subsequently exerted upon the object from the outside are replaced by corresponding symbols in the framework of the world-picture. Thus we are provided with the external forces acting on the

systems, or with the boundary conditions. By these data the behaviour of the system is for all time unambiguously defined and can be calculated with absolute accuracy from the differential equations of the theory. Thus the coordinates and the momenta of all particles of the system result in quite definite functions of time. Now if for any later time we transfer back into the world of the sense the symbols used for the world-picture, we obtain a connection between a later event in the world of sense and an earlier event in the world of sense. This connection can then be utilized for the approximate prediction of the later event.

To summarize: while in the world of sense the prediction of an event is always affected by something of an uncertainty, in the physicist's world-picture all events follow certain definable laws; they are strictly determined causally. Therefore the introduction of the world-picture reduces the uncertainty in the prediction of an event of the world of sense, to the uncertainty of the translation of the event from the world of sense to the world-picture and vice versa. Herein lies the significance of the physicist's world-picture.

In classical theory, without much bothering about this uncertainty, attention was concentrated on the elaboration of the causal view of what is going on in the ideal world-picture. That is how it has achieved its great successes. In particular, it has succeeded in finding a satisfactory interpretation for the irregular fluctuations mentioned above, which correspond to the pressure of a gas or to the Brownian movement—an interpretation that was based on the assumption of strict causality. For the indeterminists no real problem existed here. As they seek irregularity behind every rule, statistical law is what immediately satisfies them. Therefore they content themselves with the assumption that the collision of two individual molecules, as well as the impact of the molecules on the sides of the vessel, occurs only according to statistical laws. However, there is as little conclusive reason for such an assumption as there is for assuming that, because the electrons gather on the surface in a charged conductor, the charge of an individual electron must be on its surface too. On the other hand, the determinists, who conversely seek a rule behind every irregularity, were led to the task of building up a theory of the gas laws on the assumption that the collision of two individual molecules is strictly conditioned causally. The achievement of this task is the life-work of Ludwig Boltzmann. It forms one of the greatest triumphs of theoretical research. For Boltzmann's theory leads to the statement—confirmed by measurements—that the average energy of the fluctuations around the position of equilibrium is proportional to the absolute temperature. And further: from the measurement of such oscillations, for instance those of an extremely sensitive torsion-balance, this theory makes possible a remarkably accurate calculation of the absolute number and mass of the striking molecules.

In view of these and other great successes, reasonable hope prevailed that the world-picture of classical physics would on the whole be equal to its task, and that the uncertainties remaining after the transfer into and from the world of the senses would lose their importance as experimental methods improved in refinement. But with one stroke this hope has for all time been destroyed by the appearance of the elementary quantum of action.

The so-called principle of uncertainty, discovered and formulated by Heisenberg, is characteristic of quantum physics. It states that of two canonically conjugated quantities, such as position and momentum, or time and energy, only one can be measured with absolute accuracy, and that only by the sacrifice of accuracy in the other. That is to say that by increasing the accuracy with which one of them is measured you diminish the accuracy of the other, the product of the two errors being constant. Hence, if one of the two is determined with absolute accuracy, the other remains absolutely undetermined.

It stands to reason that this statement makes it on principle impossible to transfer with any accuracy into the world of the senses the simultaneous values of coordinates and momenta which play the predominant part in the world of classical physics. For the strictly causal view of the world this fact raises a difficulty, which has already led some indeterminists to affirm that the law of causality in physics is definitely disproved. However, on closer consideration this conclusion, which is due to confusion of the world-picture with the world of sense, must be called at least premature. For there is at hand, for overcoming this difficulty, a means which has often done excellent service in similar cases. It is the assumption that the question as to the simultaneous values of the coordinates and of the momenta of a particle has no meaning in physics. The law of causality must not be blamed for the impossibility of answering a meaningless question. The blame must rather be laid on the assumptions which have led to the putting of that question, that is to say on the assumed structure of the physicist's world-picture. And as the classical world-picture has failed, it must be replaced by another.

In fact this has been done. The new world-picture of quantum physics has arisen from the desire to render possible the accomplishment of a strict determinism in spite of the existence of the quantum of action. For this end the material particle, which had hitherto formed the primary component of the world-picture, has had to be divested of its elementary character: it has been dissolved into a system of material waves, which form the elements of the new mental picture of the world. The particle in its old meaning now forms only a special limiting case as an infinitely narrow wave-group, of which—following Heisenberg's principle of uncertainty—the momentum in a specified position of the particle is quite indefinite. If we allow a certain latitude to the position of the particle, the momentum will also attain an approximately definable value. The laws of classical mechanics are then approximately valid for both position and momentum.

In general the laws of wave mechanics are, as everybody knows, quite different from those applying to particles in classical mechanics. But the most important point is that the function which is characteristic for the material waves, the wave function or the probability function (the name is irrelevant here), is completely determined for all places and times by the initial conditions and the boundary conditions. We can calculate it by quite definite rules, employing either Schrödinger's operators or Heisenberg's matrices or Dirac's q-numbers.

Thus we see that the world-picture in quantum physics is governed by the same rigorous determinism which rules classical physics. It is only that the symbols are

different, and that we operate with other rules of calculation. Accordingly in quantum physics, as formerly in classical physics, the uncertainty in the prediction of events of the world of sense is reduced to the uncertainty of the connection between the world-picture and the world of sense; that is to say, to the uncertainty of the translation of the symbols of the world-picture into the world of sense and vice versa. The fact that this double uncertainty is put up with forms the most impressive proof of the importance of the task of maintaining determinism in the world-picture.

Yet but a superficial glance enables us to recognize how very far in quantum physics the world-picture has diverged from the world of the senses, and how much more difficult it is to transfer an event from the world-picture to the world of the senses, or vice versa, in quantum physics than it formerly was in classical physics. In classical physics the meaning of every symbol was immediately comprehensible: the position, the velocity, the momentum, the energy of a particle could be stated more or less directly from measurements. There was no evident reason for not assuming that one should be able to reduce the remaining uncertainty below any limit, as the refinement of experimental methods progressed. On the other hand, in quantum mechanics the wave function yields no means whatever whereby this function can be interpreted directly in the world of sense, the first obstacle being that it does not refer to ordinary space but to configuration space, which possesses as many dimensions as there are independent coordinates in the physical system under discussion. Furthermore—and this is the important point—the wave function does not furnish the values of the coordinates as functions of time; it only furnishes the probability for the coordinates possessing any given values at a definite given time.

This circumstance has again incited the indeterminists to an attack upon the law of causality. And this time the attack seems to promise a positive success; for from all measurements nothing more than a statistical significance of the wave function can be deduced. But again the same loop-hole for escape is open to champions of strict causality. They assume that the question as to the significance of a definite symbol in the world of quantum physics—for instance a material wave—has no definite meaning, as long as we are not at the same time told how to ascertain this meaning—not told in what condition is the special instrument which is employed for transferring the symbol into the world of sense. We therefore also speak of a causal effect of this instrument. Thereby we imply that the inaccuracy under discussion is at least in part conditioned by the fact that the amount of the quantity to be measured depends in a certain manner, subject to law, on the nature of the measuring-process.

With this auxiliary assumption the whole question has been led into channels the further course of which still remains dark. For now the indeterminists are justified in putting forward the question whether any sensible meaning can be attributed to the idea that the measuring-instrument should exert a causal influence on the process to be measured; for any attempt to test this influence would require new measurements, which would involve a new causal interference and would therefore bring a new feature of uncertainty into the problem.

And yet this objection does not finish the matter. For, as every experimental

physicist knows, there are not only direct but also indirect testing methods. In many cases the latter have done good service where the former had failed. Above all I wish to oppose the now widespread and seemingly plausible opinion, that a question in physics is only worth investigation if from the outset the fact that it admits of a definite answer is established. If the physicists had always followed this precept, the celebrated experiment of Michelson and Morley on the measurement of the so-called absolute velocity of the earth would never have been made. We should then perhaps not even to-day be in possession of the theory of relativity. So our efforts to ascertain the absolute velocity of the earth have proved exceedingly fruitful for science, although nowadays the question itself is almost universally considered to be meaningless. Then are we not justified in expecting even much greater profit from investigating the problem of causality, the roots of which have certainly not been reached hitherto—a problem quite pre-eminent in its fertilizing influence on research?

But how to come to a decision? Evidently there is nothing for it but to take one's choice between the opposing standpoints, to adopt one, and then to see whether from this starting-point we attain valuable or useless results. In this respect we must welcome the fact that the physicists who are interested in this subject are divided into two camps, one inclining to determinism, the other to indeterminism. As far as I see, the latter are at present in the majority. But it is hard to tell and the situation may easily change in the course of time. In between there seems to be room for a third party, occupying an intermediate position. They attribute to certain concepts, such as electrical attraction and gravitation, an immediate significance and a strict rule of law, while ascribing to other concepts, such as the light wave and the particle wave, only a statistical significance for the world of sense. This notion however appears at the outset rather unsatisfactory on account of its want of uniformity. So I shall leave it aside and confine myself to the elucidation of the two absolutely logical standpoints.

The indeterminists' yearning after knowledge is satisfied by the statement that the wave function of quantum physics is only a probability scheme; for him there is no further question to put. On the other hand he looks upon a definite law of nature, such as Coulomb's law of electrical attraction, as an unsolved problem. He cannot content himself with Coulomb's law of force or potential, but must try to find exceptions. He will not be satisfied unless he succeeds in establishing what the probability is that the electrical force will diverge from Coulomb's law to any specified intent.

The determinist takes the opposite view in all these matters. He is quite content to look upon Coulomb's law as an ultimate and fundamental law of nature. The interpretation of the wave function as a probability function he will only admit as long as no account is taken of the particular apparatus with which the wave is generated or analysed. He seeks relations strictly subject to law between what is going on in the bodies that interact with the wave, and the form of the wave function. For this purpose he is of course obliged to begin by making the measuring-apparatus as well as the wave function the object of his research. That is to say, he must

transfer into his world-picture the whole experimental arrangement for generating the material waves (for instance the high-potential battery, incandescent wire, or radio-active substance) and also the whole of the measuring-apparatus such as the photographic plate, ionization chamber, or Geiger counter, with all that is going on in them; he must treat all these objects together as one single system, as a closed unity. This, of course, is not sufficient to solve the problem, which on the contrary has become even more complicated. For since one is allowed neither to divide the total system nor to expose it to any interference from outside, lest it should lose its original character, no direct test whatever is feasible. On the other hand it now becomes possible to set up certain hypotheses of a new kind with regard to the internal occurrences, and then to examine their consequences. The future will show whether we are able to advance in this way; up to now we cannot distinctly discern in what direction progress will be accomplished. But this much may be safely affirmed: the elementary quantum of action sets an objective insuperable limit to the sensitiveness of the physical measuring-apparatus at our disposal, and the progress we strive for will consequently exhibit this limit more plainly than heretofore.

This really brings us to the end of our considerations. They have shown us that the standpoint even of modern physics does not prevent us from achieving a strictly causal view—the word "causal" being understood in the modified sense explained above—although the necessity for such a view can be proved neither from the outset nor afterwards. Yet even the convinced determinist—and he perhaps more than anybody else—is overcome by a scruple which hinders him from being quite satisfied with the interpretation of causality introduced here. For even if we should succeed in further developing the concept of causality on the lines laid down, it would be affected by a grave deficiency. One might suppose that a relation of so deep a significance as the causal connection between two successive events would in its essence be independent of the human mind which considers it. The reverse is true. At the outset we had to attach the concept of causality to the human intellect, with reference to the capability of predicting an event; furthermore, we were not able to enforce the adoption of the deterministic view otherwise than by substituting for the given world of our senses the physical world-picture. The latter is a creation of human imagination of a provisional and changeable character. Anthropomorphisms of that sort are ill suited to form a fundamental physical concept. So the question arises whether there is no way of giving the concept of causality a deeper significance, by divesting it as much as possible of its anthropomorphic character and by making it independent of the introduction of an artefact, such as the physicist's world-picture. We must of course retain our original proposition, that an event is causally conditioned if it can be safely predicted; otherwise we shall lose our only foothold. But we must also adhere to the second proposition, that in not a single case is it possible to predict an event exactly. It then follows, as before, that in order to be able to speak of causality in nature we must apply some modification to the first statement. So far everything remains as it was. But the modification we had applied above can be replaced by one of quite a different kind, in one sense quite an opposite modification.

What we modified there was the object of prediction, the event. We did not refer the events to the immediately given world of the senses, but to the fictitious world-picture. Thus we were able accurately to determine the events. But instead of the object we may modify the subject of prediction, the predicting mind. For every prediction necessitates the existence of a predictor. In the following we shall therefore limit our attention to the predicting subject, and look upon the immediately given events of the world of sense as the object of prediction, without introducing an artificial world-picture.

It is obvious that the certainty of the prediction depends in a high degree upon the individuality of the predictor. Let us again refer to weather forecasts: it is evident that it makes a great difference who provides us with the forecast for to-morrow—whether it is an ignorant person, who knows nothing about to-day's atmospheric pressure, direction of the wind, atmospheric temperature and humidity, or a practical farmer, who has noted all these data and has a wide experience, or again a scientifically trained meteorologist, who besides the local data has at his disposal numerous weather-charts from all parts with exact information. With each of these successive prophets the uncertainty of the prediction is more and more diminished. It is therefore an obvious thought to assume that an ideal mind, apprehending everywhere all the physical occurrences of to-day in their minutest points, should be able to predict with absolute accuracy the weather of to-morrow in all its details. And the same argument can be applied to every other prediction of physical events.

Such an assumption means an extrapolation, a generalization, which can neither be maintained by a logical conclusion nor refuted *a priori*. It must therefore not be judged according to its truth but rather according to the value that is inherent in it. In the light of this view, the actual impossibility of accurately predicting an event in even one single case, either from the standpoint of classical physics or from that of quantum physics, is a natural consequence of the circumstance that man with his senses and his measuring-instruments is a part of nature. He is subject to her laws and cannot escape from her, while such a tie does not exist for the ideal mind. The objection that this ideal mind is only a product of our thoughts, and that our thinking brain also consists of atoms following physical laws, is not able to withstand a closer test. For it is indubitable that our thoughts can lead us beyond every law of nature known to us, and that we are able to imagine relations which transcend the realm of physics.

It is true that in order to accomplish such a notion, logically we must subject ourselves to a severe restriction: we must forego making the ideal mind the object of a scientific investigation. We must not consider the ideal mind akin to us, and must not demand of it how it procures the knowledge enabling the exact prediction of future events. For the inquisitive questioner might easily, like Faust, be awed by the answer: "Thou'rt like the spirit which thou comprehendest, not me!" And if the questioner nevertheless remains obdurate and declares that the notion of an ideal mind, if not illogical, is yet empty and superfluous, he must be met with these arguments: Not all statements eluding logical reasoning are scientifically valueless,

and such a short-sighted formalism chokes up the source at which men like Galilei, Kepler, Newton and many other great physicists have slaked their scientific thirst for knowledge. For all these men, consciously or unconsciously, the devotion to science was a matter of faith, of unwavering faith in a rational scheme of the universe.

It is true, this faith can be forced upon nobody, just as one cannot command truth or forbid error. But the simple fact that up to a certain degree we are able to subject future natural events to our thoughts and to guide them at our will would remain a complete riddle, if it did not at least point to the existence of a certain harmony between the outer world and the human mind. And the question to what depths one imagines the sphere of this harmony to be extended is only of secondary importance.

In conclusion we may therefore say: the law of causality is neither right nor wrong, it can be neither generally proved nor generally disproved. It is rather a heuristic principle, a sign-post (and to my mind the most valuable sign-post we possess) to guide us in the motley confusion of events and to show us the direction in which scientific research must advance in order to attain fruitful results. As the law of causality immediately seizes the awakening soul of the child and causes him indefatigably to ask "Why?" so it accompanies the investigator through his whole life and incessantly sets him new problems. For science does not mean contemplative rest in possession of sure knowledge, it means untiring work and steadily advancing development.

THE AXIAL SOUND-PRESSURE DUE TO DIAPHRAGMS WITH NODAL LINES

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ABSTRACT. A formula is found for the axial sound-pressure due to a disc having a nodal circle and vibrating in an infinite rigid plane. Beyond a certain axial distance, when the nodal circle occurs at $r = a/\sqrt{2}$, the pressure vanishes owing to interference caused by the inner and outer portions of the disc vibrating in opposite phase. The case of n nodal circles of arbitrary radii is treated by an approximate method. A rigid disc is imagined to be severed around each nodal circle, whilst contiguous annuli vibrate with equal amplitudes in opposite phases. Finally, the pressure on the axis of a conical shell having nodal circles is treated as in the previous case. When the semi-apical angle of the cone is $\frac{1}{2}\pi$ and there are no nodal circles, the formula reduces to that for a rigid disc.

§ 1. FREE-EDGE DISC WITH NODAL CIRCLE

p, r_1, s THE sound-pressure dp at a point P distant r_1 from an elementary area ds vibrating in an infinite rigid plane as shown in figure 1, is given by*

$$dp = \frac{\rho}{2\pi}\frac{e^{-ikr_1}}{r_1}\frac{\partial^2 w}{\partial t^2}\,ds \qquad \text{.....(1),}$$

ρ, w ω, c where ρ is the density of the medium, $\partial^2 w/\partial t^2$ the acceleration of the element normal to the plane, $k = \omega/c$, c being the velocity of sound, while $\omega = 2\pi \times$ frequency.

Let the dynamic deformation curve of the disc be of the form

$$w = A\,(1 - 2r^2/a^2) \qquad \text{.....(2),}$$

A, X where $A = X\cos\omega t.$

r, R_1 From figure 1 $r^2 = r_1^2 - R_1^2$ .....(3).

Substituting for r^2 from (3) in (2) we get

$$w = A\,\{1 - 2\,(r_1^2 - R_1^2)/a^2\}$$

and, therefore, the axial acceleration†

$$\frac{\partial^2 w}{\partial t^2} = \ddot{A}\,\{1 - 2\,(r_1^2 - R_1^2)/a^2\} \qquad \text{.....(4),}$$

where $\ddot{A} = -\omega^2 X\cos\omega t.$

Differentiating (3) we have

$$2r\,dr = 2r_1\,dr_1,$$

so that $ds = 2\pi r\,dr = 2\pi r_1\,dr_1$ .....(5),

R_2 the limits of r_1 being R_2 and R_1.

* Rayleigh, *Sound*, 2, 162 (1894), equation (1).

† The acceleration of any point on the disc, parallel to the axis.

Substituting from (4) and (5) in (1), the axial pressure at P due to the whole disc is, therefore, given by

$$p = \rho \dot{A} \int_{R_1}^{R_2} e^{-ikr_1} \left\{ 1 - \frac{2(r_1^2 - R_1^2)}{a^2} \right\} dr_1 \qquad \text{.....(6).}$$

Performing the integration in (6), inserting the limits, and arranging terms, we ultimately find that

$$p = \frac{2\rho \dot{A} e^{-i\alpha}}{k} \left[\cos\beta - \frac{2}{ka^2}(R_2 + R_1)\sin\beta + i\left\{ \frac{4}{k^2a^2}\sin\beta - \frac{2}{ka^2}(R_2 - R_1)\cos\beta \right\} \right] \qquad \text{.....(7),}$$

where $\alpha = \frac{1}{2}k(R_2 + R_1)$, α

and $\beta = \frac{1}{2}k(R_2 - R_1)$. β

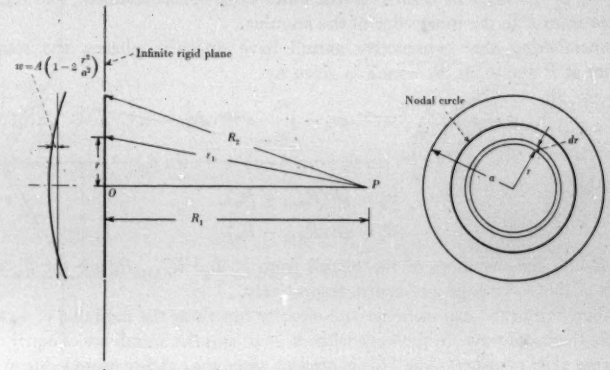

Fig. 1. Diagram illustrating free-edge disc vibrating with one nodal circle, and set in an infinite rigid plane. The dynamic deformation curve is $w = A(1 - 2r^2/a^2)$.

At a great distance when $R_1 \gtrsim a$ we can write

$$R_2 + R_1 = 2R_1, \quad R_2 - R_1 = a^2/2R_1,$$

so that $\alpha = kR_1$ and $\beta = ka^2/4R_1$.

Since β is very small $\cos\beta \doteqdot 1$ and $\sin\beta = ka^2/4R_1$.

On insertion of these values in (7) the value of p is zero. This is due to the interference arising from the inner and outer portions of the disc vibrating in opposition. In equation (2) we tacitly assumed the nodal circle to occur where $r = a/\sqrt{2}$. If some other value were chosen, the pressure would have a value different from zero.

The modulus of p found in the usual way is

$$|p| = \frac{2\rho \dot{A}}{k}\left[\frac{4}{k^2a^4}\{(a^2 + 2R_1^2) - 2R_1\sqrt{a^2 + R_1^2}\cos 2\beta\} + \cos^2\beta + \frac{16\sin^2\beta}{k^4a^4} - \frac{2\sin 2\beta}{ka^2}\left\{ (\sqrt{a^2 + R_1^2} + R_1) + \frac{4}{k^2a^2}(\sqrt{a^2 + R_1^2} - R_1) \right\} \right]^{\frac{1}{2}} \qquad \text{.....(8).}$$

§ 2. FREE-EDGE DISC WITH *n* NODAL CIRCLES

In this case we shall use an approximate artifice to simplify the analytical treatment. Accordingly we take a rigid disc having n concentric nodal circles of arbitrary radii. The disc is assumed to be severed at each circle and contiguous annuli vibrate in opposite phase. The amplitude* of each annulus is represented by w, where $w = A = X \cos \omega t$.

From expressions (1) and (6) we have for any particular annulus

$$p_\mu = \rho \ddot{A} \int_{R_{\mu-1}}^{R_\mu} e^{-ikr_1}\, dr_1 \qquad \text{.....(9)},$$

where R_μ is the distance from P to the outer edge of the annulus, and $R_{\mu-1}$ the distance from P to the inner edge of the annulus.

Remembering that consecutive annuli have opposite phases, the resultant pressure at P due to all the annuli is given by

$$p = \rho \ddot{A} \left[\int_{R_n}^{R_{n+1}} e^{-ikr_1}\, dr_1 - \int_{R_{n-1}}^{R_n} e^{-ikr_1}\, dr_1 + \ldots \right] \qquad \text{.....(10)}$$

$$= 2\rho \ddot{A} \{ e^{-i\alpha_n} \sin \beta_n + \ldots (-1)^n e^{-i\alpha_0} \sin \beta_0 \} \qquad \text{.....(11)},$$

where

$$\alpha_n = \tfrac{1}{2}k\,(R_{n+1} + R_n),$$

$$\beta_n = \tfrac{1}{2}k\,(R_{n+1} - R_n).$$

$R_n \ldots R_1$ are the distances of the annuli from P, and R_{n+1}, R_0 are the distances from P of the outer edge and centre respectively.

By converting the exponentials into circular functions the modulus of p can be found in the usual way. In the limit when $n \to \infty$ and the annuli are of equal radial width, the axial pressure $\to 0$. This is obvious since the axial pressures due to contiguous annuli are equal and opposite.

When there is only one nodal circle, formula (11) reduces to

$$p = 2\rho \ddot{A} \{ e^{-i\alpha_1} \sin \beta_1 - e^{-i\alpha_0} \sin \beta_0 \} \qquad \text{.....(12)}.$$

Taking the radius of the nodal circle as $r = a/\sqrt{2}$, then at a great axial distance where $R_1 \gtrsim a$ we have, from the geometry of figure 1, $\alpha_0 = \alpha_1$ and $\beta_0 = \beta_1$. Thus the pressure vanishes as in the preceding section, where a different procedure was used. By varying the radius of the nodal circle its influence on the axial pressure can be determined.

§ 3. CONICAL SHELL WITH NODAL CIRCLE

As in the previous section we shall adopt the artifice of a rigid shell severed at the nodal circle, the inner and outer portions vibrating in opposition with equal amplitudes.

* Cases can be treated when the amplitudes of the annuli are different; the phases can also be suitably chosen to fit special conditions.

From the geometry of figure 2 when $R \gtrsim a$ with adequate approximation we can write

$$r_1 = R - r \cot \tfrac{1}{2}\psi \qquad \text{.....(13),}$$

ψ being the plane apical angle of the shell. ψ

From expression (1) the pressure at P due to an elementary ring of radius r is

$$dp = \rho \breve{A} r_1^{-1} e^{-ikr_1} r\, dr \qquad \text{.....(14).}$$

So far as the amplitude of the sound-pressure is concerned the distance r_1 can be assumed constant and equal to R over the surface of the shell. The phase being paramount, since it controls the interference, it is essential to use the value of r_1 from (13) in the exponential index of (14).

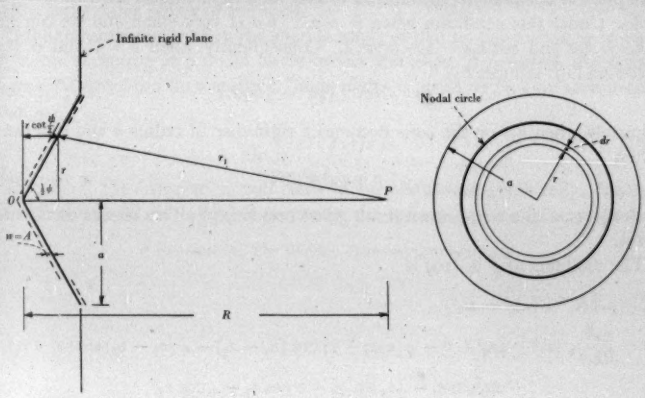

Fig. 2. Diagram illustrating conical shell vibrating with one (hypothetical) nodal circle, in an infinite rigid plane. The shell is severed at the nodal line; the inner and outer portions vibrate in opposite phase.

Thus (14) becomes

$$dp = \rho \breve{A} R^{-1} e^{-ikR} e^{ik_1 r} r\, dr \qquad \text{.....(15),}$$

where $k_1 = k \cot \tfrac{1}{2}\psi$. k_1

If the nodal radius is b the axial pressure at P due to the inner portion of the shell is p_1, where b p_1

$$p_1 = \frac{\rho \breve{A}}{R} \int_0^b e^{ik_1 r} r\, dr \qquad \text{.....(16),}$$

the factor e^{-ikR} being omitted since it merely affects the phase.

The axial pressure at P due to the outer ring of the shell is

$$p_2 = \frac{\rho \breve{A}}{R} \int_b^a e^{ik_1 r} r\, dr \qquad \text{.....(17).}$$

Moreover, the resultant pressure due to the whole shell is the difference between (17) and (16). Expanding $e^{ik_1 r}$ in circular functions we get for (17) and (16),

$$p = \frac{\rho \breve{A}}{R} \left\{ \int_b^a r \cos k_1 r\, dr - \int_0^b r \cos k_1 r\, dr + i \left[\int_b^a r \sin k_1 r\, dr - \int_0^b r \sin k_1 r\, dr \right] \right\} \qquad \text{.....(18).}$$

Integrating (18) and inserting the various limits we ultimately arrive at the formula*

$$p = \frac{\rho \ddot{A}}{R}\left\{\frac{1}{k_1}(a \sin k_1 a - 2b \sin k_1 b) + \frac{1}{k_1^2}(\cos k_1 a - 2 \cos k_1 b + 1)\right.$$
$$\left. - i\left[\frac{1}{k_1}(a \cos k_1 a - 2b \cos k_1 b) - \frac{1}{k_1^2}(\sin k_1 a - 2 \sin k_1 b)\right]\right\} \quad \ldots\ldots(19)$$
$$= \frac{\rho \ddot{A}}{R}\{A_1 + iB_1\}.$$

If we put $b = 0$, equation (19) reduces to that for a rigid conical shell vibrating as a whole. Under this condition when $\psi \to \pi/2$, $k_1 a$ is very small and we can write $\sin k_1 a = k_1 a$ and $\cos k_1 a = 1 - k_1^2 a^2/2$. Consequently when $b = 0$ and $\psi \to \pi/2$ expression (19) reduces to

$$p = \rho \ddot{A} R^{-1}(\tfrac{1}{2}a^2 + \tfrac{1}{2}ik_1 a^3) \quad \ldots\ldots(20).$$

Ultimately when $k_1 = 0$ the cone becomes a rigid disc of radius a and (20) can be written

$$p = \rho \ddot{A} a^2/2R \quad \ldots\ldots(21),$$

which agrees with a well-known result given previously†. This checks the accuracy of (19).

The modulus of p in (19) is

$$|p| = \rho \ddot{A} R^{-1}\sqrt{(A_1^2 + B_1^2)}$$
$$= \frac{\rho \ddot{A}}{Rk_1^2}\sqrt{\{z_1^2 + 4z_2^2 + 6 - 4(z_1 z_2 + 1)\cos(z_1 - z_2) - 4(z_1 - z_2)\sin(z_1 - z_2)}$$
$$+ 2z_1 \sin z_1 - 4z_2 \sin z_2 + 2 \cos z_1 - 4 \cos z_2\} \quad \ldots\ldots(22),$$

z_1 where $z_1 = k_1 a$,

z_2 $z_2 = k_1 b$.

The case of a rigid conical shell is obtained when $b = 0$; i.e. when $z_2 = 0$. Then expression (22) reduces to

$$p = \frac{\rho \ddot{A}}{Rk_1^2}\sqrt{\{z_1^2 + 2 - 2(\cos z_1 + z_1 \sin z_1)\}} \quad \ldots\ldots(23),$$

which is identical with that found by Stenzel‡. In this case the axial pressure is not zero. It decreases slowly with increase in frequency, i.e. increase in k_1 or

$$\omega c^{-1} \cot \tfrac{1}{2}\psi,$$

c where c is the velocity of sound.

* In obtaining the various formulae in this section, the influence of the air column vibrations within the shell and horn action of the shell in response to its own radiation at higher frequencies have been neglected.

† *Wireless World*, March 30, 1927; *Proc. R. S.* A, **122**, 606 (1929), equation (6).

‡ *Z. f. Tech. Phys.* **10**, 569 (1929).

When there are n nodal circles of radii a_1, a_2, ... a_n it is easy to show that

$$A_1 = \frac{1}{k_1}\{a \sin k_1 a - 2a_n \sin k_1 a_n + 2a_{n-1} \sin k_1 a_{n-1} - \ldots\}$$
$$+ \frac{1}{k_1^2}\{\cos k_1 a - 2 \cos k_1 a_n + 2 \cos k_1 a_{n-1} - \ldots\} \quad \ldots\ldots(24),$$

$$B_1 = -\frac{1}{k_1}\{a \cos k_1 a - 2a_n \cos k_1 a_n + \ldots\}$$
$$+ \frac{1}{k_1^2}\{\sin k_1 a - 2 \sin k_1 a_n + \ldots\} \quad \ldots\ldots(25),$$

where a is the outer radius of the shell.

If the transmission loss* in the shell is small, so that no phase changes in amplitude occur excepting at a nodal circle, where the phase is reversed, the dynamic deformation curve can be measured. Then if this is given by $f(r)$ the axial pressure becomes

$$\frac{\rho \dot{A}}{R}\int_0^a e^{ik_1 r} f(r)\, r\, dr \quad \ldots\ldots(26).$$

In general the integration will have to be performed graphically. Assuming that with one nodal circle $f(r)$ is parabolic, the integral can easily be evaluated.

* See letter to *The Wireless Engineer*, October, 1931.

THE ACCESSION TO INERTIA OF FLEXIBLE DISCS VIBRATING IN A FLUID

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ABSTRACT. In this paper formulae are obtained for the velocity-potential at the surface of a free-edge disc vibrating with nodal lines in a fluid. These formulae are used to ascertain the accession to inertia due to the fluid when the disc is set in an infinite rigid plane. The equivalent mass and the mass coefficient of the disc vibrating *in vacuo* are found also. By means of these results, the influence of the fluid on the frequency of vibration with (*a*) one nodal circle, (*b*) one nodal diameter, (*c*) stationary centre, is evaluated. In air the alteration in frequency is almost negligible, whereas in water the frequency is reduced to a small fraction of its value *in vacuo*.

§ 1. INTRODUCTION

THE additional inertia of a body when it moves in a fluid is very familiar in hydrodynamics. Treatment of the problem from an acoustical and, therefore, vibrational viewpoint, has been confined mainly to rigid bodies, although H. Lamb* has examined the case of a clamped-edge flexible disc vibrating in its gravest mode in water†.

In the present paper it is proposed to deal with the accession to inertia of free-edge flexible discs having nodal lines. Although vibration in air is of primary interest, the analysis also holds for water or any other fluid. Computations will be made to show the enormous difference between the vibrational frequencies in air and in water.

The analysis is based on the following assumptions:

(1) The ratio of the radius of the disc to the wave-length is such that the distribution of radiation is substantially spherical.

(2) The disc vibrates in an infinite rigid plane situated in free fluid.

(3) The amplitude of the disc at any radius varies sinusoidally with time.

(4) Transmission loss in the disc is neglected.

(5) The shape of the disc is unaffected by fluid pressure, and true nodal lines —not merely positions of minimum amplitude‡—exist.

* *Proc. R. S.* A, **98**, 205 (1920).

† This has been treated experimentally by J. H. Powell and J. H. T. Roberts, *Proc. Phys. Soc.* **35**, 170 (1923).

‡ *Phil. Mag.* **12**, 771 (1931).

§ 2. FREE-EDGE DISC WITH ONE NODAL CIRCLE

Let the dynamic deformation curve be of the type

$$w = A(1 - p_1 r^2/a^2), \qquad w$$

then

$$\partial w/\partial t = \dot{A}(1 - p_1 r^2/a^2),$$

where

$$A = X \cos \omega t, \qquad A, X, \omega,$$

$$\dot{A} = -\omega X \sin \omega t;$$

a is the radius of disc; a

p_1 a parameter governing the radius of the nodal circle; p_1

and r is the distance of any point on the disc from the centre. r

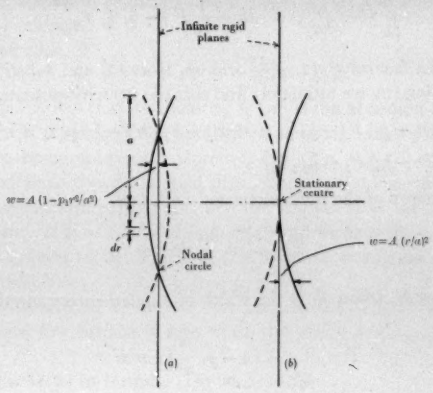

Fig. 1. Diagram illustrating free-edge disc vibrating in an infinite rigid plane (*a*) with one nodal circle; (*b*) with stationary centre. The dynamic deformation curve for (*a*) is $w = A(1 - p_1 r^2/a^2)$, and for (*b*) $w = A(r/a)^2$.

The velocity-potential ϕ at points on the disc is given* by ϕ

$$\phi = \int_0^\infty J_0(kr)\, dk \int_0^a J_0(kr) \frac{\partial w}{\partial t} r\, dr \qquad \ldots\ldots(1),$$

where

$k = \omega/c$,

$\omega = 2\pi$ frequency,

$c =$ velocity of longitudinal waves in fluid.

This formula is applicable only when the deformed shape of the disc is symmetrical about its axis, e.g. it cannot be used for nodal diameters. Inserting the value of $\partial w/\partial t$ in (1) we obtain

$$\phi = \dot{A} \int_0^\infty J_0(kr)\, dk \int_0^a J_0(kr) \left(1 - p_1 \frac{r^2}{a^2}\right) r\, dr \qquad \ldots\ldots(2).$$

Taking the first integral in (2) we find after a partial integration that

$$\int_0^a J_0(kr) \left(1 - p_1 \frac{r^2}{a^2}\right) r\, dr = a^2 \left\{(1 - p_1) \frac{J_1(ka)}{ka} + 2p_1 \frac{J_2(ka)}{(ka)^2}\right\} \qquad \ldots\ldots(3).$$

* H. Lamb, *Hydrodynamics*, p. 129 (1906).

To complete the integration of (2) we require a formula pertaining to infinite integrals*, namely,

$$\int_0^\infty \frac{J_0(kr)\,J_n(ka)}{(ka)^n}\,dk = \frac{\pi^{\frac{1}{2}}}{2^n\,\Gamma(n+\frac{1}{2})}\,\frac{1}{a}\,F\left[-n+\tfrac{1}{2},\ \tfrac{1}{2},\ 1,\ \frac{r^2}{a^2}\right] \qquad \text{.....(4).}$$

Putting $\left.\begin{matrix} n=1 \\ n=2 \end{matrix}\right\}$ in (4) we obtain

$$\int_0^\infty \frac{J_0(kr)\,J_1(ka)}{ka}\,dk = \frac{1}{a}\,F\left(-\tfrac{1}{2},\ \tfrac{1}{2},\ 1,\ \frac{r^2}{a^2}\right) \qquad \text{.....(5),}$$

and

$$\int_0^\infty \frac{J_0(kr)\,J_2(ka)}{(ka)^2}\,dk = \frac{1}{3a}\,F\left(-\tfrac{3}{2},\ \tfrac{1}{2},\ 1,\ \frac{r^2}{a^2}\right) \qquad \text{.....(6).}$$

Introducing the factors a^2, $(1-p_1)$ and $2p_1$ from (3) and substituting from (5) and (6) in expression (2), we ultimately find that the velocity-potential ϕ is given by

$$\phi = \dot{A}a\,\{(1-p_1)\,F(-\tfrac{1}{2},\ \tfrac{1}{2},\ 1,\ r^2/a^2) + \tfrac{2}{3}p_1\,F(-\tfrac{3}{2},\ \tfrac{1}{2},\ 1,\ r^2/a^2)\} \qquad \text{.....(7)}$$

$$= \dot{A}a\,\{(1-p_1)\,F_1 + \tfrac{2}{3}p_1 F_2\} \qquad \text{.....(8).}$$

The kinetic energy of the fluid associated with *both* sides of the disc is†

$$T = \rho \iint \phi\,\frac{\partial w}{\partial t}\,dS,$$

and since $dS = 2\pi r\,dr$, where dr is the width of an elementary annulus of radius r, we have

$$T = \rho \int_0^a \phi\dot{A}\left(1 - p_1\frac{r^2}{a^2}\right) 2\pi r\,dr \qquad \text{.....(9).}$$

Inserting the value of ϕ from expression (8) in (9) above we obtain

$$T = 2\pi\rho\dot{A}^2 a \int_0^a \{(1-p_1)\,F_1 + \tfrac{2}{3}p_1 F_2\}\left(1 - p_1\frac{r^2}{a^2}\right) r\,dr \qquad \text{.....(10).}$$

It is easy to show that

$$\int_0^a F\left(\alpha,\ \beta,\ 1,\ \frac{r^2}{a^2}\right) r\,dr = \frac{a^2}{2}\,F(\alpha,\ \beta,\ 2) \qquad \text{.....(11),}$$

and that

$$\int_0^a F\left(\alpha,\ \beta,\ 1,\ \frac{r^2}{a^2}\right) r^3\,dr = \frac{a^4}{2}\,\{F(\alpha,\ \beta,\ 2) - \tfrac{1}{2}\,F(\alpha,\ \beta,\ 3)\} \qquad \text{.....(12).}$$

Applying the relationships of (11) and (12) to the integral in (10) the kinetic energy can be written

$$T = 2\pi\rho\dot{A}^2 a^3\,\{\tfrac{1}{2}\,(1-p_1)\,[(1-p_1)\,F(-\tfrac{1}{2},\ \tfrac{1}{2},\ 2) + \tfrac{1}{2}p_1\,F(-\tfrac{1}{2},\ \tfrac{1}{2},\ 3)]$$
$$+ \tfrac{1}{3}p_1\,[(1-p_1)\,F(-\tfrac{3}{2},\ \tfrac{1}{2},\ 2) + \tfrac{1}{2}p_1\,F(-\tfrac{3}{2},\ \tfrac{1}{2},\ 3)]\} \qquad \text{.....(13).}$$

* G. N. Watson, *Theory of Bessel Functions*, formula (2), p. 401 (1922).
† H. Lamb, *Hydrodynamics*, formula (4), p. 44.

Using Gauss's theorem for the reduction of the hypergeometric functions we get

$$T = 2\pi\rho\dot{A}^2a^3\left\{\frac{(1-p_1)}{2}\left[(1-p_1)\frac{\Gamma(2)\,\Gamma(2)}{\Gamma(\frac{5}{2})\,\Gamma(\frac{3}{2})}+\frac{p_1}{2}\frac{\Gamma(3)\,\Gamma(3)}{\Gamma(\frac{7}{2})\,\Gamma(\frac{5}{2})}\right]\right.$$

$$\left.+\frac{p_1}{3}\left[(1-p_1)\frac{\Gamma(2)\,\Gamma(3)}{\Gamma(\frac{5}{2})\,\Gamma(\frac{3}{2})}+\frac{p_1}{2}\frac{\Gamma(3)\,\Gamma(4)}{\Gamma(\frac{7}{2})\,\Gamma(\frac{5}{2})}\right]\right\} \quad \text{.....(14)}$$

$$= \tfrac{8}{5}\rho a^3\dot{A}^2\,(1-\tfrac{14}{15}p_1+\tfrac{5}{21}p_1^2) \quad \text{.....(15)*.}$$

But $$T = \tfrac{1}{2}M_1\,(\partial w/\partial t)^2 = \tfrac{1}{2}M_1\dot{A}^2,$$

where M_1 is the accession to inertia due to the fluid on *both* sides of the disc, and $\dot{A}$ is the central velocity.

Thus we find that

$$M_1 = \tfrac{16}{3}\,\rho a^3\,(1-\tfrac{14}{15}p_1+\tfrac{5}{21}p_1^2) \quad \text{.....(16).}$$

Rayleigh† has shown that the quantity $\frac{16}{3}\,\rho a^3$ is the accession to inertia for *both* sides of a rigid disc vibrating in an aperture in an infinite rigid plane—the distribution of radiation being spherical. Moreover $(1-\frac{14}{15}p_1+\frac{5}{21}p_1^2)$ represents the ratio of M_1 for a flexible to that for a rigid disc. When $p_1 = 0$ the rigid-disc formula is reproduced as we should expect, for then the amplitude at any radius is given by $w = A$, a constant. If $p_1 = 1$, $w = A\,(1-r^2/a^2)$ and we have the case of a membrane vibrating in its gravest mode. The accession to inertia is $1{\cdot}625\rho a^3$, this being 0·305 that of a rigid disc.

For our particular case where the disc has one nodal circle, the integral of the momentum‡ over the surface is zero when the radius r of this circle is $a/\sqrt{2}$ or $p_1 = 2$.

Inserting this value in formula (16) we find that

$$M_1 = \tfrac{16}{35}\,\rho a^3 \quad \text{.....(17),}$$

this being approximately 8·6 per cent of that for a rigid disc. The very large reduction in M_1 is due to the two equal areas on either side of the nodal circle vibrating in opposite phase.

By allotting any desired value to the parameter p_1, formula (16) can be used to ascertain the influence of the radius of the nodal circle on M_1 in relation to the central velocity $\dot{A}$. If $p_1 > 2$ the amplitude at the edge exceeds that at the centre. For this condition M_1 should be evaluated in terms of the edge velocity $\dot{A}\,(1-p_1)$. Then $T = \frac{1}{2}M_1\dot{A}^2\,(1-p_1)^2$, so that (16) must be divided by $(1-p_1)^2$. For example, if the thickness of the disc varies as $(1-r^2/a^2)$, p_1 for zero momentum is 3. The value of M_1 is identical with (17) but T is four times that for a uniform disc where $p_1 = 2$.!

* T is a minimum when $p_1 = 1{\cdot}96$.

† *Sound*, **2**, 162 (1894).

‡ This is the condition for a vibrational mode when the *edge is free* and the centre moving.

§ 3. FREE-EDGE DISC WITH ONE NODAL DIAMETER

Let the dynamic deformation curve be of the form

$$w = A\frac{r}{a}\cos\theta = \frac{A}{a}x \qquad \text{.....(18),}$$

x where $x = r\cos\theta$ as in figure 2.

For finding the velocity-potential at points on the disc, equation (1) is barred by the asymmetry of (18), so we proceed as follows: When the dynamic deformation curve is of the type $w = A(1 - r^2/a^2)^{n+1}$, the velocity-potential from (1) is

$$\phi = \dot{A}\int_0^\infty J_0(kr)\,dk\int_0^a J_0(kr)\left(1-\frac{r^2}{a^2}\right)^{n+1} r\,dr \qquad \text{.....(19).}$$

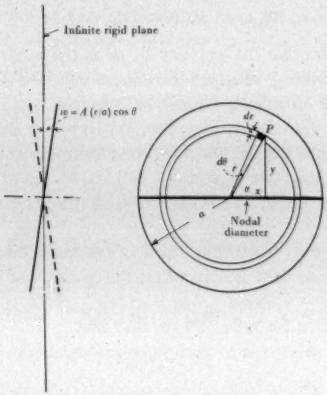

Fig. 2. Diagram illustrating flexible disc vibrating with one nodal diameter in an infinite rigid plane. The dynamic-deformation curve is $w = A(r/a)\cos\theta$, and the elementary area at P is $r\,dr\,d\theta$.

Substituting $r = a\sin\chi$ in the first integral of (19) and altering the limits accordingly, we have

$$a^2\int_0^{\frac{1}{2}\pi} J_0(ka\sin\chi)\cos^{2n+3}\chi\sin\chi\,d\chi \qquad \text{.....(20).}$$

The form in expression (20) tallies with that of Sonine's first finite integral and its value* is

$$2^{n+1}\Gamma(n+2)\,a^2\frac{J_{n+2}(ka)}{(ka)^{n+2}} \qquad \text{.....(21).}$$

Thus substituting (21) in (19) we obtain

$$\phi = 2^{n+1}\Gamma(n+2)\,\dot{A}\,a^2\int_0^\infty \frac{J_0(kr)\,J_{n+2}(ka)\,dk}{(ka)^{n+2}},$$

* G. N. Watson, *Theory of Bessel Functions*, formula (1), p. 373 (1922).

and by formula (4) above this reduces to

$$\phi = \frac{\pi^{\frac{1}{2}}}{2}\frac{\Gamma(n+2)}{\Gamma(n+\frac{3}{2})} Aa F\left[-(n+\tfrac{3}{2}), \tfrac{1}{2}, 1, \frac{r^2}{a^2}\right] \qquad \text{.....(22).}$$

Since $r^2 = x^2 + y^2$ (figure 2), we find that*

$$\frac{\partial}{\partial x}\left\{A\left(1-\frac{r^2}{a^2}\right)^{n+1}\right\} = -2(n+1)\frac{Ax}{a^2}\left(1-\frac{r^2}{a^2}\right)^n \qquad \text{.....(23).}$$

Also, since $$\frac{\partial}{\partial x} F(\alpha, \beta, \gamma, z) = \frac{\alpha\beta}{\gamma} F[\alpha+1, \beta+1, \gamma+1, z]\frac{dz}{dx},$$

we obtain $$\frac{\partial\phi}{\partial x} = -\frac{\pi^{\frac{1}{2}}}{2}\frac{\Gamma(n+2)}{\Gamma(n+\frac{3}{2})}\frac{\dot{A}x}{a} F\left[-(n+\tfrac{1}{2}), \tfrac{3}{2}, 2, \frac{r^2}{a^2}\right] \qquad \text{.....(24).}$$

Putting $n = 0$ in (23) we have $-2Ax/a^2$, whilst $n = 0$ in (24) yields

$$-\dot{A}(x/a) F(-\tfrac{1}{2}, \tfrac{3}{2}, 2, r^2/a^2) \qquad \text{.....(25).}$$

Thus, if the deformation curve be of the form $A(x/a)$, the velocity-potential is found when (25) is multiplied by $-a/2$. Moreover, in this case

$$\phi = \frac{A}{2} x F\left(-\tfrac{1}{2}, \tfrac{3}{2}, 2, \frac{r^2}{a^2}\right) \qquad \text{.....(26).}$$

The kinetic energy associated with *both* sides of the disc is

$$T = \rho\int_0^a\int_0^{2\pi}\phi\left(\frac{\partial w}{\partial t}\right) r\,dr\,d\theta \qquad \text{.....(27),}$$

where $r\,dr\,d\theta$ is an elementary area on the disc (figure 2).

Substituting for the various quantities in the integrand of (27) and writing $x = r\cos\theta$ we have, for the kinetic energy,

$$T = \rho\frac{\dot{A}^2}{2a}\int_0^a F\left(-\tfrac{1}{2}, \tfrac{3}{2}, 2, \frac{r^2}{a^2}\right) r^3\,dr\int_0^{2\pi}\cos^2\theta\,d\theta \qquad \text{.....(28).}$$

The first integral is π, whilst by expanding and integrating term by term the second integral is found to be $\frac{a^4}{4} F(-\frac{1}{2}, \frac{3}{2}, 3)$. Substituting these values in (28) we have

$$T = \tfrac{1}{8}\pi\rho a^3\dot{A}^2 F(-\tfrac{1}{2}, \tfrac{3}{2}, 3) \qquad \text{.....(29).}$$

But $$F(-\tfrac{1}{2}, \tfrac{3}{2}, 3) = \frac{\Gamma(3)\Gamma(2)}{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2})} = \frac{32}{15\pi},$$ and therefore $T = \frac{1}{15}A^2\rho a^3$.

Thus the accession to inertia for both sides of the disc is

$$M_1 = \tfrac{8}{15}\rho a^3 \qquad \text{.....(30),}$$

which is precisely $\frac{1}{10}$ that for a rigid disc.

* See N. M. Ferrers, *Quarterly Journal of Mathematics*, **14**, 7 (1877). This procedure involving differentiation is valid only when the amplitude at the edge of the disc is zero, before the derivative is obtained. Thus the original formula being $w = A(1 - r^2/a^2)^{n+1}$, the necessary condition is fulfilled, since $w = 0$ when $r = a$. It is not possible, however, to obtain the value of ϕ corresponding to $r\cos\theta$ by differentiating $w = A(r^2/a^2)$ with respect to x, since $w \neq 0$ when $r = a$. Obviously the differentiation can be extended to higher derivatives provided the above condition is satisfied.

From formulae (17) and (30) we see that the ratio of the values of M_1 for a nodal circle and a nodal diameter is $\frac{30}{35}$ or 0·86. Moreover, there is not a wide divergence in M_1 for the two modes of vibration. In the above analysis the value $p_1 = 2$ was used, this corresponding to a nodal circle whose radius $r = a/\sqrt{2}$. Actually when there are no diametral nodes and the disc vibrates freely, $r = 0{\cdot}68a$ and $p_1 = 2{\cdot}16$. Using this value of p_1 in expression (16) and dividing by $(p_1 - 1)^2$, since $p_1 > 2$, we find $M_1 = 0{\cdot}371\rho a^3$ and the ratio (nodal circle)/(nodal diameter) = 0.

§ 4. FREE-EDGE DISC WITH STATIONARY CENTRE

Let the dynamic deformation curve be of the form $w = A(r/a)^2$, figure 1 (*b*). Then the velocity-potential at points on the disc is, by (1),

$$\phi = \frac{\dot{A}}{a^2}\int_0^\infty J_0(kr)\,dk\int_0^a J_0(kr)\,r^3\,dr \qquad \text{.....(31).}$$

After a partial integration the first integral becomes

$$\int_0^a J_0(kr)\,r^3\,dr = a^4\left\{\frac{J_1(ka)}{ka} - \frac{2J_2(ka)}{(ka)^2}\right\} \qquad \text{.....(32).}$$

From the results in (5), (6) and (32), (31) gives

$$\phi = \dot{A}a\{F(-\tfrac{1}{2}, \tfrac{1}{2}, 1, r^2/a^2) - \tfrac{2}{3}F(-\tfrac{3}{2}, \tfrac{1}{2}, 1, r^2/a^2)\} \qquad \text{.....(33)}$$

$$= \dot{A}a\{F_1 - \tfrac{2}{3}F_2\} \qquad \text{.....(34).}$$

Thus the kinetic energy associated with *both* sides of the disc is given by

$$T = 2\pi\rho\frac{\dot{A}^2}{a^2}\int_0^a \phi r^3\,dr,$$

which on substitution for ϕ from (34) yields

$$T = 2\pi\rho\frac{\dot{A}^2}{a}\int_0^a (F_1 - \tfrac{2}{3}F_2)\,r^3\,dr \qquad \text{.....(35).}$$

The evaluation of (35) is accomplished by using the result given in (12), so that we obtain

$$T = 2\pi\rho\frac{\dot{A}^2}{a}\left\{\frac{a^4}{2}[F(-\tfrac{1}{2}, \tfrac{1}{2}, 2) - \tfrac{1}{2}F(-\tfrac{1}{2}, \tfrac{1}{2}, 3) - \tfrac{2}{3}F(-\tfrac{3}{2}, \tfrac{1}{2}, 2) + \tfrac{1}{3}F(-\tfrac{3}{2}, \tfrac{1}{2}, 3)]\right\},$$

which on reduction by Gauss's theorem gives the kinetic energy of the fluid as

$$T = \tfrac{40}{63}\dot{A}^2\rho a^3 \qquad \text{.....(36).}$$

Thus the accession to inertia

$$M_1 = \tfrac{80}{63}\rho a^3 \qquad \text{.....(37)*,}$$

which is $\frac{5}{21}$ or nearly $\frac{1}{4}$ that of a rigid disc. This exceeds the values obtained for a nodal circle or diameter, since in the present case the whole of the disc, at any particular epoch, is moving in the same direction. It is less than that for a rigid disc, since the amplitude of vibration gradually decreases from the periphery inwards.

* Dividing (16) by $(1 - p_1)^2$ and making $p_1 = \infty$ yields (37).

§ 5. INFLUENCE OF M_1 ON FREQUENCY OF VIBRATION

Since the curve of deformation *in vacuo* is maintained during vibration in the fluid, it follows that the potential energy is identical in both cases. Moreover, the kinetic energy of the system is alone affected and with it the frequency of vibration. The latter being inversely proportional to the square root of the total equivalent mass it follows that

$$\frac{f_1}{f_0}=\sqrt{\left(\frac{M_q}{M_q+M_1}\right)}=\sqrt{\left(\frac{1}{1+M_1/M_q}\right)} \qquad \text{.....(38),}$$

where M_q is the equivalent mass of the disc during vibration *in vacuo*, M_q

f_0 is the frequency *in vacuo*, f_0

and f_1 is the frequency in fluid. f_1

To estimate the magnitude of f_1/f_0 it is essential to determine the value of M_q.

§ 6. EVALUATION OF THE EQUIVALENT MASS M_q

(*a*) *Free-edge disc with nodal circle.* The kinetic energy of the disc *in vacuo* is $\Sigma \frac{1}{2}mv^2$ over its surface, and this is

$$\tfrac{1}{2}\rho_1\tau\int_0^a \dot{w}^2\,2\pi r\,dr=\frac{\pi\rho_1\tau a^2\dot{A}^2}{2}\left(1-p_1+\frac{p_1^2}{3}\right)=\frac{\dot{A}^2}{2}M\left(1-p_1+\frac{p_1^2}{3}\right),$$

or
$$T_v=\tfrac{1}{2}\dot{A}^2M_q=\tfrac{1}{2}\dot{A}^2Mk_q \qquad \text{.....(39),}$$

where T_v is the kinetic energy of the disc *in vacuo*, T_v

M is the natural mass, M

k_q, the mass coefficient, is $(1-p_1+p_1^2/3)$*, k_q

ρ_1 is the density of disc, and ρ_1

τ is the thickness of disc. τ

Putting $p_1=2$ in the above, we find that for one nodal circle

$$M_q=\tfrac{1}{3}M \qquad \text{.....(40),}$$

and $k_q=\frac{1}{3}$.

(*b*) *Free-edge disc with one nodal diameter.* It will be of greater interest if we consider the general case of n diameters. Let the dynamic deformation curve be n
$w=A(r/a)^n\cos n\theta$. Then the kinetic energy T_v of the disc *in vacuo* is (using figure 2 and remembering that there are n diameters) given by

$$T_v=\frac{\dot{A}^2\rho_1\tau}{2a^{2n}}\int_0^a r^{2n+1}\,dr\int_0^{2\pi}\cos^2 n\theta\,d\theta$$

$$=\frac{\dot{A}^2}{2}\,\frac{\pi\rho_1\tau a^2}{2(n+1)},$$

so that the equivalent mass

$$M_q=M/2(n+1) \qquad \text{.....(41),}$$

and
$$k_q=1/2(n+1) \qquad \text{.....(42).}$$

For one nodal diameter $n=1$, so that $M_q=\frac{1}{4}M$ and $k_q=\frac{1}{4}$. These formulae are

* When $p_1>2$, $k_q=(1-p_1+p_1^2)/(1-p_1)^2$ as is shown in § 2. By making p_1 infinite the value of k_q in (*c*) is obtained.

also valid in the case of a conical shell where the deformation curve is as defined above.

(*c*) *Free-edge disc with stationary centre.* In this case $w = A(r/a)^2$ and, therefore,

$$T_e = \frac{\dot{A}^2 \pi \rho_1 \tau}{a^4} \int_0^a r^5 dr$$

$$= \tfrac{1}{2} \dot{A}^2 \pi \rho_1 \tau a^2/3,$$

whence $$M_q = \tfrac{1}{3} M \qquad \text{.....(43),}$$

and $$k_q = \tfrac{1}{3}.$$

Consequently the values of M_q and k_q for a free-edge disc with one nodal circle and with stationary centre are identical.

It is important to observe that the *equivalent* mass is quite distinct from the *effective** mass. The former refers to the whole surface and is associated with energy relationships; it is always positive and never zero. The latter is referred to the driving point on the disc and is associated with the mechanical impedance. It can be positive, negative or zero.

§ 7. COMPUTATION OF THE RATIOS M_1/M_q AND f_1/f_0

$$\frac{M_1}{M_q} = \frac{k_1}{k_q} \frac{\rho}{\rho_1} \frac{a^3}{\pi a^2 \tau}$$

$$= \frac{1}{\pi} \frac{k_1}{k_q} \frac{\rho}{\rho_1} \frac{a}{\tau} \qquad \text{.....(44),}$$

k_1 where the accession coefficient k_1 is (M_1 for flexible disc) ÷ (M_1 for rigid disc).

To illustrate the foregoing analysis concretely we shall calculate the ratios M_1/M_q and f_1/f_0 for the cases (*a*), (*b*), (*c*) in air and in water. Taking an aluminium disc for which $a = 10$ cm., $\tau = 0{\cdot}055$ cm., density of aluminium $\rho_1 = 2{\cdot}7$ gm./cm.^{3}, density of air $= 1{\cdot}3 \times 10^{-3}$ gm./cm.^{3} and of water $= 1{\cdot}0$ gm./cm.^{3}, we obtain the data set forth in tables 1 and 2 below.

Table 1. Vibration in air

Type of vibration	M_1/M_q	Reduction in frequency
		%
One nodal circle (*a*)	0·028	1·9
One nodal diameter (*b*)	0·059	3·0
Stationary centre (*c*)	0·106	5·3

Table 2. Vibration in water

Type of vibration	M_1/M_q	Frequency ratio
One nodal circle (*a*)	29·2	0·182
One nodal diameter (*b*)	45·4	0·147
Stationary centre (*c*)	81·6	0·111

* *Proc. Phys. Soc.* **44**, 88 (1932).

The data of table 1 indicate that the alteration in frequency in air is relatively unimportant from a practical standpoint. On the other hand, the results in table 2 are somewhat startling to anyone unversed in the subject of subaqueous acoustics. In case (*a*) the accession to inertia is 29·2 times the equivalent mass of the disc and nearly ten times its natural mass. If the frequency f_0 *in vacuo* were 120 ~, the frequency f_1 in water would fall to 21·8 ~! If the disc were made of steel, its mass for the same value of f_0 would be three times that of the aluminium disc, so that in this case $f_1 = 21{\cdot}8\sqrt{3} = 36{\cdot}7\sim$. From expression (44) it is evident that if the radius is constant, increases in either the density of the disc or its thickness, or both, entail a corresponding rise in the frequency ratio f_1/f_0. In other words, the higher the frequency *in vacuo*, the smaller the reduction, due to accession to inertia, in a fluid.

THE DETERMINATION OF REFRACTIVITY TEMPERATURE COEFFICIENTS FOR LIQUIDS

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ABSTRACT. In this paper attention is directed chiefly to the problem of measuring with precision the changes in the refractivity of a liquid for small alterations in temperature. An elaborated Jamin interferometer is described, as are also the several pieces of auxiliary apparatus necessary for setting up and maintaining differences in the temperature of the two interferometer tubes. Further, there is given a plan for measuring, by means of platinum resistance thermometers, differences in the temperature of the two tubes.

§ 1. INTRODUCTION

During a recent research, certain variations in the refractivity of benzene were determined with the aid of a Jamin interferometer. As the work progressed the instrument was, as described elsewhere*, in some degree elaborated and increased accuracy imparted to the required measurements. In this communication a description is given of additional devices (applicable to interferometers of the Jamin type), which facilitate the determination of refractivities with a precision not otherwise readily attained. To begin with we briefly review certain difficulties inherent in such interferometric methods.

First, we observe that in determining a refractivity there is, in general, some uncertainty as to the true temperature of the liquid. This is of little moment when the measurements are made in a laboratory having a convenient and practically constant temperature, and the influence of draughts is excluded by surrounding the containing tubes with an aluminium case; for under these circumstances both tubes are equally affected. But for the determination of a refractivity temperature coefficient, some definite difference in the temperatures of the contents of the tubes must be established and maintained. Failing this, some plan for securing a constant difference in the two temperatures is allowable. Such a plan does not necessarily involve a strict constancy in the two temperatures; but it does require that, for a given difference, any small variation in the temperature of the one tube shall be accompanied by a precisely similar, equal and simultaneous variation in that of the other.

Assuming that this difficulty is overcome, we are at once confronted with the further problem of determining with accuracy (*a*) the temperature of the contents of each tube; or (*b*) the difference in the temperature of the two liquids.

* *Phil. Mag.* 13, 249–64 (1932).

For the solution of this problem two methods are at once presented. According to the one, thermometers are introduced through the tubulures of the interferometer chambers; whilst for the second, other thermometers, having exceptionally long and narrow bulbs, are placed horizontally upon the chambers. But from experiments it was found that both plans are inconvenient and lacking in precision. An attempt to win success by surrounding the interferometer chambers with water jackets failed, owing to the impossibility of circulating the water with the necessary rapidity. Even with the quickest flow obtainable, a distinct temperature-gradient was observable between the extremities of the jacket. Theoretically, success by this simple method can be achieved only by establishing an infinitely rapid circulation of the water. The difficulty was evaded in the following way.

§ 2. APPARATUS

First, a new interferometer, having a gap for tubes some 60 cm. long, was designed and built. The glass prisms of this instrument were such that the interfering beams of light were widely separated. When the instrument was received the usual glass tubes were displaced by others of copper having walls 1 mm. thick.

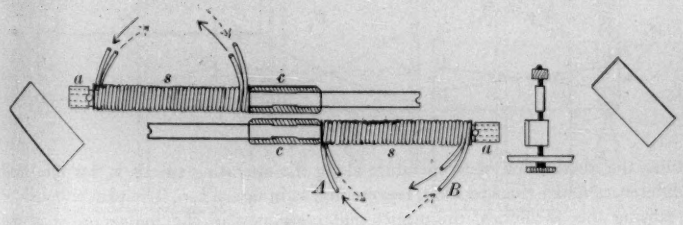

Fig. 1.

Secondly, each copper tube was cut into two unequal lengths which were then trued in a lathe. Subsequently these were reunited with a tightly fitting boxwood cylinder *c*, figure 1, so that the enclosed ends were 15 mm. apart as indicated in the figure.

Thirdly, the longer limb of each compound tube was, as shown in figure 1, very closely and simultaneously wound with twin lengths of "compo" tubing, the bore of which was 2 mm. Then by means of tape the tubular spirals *s*, *s* were firmly bound to the encircled copper tube. Finally they were covered with bright tinfoil and radiation effects were thus reduced to a minimum.

Lastly, the metal pillars provided for supporting the interferometer tubes were somewhat shortened and then fitted with thin adjustable plates of non-conducting material. In these plates were cut the necessary V grooves for the reception of the compound tubes or jackets (*vide supra*).

Given the above-described apparatus and confining ourselves to temperatures not greatly removed from that of the laboratory, we can readily set up and maintain any one of a series of small and constant temperature-differences as between and

within the two compound interferometer chambers. That this is so, will at once be seen from the following considerations.

Referring again to figure 1, let a stream of water constant in temperature and velocity pass through the spiral A in the direction shown. Under these conditions the extremities of the enclosed copper tube attain certain invariable temperatures θ_1, θ_2 θ_1, θ_2; and for small differences the gradient may be represented by ab, figure 2, α. When the other spiral also is used alone, and the experiment repeated by passing the water in at B, the temperature-gradient will be cd. On adding the opposed but precisely equal gradients, we obtain the result represented by the dotted graph mn; whence it follows that by operating the two spirals simultaneously and in opposite directions, we attain the desired end and secure uniformity in temperature throughout the entire length of the interferometer chamber. This is for small differences dt. But when the difference $\theta_1 - \theta_2$ is large, the result is that represented in figure 2, β. Here it is seen that for the central section of the tube the temperature is lower than that of the ends. When, however, $\theta_1 - \theta_2$ does not exceed 5° C., any inequality of temperature becomes, owing to the high conductivity of copper, vanishingly small and altogether negligible. In the absence of the copper

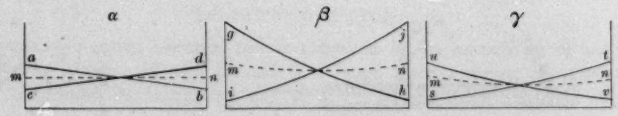

Fig. 2.

tube, the distribution of temperature along the operating spirals is, for the small differences under consideration, represented as in figure 2, γ. The plan adopted for realizing this theoretical uniformity and constancy in the temperatures of the interferometer chambers was as follows.

First, two thermostats were set up and conveniently placed, the one for serving the twin spirals of the first chamber and the other for serving those of the second. The temperature of each thermostat was indicated by a Baudin thermometer graduated in $\frac{1}{50}$° C., and easily readable to $\frac{1}{200}$° C. With the aid of regulators charged with xylene, the usual electrical devices, and centrifugal stirrers, the two temperatures were maintainable within limits approximating to ± ·005° C.

Secondly, each thermostat T was fitted with two siphons p, p of "compo" tubing, and an air injector J as shown diagrammatically in figure 3. The inlets of the twin spirals were connected, the one with the first siphon and the other with the second, whilst the outlets were similarly attached to the tubes t, t of the injector. The vertical limb L was led away and joined to the air compressor shown in the explanatory figure 4.

Thirdly, in each case the temperature of the thermostat was adjusted and the apparatus made ready for action by first closing the stopcock c and the side-tube s, and then applying suction to the tube e. The siphons were thus activated and the spirals filled with water.

Lastly, a stream of air governed by the tap *c* was admitted from the compressor; and this on its way through *J* caused an intermittent flow of water through the spirals.

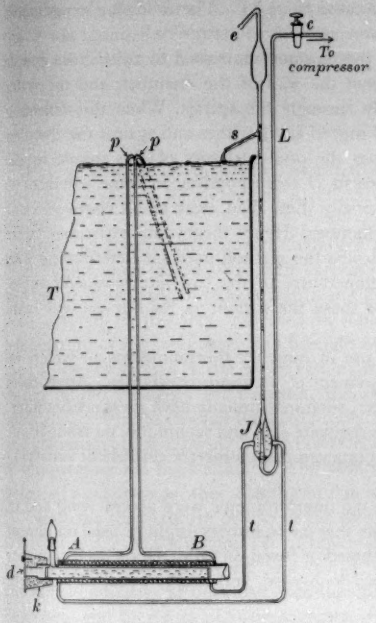

Fig. 3.

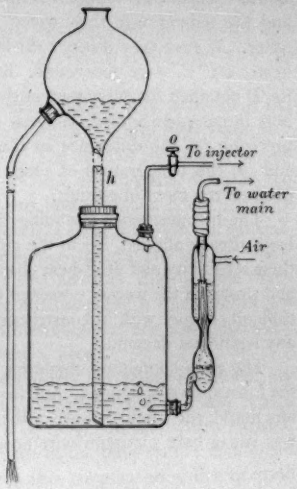

Fig. 4.

It may be noted that the temperature to which an interferometer chamber is raised depends largely upon the rate of flow; the maximum increase being obtained when the rate also is a maximum. Thus for each given rate there is a corresponding constant temperature. In illustration of this, the following data are here tabulated:

Air stream	Temperature of thermostat (° C.)	Temperature of chamber (° C.)	Temperature of laboratory (° C.)
Passing slowly	19·7	15·7	13
,, moderately slowly	19·7	16·5	13
,, rapidly	19·7	17·7	13

The apparatus having been completed, it was deemed advisable to obtain experimental proof of the correctness of the conclusions, reached theoretically, as to the uniformity of the temperature within the interferometer chambers.

Accordingly use was made of a thermoelectric couple consisting of fine wires of nickel and iron, and a low resistance reflecting galvanometer. The couple having been standardized, the galvanometer and its scale were so adjusted that the minimum reading obtainable corresponded to $\frac{1}{100}$° C. The following experiments were then carried out.

The temperature of one of the thermostats was raised to 19° C., the couple was placed centrally within and against the wall of the chamber, and the water was passed continuously and rapidly through the spirals. When the deflection
D *D* was constant, the couple was slowly moved along within and against the chamber and the temperature thus tested from the one extremity of the spirals to the other. At first very distinct variations in *D* corresponding in some instances to nearly 0·1° C. were observable. But finally these died away and a common value for *D* obtained for every part of the chamber. Hence theory and experiment were here in complete accord. We now describe the method used for determining with precision (*a*) the difference in the temperature of the two interferometer tubes, and (*b*) the temperature of each. Of these the former is for our present main purpose the more important.

The inconvenience attending the use of mercury thermometers, to which we have already alluded, led to the employment of platinum resistances. Apart from their sensitivity and dead-beat character, such instruments have great adaptability; and provided the worker possesses the requisite skill and technique, no insuperable difficulty is met with in constructing platinum thermometers capable of satisfying any legitimate demand.

The glass containing chambers of the interferometer were 20 cm. long and of the form shown in figure 5. And in order that the chambers might be used for almost any liquid, the windows were of the adhesed or fused-on type. Upon each chamber was laid a bare platinum wire arranged loop-wise, and nearly equal in extent to

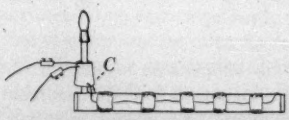

Fig. 5.

the length of the chamber. To the ends of the wire, which was 0·03 mm. in diameter, gold leads were fused; and these received ample support from a large cork *C* through which they were threaded. The arranged platinum loop was, as shown in the figure, secured to the chamber by means of narrow white ribbon, the ends of which were fastened by "Durofix" cement.

Prior to the fixing *in situ* of the thermometers, data for the calculation of their constants were obtained by successively measuring the resistance *R* of one of the wires when the wire was placed (*a*) in melting ice, (*b*) in steam, and (*c*) in sulphur vapour. It was thus found that in the expression $R_t = R_0(1 + \alpha t - \beta t^2)$, $\alpha = 3926 \times 10^{-6}$ and $\beta = 5956 \times 10^{-10}$. (Incidentally, these values indicate that

the wire was, as stated by the maker, "exceptionally pure.") We observe that for small changes in temperature the influence of the constant β is wholly negligible.

At 0° C., the resistance R_0 of thermometer No. 1 was 6·5824 Ω, whilst that of No. 2 was 6·6349 Ω. At 1° C. the resistance of No. 1 was increased by 0·0263 Ω. By trial it was found that variations in R as small as 4×10^{-5} Ω were determinable. Hence a difference in temperature equal to 0·002° C. was detectable.

§ 3. PROCEDURE

Having now set forth certain essential details, we next briefly describe the procedure observed in measuring a refractivity temperature coefficient. It is as follows.

First, the glass interferometer chambers are each charged with portions of the given liquid, then closed with their capsules and introduced into their respective and closely-fitting water-jackets shown in figure 1. The ends of the jackets having been covered in the way indicated in figure 3*, the thermostats, both at the temperature of the laboratory, are brought into use, and the water is circulated in the two sets of spirals. Later, observations are taken at short intervals. In general, owing to the changing temperature, the interference fringes are at first hazy, somewhat indefinite or even markedly wavy. Finally, however, they become sharp and stationary, thus showing in the most decisive manner possible that equilibrium of temperature has been attained. The platinum thermometers are now electrically balanced against each other and the bridge reading r_1 noted.

Next, the temperature of one of the two thermostats is suitably increased, say by 3° or 4° C., and a second set of observations carried out and the new bridge reading r_2 obtained. Knowing now the value of $(r_1 - r_2)$, we can, with the aid of the allied constants, at once calculate the difference in the temperature of the two chambers and therefore of their contents. Given this temperature-difference and the corresponding shift in the fringes, the sought-for refractivity temperature coefficient may then be found†.

When the value of a coefficient k is required for temperatures higher than those indicated above, both thermostats are first set for the chosen minor temperature and brought under automatic control. The necessary data having been obtained, the temperature of one of the two thermostats is then increased by some 3° or 5° C., and a second set of observations is carried out as already described: the value of k may then be calculated.

* The caps of the copper tubes are of cork covered with tinfoil, and the pivoted shutter of polished aluminium. The success of the device depends, as will be seen, upon the high non-conductivity of cork and the high reflectivity of the two metals for heat.

† The success of this plan, in which the platinum thermometers are used differentially, largely depends upon a fairly close equalization of the two resistances r_1, r_2. Failing this, the ratio r_1/r_2 will vary as the common temperature varies. In the present instance $(r_1 - r_2) = 0{\cdot}05\ \Omega$; and the ratio $r_1/r_2 = 1{\cdot}007976$. For a range of 10° C. this ratio is, as calculation shows, quite unaffected.

§ 4. CONCLUSION

It will be seen that the differential method now advanced and detailed is theoretically, entirely correct. For although the temperature of the cooler thermostat may and probably does to some extent vary whilst its companion is being warmed, the only effect is one that either increases or diminishes the final difference in the temperature of the two interferometer chambers. That this is so follows from the fact that the ratio of the two thermometer resistances is, as we have shown, invariable.

When circumstances are such that a knowledge of the actual temperature of each chamber is indispensable, the resistance thermometers are, for convenience, joined to the usual measuring apparatus by an appropriate mercury switch, and the necessary data obtained by well-known methods.

In a future communication our purpose will be to give an account of an investigation for which the interferometer dealt with here has been designed and built.

In conclusion, it may be remarked that the apparatus used for maintaining the two interferometer chambers at various constant temperatures is equally applicable to the tubes of polarimeters and the like.

A METHOD FOR DEDUCING ACCURATE VALUES OF THE LATTICE SPACING FROM X-RAY POWDER PHOTOGRAPHS TAKEN BY THE DEBYE-SCHERRER METHOD

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ABSTRACT. A method has been tried for obtaining accurate values of lattice spacings from X-ray powder photographs taken in the usual circular type of camera. There are two essential features of the method, (*a*) the calibration process, (*b*) the extrapolation process.

The calibration process: The exposed portion of the film is limited by sharp knife edges, and the length S_k is measured for each film at the same time as the distance S between corresponding pairs of lines. S_k corresponds to an angle θ_k in the same way that S corresponds to a glancing angle θ, so that $\theta_k = S_k/4R$, where R is the radius of curvature of the film. R is an uncertain quantity depending on the amount of film-shrinkage, and so in calculating θ from S we replace R by θ_k, which is a constant for a particular camera. θ_k having been determined in a preliminary experiment, it is possible to calculate θ for a given reflection, from the formula

$$\theta = \frac{S}{S_k}\theta_k.$$

Errors due to film-shrinkage are hereby eliminated.

The extrapolation process: Other errors due to absorption by the specimen and eccentricity of the specimen may be eliminated by plotting the values of the lattice spacing calculated from a given pair of lines, against the corresponding values of $\cos^2\theta$. For small values of $\cos^2\theta$, the curve is almost linear, and is easily extrapolated to $\cos^2\theta = 0$, where the correct value of the lattice spacing is to be found. Only a few accurate measurements at angles where $\cos^2\theta$ is small are necessary in order to carry out this process.

As examples of the method, results are given for the lattice spacing of iron taken in different cameras with specimens of different diameters. They are consistent to 1 part in 15,000, the mean value being 2·8605. A specimen of electrolytic nickel (3·5162 Å.) was found to give a different value from a specimen of Mond nickel which had been degassed (3·5170 Å.).

§ 1. INTRODUCTION

It is often necessary to obtain accurate values of lattice spacings, while at the same time a complete powder photograph is required. If the Debye-Scherrer method is used with the usual type of circular camera, it is difficult to make sure of the accuracy of the results since there may be a large number of systematic errors. It is possible to correct for each known source of inaccuracy, but in practice this is troublesome and the results are uncertain. A more satisfactory method has been described by Kettmann*. A value of the lattice spacing is calculated for each line on the film. These values are plotted against the corresponding value of the

* G. Kettmann, *Z. f. Phys.* **53**, 198 (1929).

θ glancing angle θ, and a smooth curve is drawn through the points so obtained. On extrapolating this curve to $\theta = 90°$ the true value of the spacing is obtained.

The form of the Kettmann curve is somewhat uncertain as it depends on the nature of the errors, but under certain conditions it is possible to reduce the number of errors to such an extent that the process of extrapolation is no longer arbitrary. The present paper describes a method of calibration which in many instances removes the need for correction. Any systematic errors which may occur can be easily eliminated by plotting the values of the spacing against $\cos^2\theta$, whereupon the process of extrapolation is greatly simplified, as will be seen from the following account.

§ 2. THE METHOD OF CALIBRATION

In the Debye-Scherrer type of camera the powder specimen to be examined is placed at the centre of the camera, and the diffracted beams spreading out from this specimen strike the cylindrical film wrapped round the circumference of the camera. The object of the experiment is to determine the angle (2θ) between the direction of the incident beam and that of the diffracted beam, and so to find the

a lattice spacing (a) of the powdered crystal. For a cubic crystal the lattice spacing is finally calculated from the formula

$$a = \lambda\sqrt{(h^2 + k^2 + l^2)}/2 \sin\theta \qquad \ldots\ldots(1),$$

λ where λ is the wave-length of radiation used and θ is the glancing angle of incidence

h, k, l for the plane (hkl).

From the above equation it may be seen that the accuracy with which it is possible to determine a depends on the knowledge of λ and evaluation of θ. The accuracy with which λ is known is in most cases extremely high owing to the work of Siegbahn* and others. For this reason the accuracy of a is determined almost

S entirely by θ, which may be obtained from a measurement of the distance (S) between corresponding pairs of lines at opposite ends of the photographic film,

R and by ascertaining the film radius R. Then

$$S = 4R\theta \qquad \ldots\ldots(2).$$

It is difficult to apply the above formula directly since the value of R which satisfies it varies with each photograph, depending on the thickness of the photographic film and the manner in which the film is handled.

This difficulty is especially enhanced by the uncertain amount of contraction which occurs during the handling of the film, from the time it is placed in the camera until the time it is measured, this contraction being equivalent to a reduction in R. The uncertainty is eliminated by using a method of calibrating the film which permits the replacement of a fixed value R by a value which allows for the contraction in each case. Two types of method are available. In *the mixture method*†

* Siegbahn, *Spektroskopie der Röntgenstrahlen* (1931).

† Havighurst, Mack and Blake, *J. Amer. Chem. Soc.* **46**, 2368 (1924). F. C. Blake, *Phys. Rev.* 2, 26, 60–70 (1925).

the powder is mixed with a substance of known lattice constant. The relation between the spacings and measured values of S for the standardizing substance is plotted graphically, and the values for the substance under investigation may be read off. This has the advantage of eliminating all systematic errors simultaneously, but the film is less easy to interpret.

As an alternative *the circumference of the camera may be marked in some way to correspond to definite values of* θ. These marks then provide a means of correlating values of θ and S without requiring a direct determination of R.

We have employed a modification of this method which will now be described. The camera is provided with two stops A and B, figure 1, which provide permanent points of reference. These stops form the two ends of the exposed portion of the film. The length S_k of the exposed portion of the film AYB corresponds to an angle $4\theta_k$, defined by

S_k
θ_k

$$S_k = 4R\theta_k \qquad \text{.....(3),}$$

R being the "virtual radius" of the camera as defined by equation (2). It is less than the true radius owing to film shrinkage. Combining equations (2) and (3) we get

$$\frac{\theta}{S} = \frac{\theta_k}{S_k} \qquad \text{.....(4).}$$

Instead of the uncertain value R, we now have the fixed angle θ_k, which is a constant for a particular camera. The value of S_k is carefully measured on every film at the same time as S. It is then possible to obtain θ from S, provided θ_k is known, and the film shrinkage is automatically allowed for.

The calibration of the camera thus consists in determining the angle θ_k. This is a constant for a particular camera, and need only be determined once. It is most important to ascertain this angle with accuracy, since it enters into all further work. For this purpose two methods have been used.

In the first method the camera is measured up directly. We require the ratio of the arc AYB to the radius R. It is more accurate to measure the smaller arc AXB, figure 1. Then

$$\theta_k = (\tfrac{1}{2}\pi - AXB/4R).$$

Since θ_k exceeds 80°, the percentage error in θ_k is far less than the percentage error in the measurement of AXB. Also, since AXB is far smaller than $2R$, the accuracy of AXB ultimately determines the accuracy of θ_k.

Alternatively, a photograph may be taken of a substance of known lattice spacing, and the positions of the lines on the film can then be measured, together with the length S_k. Each line corresponds to a given value of θ and we may thus calculate a series of values for θ_k by means of equation (4). This method would, under suitable conditions, be the most satisfactory way of obtaining the value of θ_k. In practice difficulties may arise, which sometimes makes it easier to use the first method. It has been found that the results obtained by the two methods agree in general to about 1 part in 4000 for a camera of diameter 9 cm.

Table 1 gives the data from which a camera was calibrated by the second method. Pure chemically prepared NaCl was used as the standardizing substance. This has been shown by Goldschmidt* to have a lattice spacing 5·626 Å. which is smaller than the value 5·628 Å. accepted for rocksalt. The value 5·626 Å. gives a constant series of values for θ_k. We show in the next section that this is an important confirmation of the correctness of the value. The value of θ_k found by this means was 80·945, which agrees closely with the value 80·96 obtained by the first method.

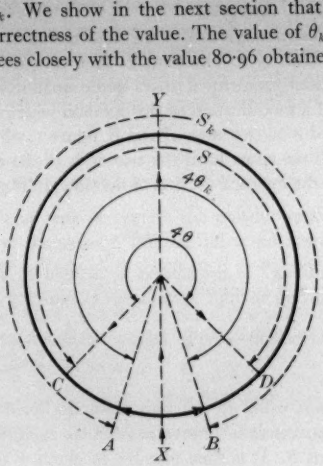

Fig. 1. Illustrating method of calibration.

Table 1. Calibration by NaCl (spacing 5·626 Å.)

Σh^2	*hkl*	Radiation	S (observed)	θ (calculated)	θ_k or $\frac{S_k}{S}\theta$	Mean value of θ_k for each α doublet
End of film	—	—	25·651 (S_k)	—	—	—
52	640	α_1	25·394	80·135	80·946	80·946
51	{711 / 551}	α_2	24·718	77·999	80·943	80·945
		α_1	24·509	77·342	80·946	
48	444	α_2	22·699	71·612	80·925	80·941
		α_1	22·555	71·185	80·956	
44	622	α_2	20·695	65·304	80·943	80·945
		α_1	20·596	64·994	80·946	
40	620	α_2	19·029	60·027	80·916	80·947
		α_1	18·936	59·779	80·977	
					Mean:	80·945

The method of calibration just described eliminates errors due to film shrinkage and simultaneously fixes a point on the camera for which the glancing angle is known with extreme accuracy. This point is chosen at an angle in the region where

* V. M. Goldschmidt, *G.V.* **8** (1927).

the α doublet is well resolved, and near a point for which there are in many instances several lines suitable for measurement. Naturally where there is a choice of radiations available, a wave-length will be selected which gives a reflection close to this reference point. For example, with α iron using Co radiation we get K_α lines for (310) at 80° or 81°. Hence with a reference point at about 82° extremely accurate measurements of lattice spacing are obtainable merely by the use of this one doublet. In some cases it is not possible to obtain lines so near the end of the film. For example, copper with Copper K_α radiation does not give an α reflection beyond 73°. In this case the measurement may be influenced by various sources of error, which, if allowed to go unchecked, would vitiate the accuracy of the results. A brief account will be given of some of the causes of trouble, and it will be shown that a simple and straightforward method of treatment which practically eliminates every difficulty may be followed.

§ 3. SOURCES OF ERROR IN DETERMINING LATTICE SPACINGS

The possible sources of inaccuracy may be deduced from equation (1). We see that if da is the error in determining a, da

$$\frac{da}{a} = \frac{d\lambda}{\lambda} - \cot\theta\, d\theta \qquad \text{.....(5),}$$

where $d\lambda$ is the error in λ, and $d\theta$ the error in θ. Assuming $d\lambda$ to be negligible, $d\lambda$, $d\theta$ we have

$$da/a = -\cot\theta\, d\theta \qquad \text{.....(6).}$$

$d\theta$ can be found from equation (4), as follows,

$$\frac{d\theta}{\theta} = \frac{d\theta_k}{\theta_k} - \frac{dS_k}{S_k} + \frac{dS}{S} \qquad \text{.....(7),}$$

where $d\theta_k$, dS_k and $d\theta$ are the errors in θ_k, S_k and θ respectively. $d\theta_k, dS_k,$

The errors in θ_k arise in the process of calibration. It has been shown that in the first method of calibration by measuring the arc AXB, figure 1, the value of da is virtually proportional to dS_k alone, the exact value found for the radius being comparatively unimportant.

In the second method of calibration, where a standard substance is employed, there are possibilities of systematic errors. It is possible that a somewhat incorrect lattice spacing may have been assumed for the substance. This error may arise if NaCl is used, since the chemically prepared compound has not the same spacing as natural rocksalt. The effect of this error is shown in table 2, in which we have used an incorrect value (5·632) for the lattice spacing, obtaining a series of values for θ_k which vary considerably. In figure 2 we have plotted the value of θ_k against the appropriate value of θ for each line on the film. This has been done for various values of a. Only in one case do we get a constant series of values for θ_k, viz. when the correct value of the spacing has been employed. This provides a useful check on the method.

Table 2. θ_k calculated from an incorrect value of the lattice spacing (5·632)

Σh^2	hkl	Radiation	S (observed)	θ (calculated)	$\frac{\theta_k}{S}$ or $\frac{S}{\theta}$	Value of θ_k for each α doublet
End of film	—	—	25·651	—	—	—
52	640	α_1	25·394	79·790	80·598	80·60
51	{711 / 551}	α_2	24·718	77·716	80·649	80·66
		α_1	24·509	77·073	80·664	
48	444	α_2	22·699	71·429	80·718	80·74
		α_1	22·555	71·006	80·753	
44	622	α_2	20·695	65·172	80·779	80·78
		α_1	20·596	64·864	80·784	
40	620	α_2	19·029	59·921	80·773	80·80
		α_1	18·936	59·674	80·835	

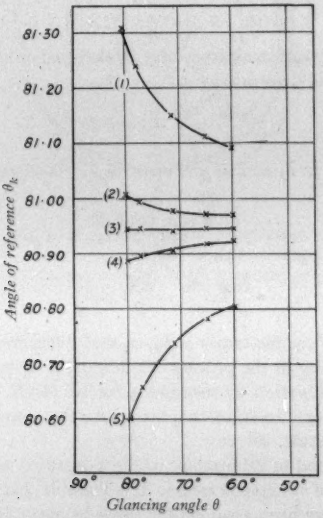

Fig. 2. Assumed lattice spacing: (1) 5·620; (2) 5·625; (3) 5·626; (4) 5·627; (5) 5·632. The correct, constant, value of θ_k is given by (3).

It is of course essential to ensure that no errors be introduced due to bad centering of the specimen, absorption, etc. Each of these errors tends to make θ_k inconstant, but the nature of the variation of θ_k depends on the type of error. By using a very thin specimen of non-absorbent material such as NaCl the absorption difficulty may be eliminated, and it is also advisable to check the results by the first method.

With the best possible precautions, the limit of accuracy for θ_k is set by S_k, and we may replace $d\theta_k/\theta_k$ by dS_k (calibration)/S_k (calibration), which is the error introduced in the actual measurement of S_k during calibration. Equation (7) may now be written:

$$\frac{d\theta}{\theta} = \left(\frac{dS_k \text{ cal.}}{S_k \text{ cal.}} - \frac{dS_k}{S_k}\right) + \frac{dS}{S} \qquad \text{.....(8).}$$

The expression in brackets represents the uncertainty in the position of the reference points.

The errors in S represented in equation (8) are of varying degrees of importance. Individual errors may arise from bad readings due to faint or fuzzy lines. These may be greatly minimized by measuring up a number of lines, but in the case of systematic errors another method of treatment must be adopted. Before describing this method we may first mention two points of minor importance, which may nevertheless in extreme cases affect the accuracy of the results.

The length of specimen in the beam influences the breadth of the line, causing a broadening of the lines at both small and large glancing angles. If measurements are made to the centre of blackening of the line, we get the two following expressions:

$$d\theta = -\frac{1+x}{96}\left(\frac{h}{R}\right)^2 \cot 2\theta \qquad \text{.....(9),}$$

$$\frac{da}{a} = \frac{1+x}{192}\left(\frac{h}{R}\right)^2 (\cot^2\theta - 1) \qquad \text{.....(10),}$$

where h is the vertical divergence of the reflected beam, R is the radius of the camera, and x is a fraction between 0 and 1 depending on whether the beam fades at the edges or is uniform. *h, R* *x*

The expression $(\cot^2\theta - 1)$ varies from infinity at $\theta = 0°$ to zero at $\theta = 45°$, when it changes sign and reaches the value -1 at $\theta = 90°$. It will be seen that if h/R is small the error is negligible except at very low angles. With fairly large values of h/R the calculated lattice spacing will be slightly depressed at high angles. It is important to keep down the value of h/R, in order to avoid this effect.

If the film shrinks unevenly or is not placed correctly in the camera, errors are introduced which may be considerable near the centre of the film, but will vanish at angles near the fixed reference point. Consequently most reliance should be placed on results obtained from measurements near this point. This is conveniently done by the method of extrapolation to be described below.

These errors are trivial compared with those due to the two main sources of inaccuracy which have now to be considered. These are (i) eccentricity of the specimen, (ii) absorption in the specimen.

(i) *Error due to eccentricity of specimen*

Let XY be the direction of the incident beam, which is assumed to be parallel, figure 3 *a*. Suppose A to be the true centre of the camera, but the specimen to have been placed at B. Let $AB = p$, and let the angle YAB be ϕ. All distances *p, φ*

on the circumference are reckoned positive away from Y. Then the displacement AB may be resolved into two components:

AB_1 or $p \cos \alpha$ in the direction of the incident beam, figure 3 *b*.

AB_2 or $p \sin \alpha$ perpendicular to the incident beam, figure 3 *c*.

It may be seen from figure 3 *b* that the displacement AB_1 introduces an error into the measured distance S between any given pair of lines. In this figure R_A and R_A' are the correct positions of the lines; they are displaced to R_B and R_B' by the displacement AB_1 of the specimen. Then if dS is the error in S,

dS

$$dS = 2R_A R_B = -2AB_1 \sin 2\theta$$
$$= -2(p \cos \phi) \sin 2\theta.$$

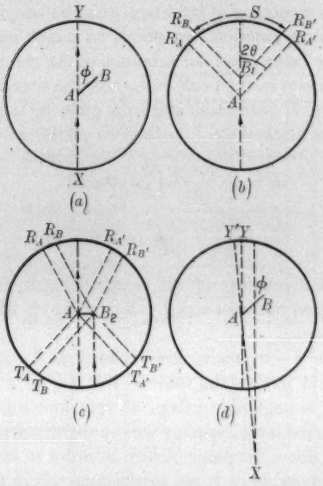

Fig. 3.

Hence
$$d\theta = \frac{\theta}{S} dS = -\frac{(p \cos \phi) \sin 2\theta}{2R} \qquad \ldots\ldots(11).$$

Combining equations (11) and (6)

$$da/a = (p \cos \phi / R) \cos^2 \theta \qquad \ldots\ldots(12).$$

A displacement of the specimen at right angles to the incident beam, figure 3 *c*, has the effect of crowding the lines together on one side of the film and spreading them out on the other side, but it has no effect on the value of S.

If the beam is divergent, figure 3 *d*, the same relations hold provided that ϕ is taken to mean the angle between the incident beam passing through B, and the direction of the displacement AB, i.e. $Y\hat{A}B$, not $Y'\hat{A}B$.

Equation (12) *therefore represents the relative error in lattice spacing introduced by a displacement of the specimen* by a distance p from the centre of the camera of radius R, while ϕ gives the direction of the displacement relative to the incident beam.

(ii) *Error due to absorption in the powder and divergence of the beam**

It is simplest to make the calculation for complete absorption. For partial absorption the effect is more complicated. In general it will result in the production of a line of varying intensity, the blackness diminishing from the outer edge of the line towards the inner edge. When the absorption coefficient is low it is convenient to measure to the centre of the line and make no correction. As the absorption coefficient increases the line becomes blacker on the outside and lighter on the inside. Finally, with very absorbent powders, only the outer portion of the line is visible, and measurements must be corrected.

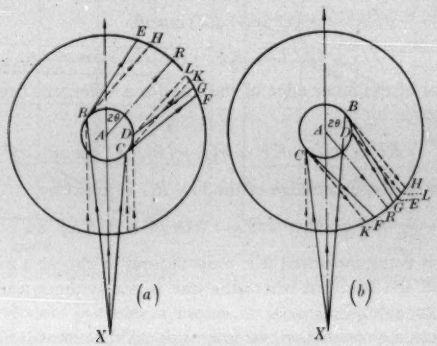

Fig. 4. Effect of absorption.

Figure 4 illustrates the effect of the error. Figure 4 (*a*) gives a small value of θ, figure 4 (*b*) a larger value. The specimen is immersed in a beam diverging from X. With a point specimen situated at A, the beam is deviated through an angle 2θ in the direction of AR, giving rise to a line on the film at R. With a specimen of finite size the ray AR is broadened to a beam, whose boundaries are defined by the rays BE and CF.

Owing to the effect of absorption, the intensity is not distributed symmetrically over the length EF, the greatest intensity being near the point F. If the substance is extremely absorbent the reflected beam is confined to a portion scattered from a narrow strip CD on the surface of the specimen, and the boundary rays are CF and DG. The beam strikes the film between F and G, and the point of measurement of the line will lie at some point between F and G, say at P. In order to estimate the error in the measured value of the lattice spacing, it is necessary to find the

* Compare O. Pauli, *Z. f. Krist.* **56**, 591 (1921); Hadding, *Centralblatt f. Min. etc.* p. 631 (1921). Hadding's expression for ds may be written $r(1 + \cos 2\theta)$.

distance RP. This is best done in two stages, a parallel beam being first considered and then the effect of divergence added.

A parallel incident beam would give rise to a reflected beam bounded by the rays BH and CK, where

$$HK = BC = 2r,$$

r being the radius of the specimen. Absorption limits the beam to the area bounded by the rays CK and DL, where $C\hat{A}D = 2\theta$, so that

$$RK = r, \quad RL = r\cos 2\theta.$$

The additional effect of divergence swings the rays CK and DL round to CF and DG respectively, where

$$G\hat{D}L = A\hat{X}D = r/AX,$$

$$F\hat{C}K = A\hat{X}C = (r/AX)\cos 2\theta.$$

Hence

$$LG = rR/AX, \quad KF = (rR/AX)\cos 2\theta.$$

The displacement of the outer edge of the line for a divergent beam RF can now be found. It is given by

$$RF = RK + KF = r\{1 + (R/AX)\cos 2\theta\} \qquad \text{.....(13).}$$

The displacement of the inner edge of the line RG is given by

$$RG = RL + LG = r(\cos 2\theta + R/AX) \qquad \text{.....(14).}$$

When a line is being measured it is most natural to choose a point somewhere near the centre of the line, but where the line is not symmetrical there is a bias towards the blacker side.

As a convenient approximation, we may suppose that measurement is made to the centre of gravity of blackening of the line. This is approximately at P, where

$$RP = \frac{r\sin 2\theta}{2\theta}\left(1 + \frac{R}{AX}\right) \qquad \text{.....(15).}$$

This is the most probable displacement of the measured position of the line. We deduce from this expression the following formulae for the errors involved in determining S, θ and a:

$$dS = 2RP = \frac{r\sin 2\theta}{\theta}\left(1 + \frac{R}{AX}\right),$$

$$d\theta = \frac{dS}{4R} = \frac{r\sin 2\theta}{4\theta}\left(\frac{1}{R} + \frac{1}{AX}\right) \qquad \text{.....(16),}$$

$$\frac{da}{a} = -\cot\theta\, d\theta = -\frac{r\cos^2\theta}{2\theta}\left(\frac{1}{R} + \frac{1}{AX}\right) \qquad \text{.....(17).}$$

§ 4. GRAPHICAL TREATMENT OF ERRORS

On comparison of equations (12) and (17), it will be seen that the factor $\cos^2\theta$ is common to both, so that the errors due to eccentricity and absorption are very similar in character. We may write a combined expression:

$$\frac{da}{a} = \left(\frac{p\cos\phi}{R} - \frac{r}{2\theta R} - \frac{r}{2\theta AX}\right)\cos^2\theta \qquad \text{.....(18).}$$

If da be plotted against $\cos^2\theta$ an approximately linear relationship will be obtained at large values of θ. The same will be true for the values of a deduced from the film measurement. In figure 5 the measured values of a are plotted for the corresponding values of $\cos^2\theta$ between 0 and 0·5. The line PQ is obtained. At P, $\cos^2\theta = 0$, so that $da = 0$. P thus gives the true value of a.

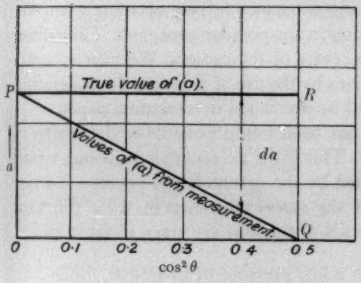

Fig. 5 (*a*). Error curve for eccentricity.

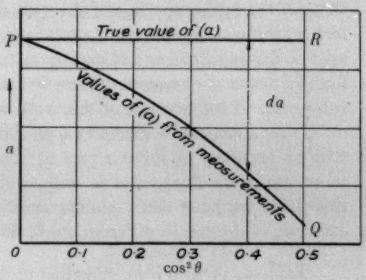

Fig. 5 (*b*). Error curve for absorption.

The above process provides a means of ascertaining the correct value of a without further calculation. The degree of accuracy with which the point P can be found depends chiefly on whether there are a sufficient number of points on the line PQ in the neighbourhood of P. If there are lines only near the region where $\theta = 0°$ and $\cos^2\theta = 1$, the position of P will be altogether uncertain. In practice a few lines in the neighbourhood of P serve to give its position with great accuracy, the lines nearer the centre of the film merely serving as pointers.

We therefore have the following rule for correcting the effects of eccentricity of specimen and absorption in the powder rod. *Plot the observed values of lattice spacing against the corresponding values of* $\cos^2\theta$. *An approximately straight line should be obtained. Extrapolation to* $\cos^2\theta = 0$ *gives the correct value of the lattice spacing.*

§ 5. DISCUSSION AND EXAMPLES

Provided all necessary precautions are taken, this method of determining lattice spacings is accurate and reliable. The essential principles of the method are as follows:

(i) In order to calculate the glancing angle θ from the measurements S of the

film, we make use of a fixed angle θ_k, figure 1, corresponding to a given length of the film S_k, and apply equation (4) as follows:

$$\theta = \frac{\theta_k}{S_k} S.$$

By this means the uncertainty in measuring the radius is eliminated.

(ii) The values of the lattice spacing obtained from different lines on the film are plotted graphically against the corresponding values of $\cos^2\theta$, and the true value of the lattice spacing is then obtained by an almost linear extrapolation to $\cos^2\theta = 0$.

The fixed angle θ_k is found either by direct measurement of the camera, or by the use of a calibrating substance. The former method has the advantage that it involves no systematic error, but the latter is found to be more accurate provided that (*a*) the lattice spacing of the calibrating substance is very accurately known, and (*b*) there are no errors due to absorption in the powder specimen. Difficulties arise if the specimen is not exactly at the centre of the camera. We have recently found a means of overcoming these limitations by the use of quartz as the calibrating substance. A full account of this work will be published in a separate paper.

Using a camera of diameter 9 cm., it has been found possible to determine θ_k with an accuracy of about 1 part in 8000. This gives an accuracy of about 1 part in 30,000 in the final value of a determined by the extrapolation process. To test this point we have taken photographs of the same substance in three different cameras, calibrated in different ways. Details of this test are given in table 3.

Table 3. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation

hkl	Radiation	*S*	θ	$\cos^2\theta$	Lattice spacing
211	α	15·893	50·01	0·41292	2·8557
220	α_1	19·706	62·015	0·22015	2·8588
	α_2	19·788	62·275	0·21643	2·8586
310	α_1	25·643	80·70	0·02610	2·8603
End of film	—	25·700 (S_k)	80·88 (θ_k)		
				Extrapolated value:	2·8605

Table 4. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation. Thin specimen, diameter 0·3 mm.

hkl	Radiation	*S*	θ	$\cos^2\theta$	Lattice spacing
211	α	15·830	49·92	0·41454	2·8595
220	α_1	19·648	61·96	0·22098	2·8605
	α_2	19·727	62·205	0·21744	2·8603
310	α_1	25·583	80·675	0·02626	2·8606
	α_2	25·835	81·47	0·02200	2·8606
End of film	—	26·164 (S_k)	82·51 (θ_k)		
				Extrapolated value:	2·8606

Table 5. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation. Thick specimen, diameter 1·5 mm.

hkl	Radiation	*S*	θ	$\cos^2\theta$	Lattice spacing
211	α	15·921	50·10	0·41145	2·852
220	α_1	19·725	62·075	0·21931	2·8575
	α_2	19·800	62·31	0·21594	2·8576
310	α_1	25·652	80·725	0·02597	2·8602
	α_2	25·905	81·52	0·02174	2·8602
End of film	—	26·220 (S_k)	82·51 (θ_k)		
				Extrapolated value:	2·8605

Table 6. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 4. Cobalt K_α radiation

hkl	Radiation	*S*	θ	$\cos^2\theta$	Lattice spacing
211	α	15·929	49·84	0·41590	2·8629
220	α_1	19·786	61·91	0·22171	2·8619
	α_2	19·862	62·145	0·21831	2·8619
310	α_1	25·783	80·675	0·02626	2·8606
	α_2	26·041	81·48	0·02197	2·8606
End of film	—	26·457 (S_k)	82·78 (θ_k)		
				Extrapolated value:	2·8604

Tables 3, 4, 5 and 6 show the results obtained from a specimen of Armco iron, remelted under a low pressure of hydrogen, the filings being annealed *in vacuo*. A photograph is given in figure 8. The absolute magnitude* of these values is not of special importance, since the iron is probably not absolutely pure, but the main point is that measurements made in different cameras give almost identical values for the lattice spacing on extrapolation to $\cos^2\theta = 0$, in spite of differences in the size and centering of the powder rod. These differences give rise to considerable variation at large values of $\cos^2\theta$, as may be seen from figure 6 where the values of the lattice spacing from tables 3 to 6 are plotted against the corresponding values of $\cos^2\theta$. The curves are wide apart when $\cos^2\theta$ is large, and come together as $\cos^2\theta$ approaches zero. Curve 2 was obtained from a fairly thin specimen, taken in camera 2; the effects of absorption and eccentricity are, in this instance, additive. Curve 3 *a* was from a fairly thin specimen, taken in camera 3; absorption and centering effects almost cancel. Curve 3 *b*, from the same camera, shows the effect of using a very thick specimen. Curve 4 was obtained from a fairly thin specimen in camera 4; the eccentricity error here acts in the opposite direction to the absorption error, and in fact quite outweighs it.

* Some previous values found for iron are: 2·861 Å., A. Westgren, *J. Iron and Steel Inst.* **1**, 383 (1928). 2·8603 ± 0·0002 Å., F. C. Blake, *loc. cit.* 2·8600 ± 0·0005 Å., Preston, *Phil. Mag.* **13**, 419 (1932). 2·8607 Å., G. Phragmèn, *J. Iron and Steel Inst.* **123**, 465.

The next examples show how the method may be used for the accurate comparison of specimens which differ only slightly in spacing. The materials compared were three specimens of nickel*: (i) a tiny specimen of electrolytic nickel as deposited on a copper wire; (ii) Mond nickel remelted under a low pressure of hydrogen and filings annealed *in vacuo*; (iii) as (ii), but comprising filings quenched from

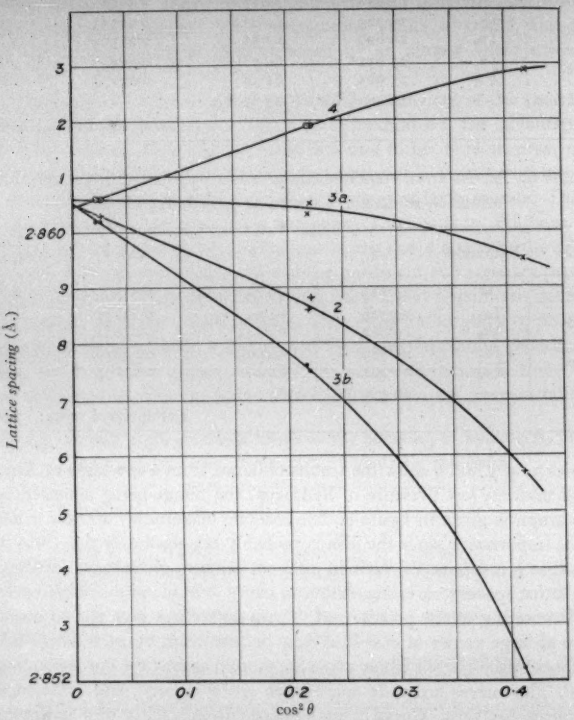

Fig. 6. Lattice spacing of iron-cobalt radiation.

700° C. after annealing. It will be seen that (ii) and (iii) give identical results, but the electrolytic nickel differs in spacing by an amount considerably greater than the experimental error. Photographs are given in figure 8.

* Some previous values found for nickel are: 3·515 ± 0·002 Å., G. Greenwood, *Z. f. Krist.* 72, 309 (1929). 3·513 ± 0·001 Å. and 3·515 ± 0·001 Å., L. Mazza and A. G. Nasini, *Phil. Mag.* 7, 301 (1929). 3·5180 Å., G. Phragmèn, *loc. cit.*

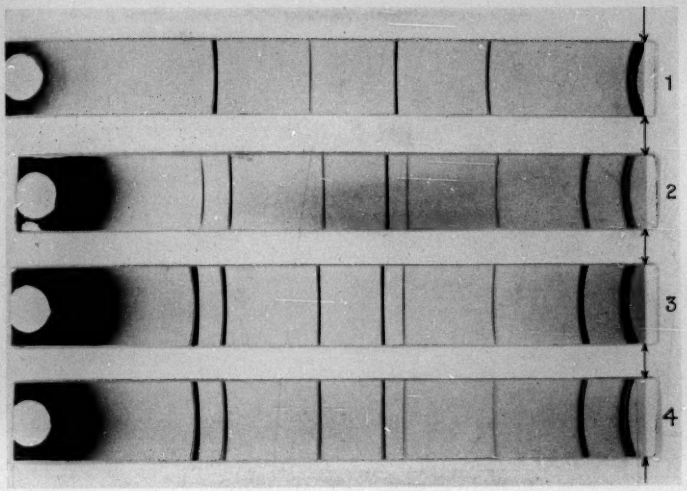

Fig. 8. Powder photographs.

(1) Iron powder annealed, Cobalt K_α radiation.
(2) Nickel electro-deposited on copper wire, Copper K_α radiation.
(3) Nickel powder annealed, Copper K_α radiation.
(4) Nickel powder quenched 700° C., Copper K_α radiation.

Marks $\downarrow$ $\uparrow$ indicate the shadow of the knife-edge which acts as a fiducial mark giving S_k and θ_k.

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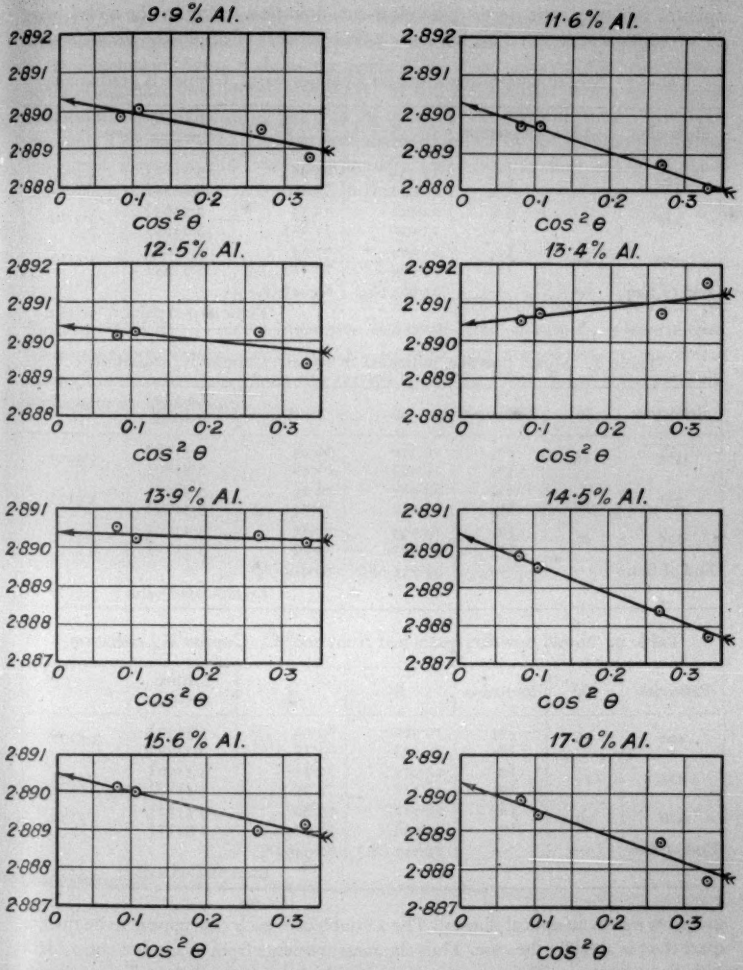

Fig. 7. Lattice spacing of iron-aluminium alloys by extrapolation.

In table 7, which gives the results from the electro-deposited nickel, an interesting point arises. The α doublets were resolved right down to (311), but in spite of this they were never quite clear-cut, doubtless owing to the nickel being in a condition of strain. This has a somewhat peculiar effect on the measurements

Table 7. Nickel electro-deposited on a copper wire. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·334	60·98	3·5162	3·5160
		α_2	19·424	61·265	3·5154	
331	19	α_1	22·940	72·35	3·5163	3·5162
		α_2	23·090	72·825	3·5159	
420	20	α_1	24·684	77·85	3·5166	3·5162
		α_2	24·924	78·61	3·5155	
End of film	—	—	25·664 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5162

Table 8. Nickel powder, annealed *in vacuo*. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·370	60·93	3·5178	3·5179
		α_2	19·452	61·19	3·5180	
331	19	α_1	22·982	72·29	3·5172	3·5173
		α_2	23·122	72·74	3·5174	
420	20	α_1	24·735	77·81	3·5169	3·5170
		α_2	24·953	78·49	3·5171	
End of film	—	—	25·731 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5170

Table 9. Nickel powder, quenched from 700° C. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·362	60·94	3·5177	3·5176
		α_2	19·443	61·20	3·5176	
331	19	α_1	22·975	72·31	3·5170	3·5171
		α_2	23·117	72·75	3·5173	
420	20	α_1	24·718	77·80	3·5172	3·5172
		α_2	24·938	78·49	3·5171	
End of film	—	—	25·717 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5170

and gives rise to an optical illusion. The α doublets in each case appear to be further apart than is actually the case. Thus the measurements from α_1 are not concordant with those from α_2. This is a phenomenon which we have observed many times, but it leads to no error if the curve for extrapolation is drawn between the α_1 and α_2 values. When this effect occurs, the values of the lattice spacing calculated from

measurement of α_1 lines are found to be greater than those from measurement of α_2 lines. Since α_1 is much stronger than α_2, the error in α_1 is likely to be less than the error in α_2, and so the curve for extrapolation is drawn nearer to α_1 than α_2. The last column of table 7 shows how the most probable value is obtained.

In conclusion, figure 7 shows the application of the extrapolation method to a series of annealed iron-aluminium alloys. It will be seen that within the range of compositions here given (9·9 per cent Al to 17 per cent Al) the lattice spacing is constant. This fact could not have been deduced except by extrapolation, since the individual measurements vary considerably from film to film owing to small differences, of the order of 0·1 mm., in the centering of the specimen.

§ 6. ACKNOWLEDGMENTS

The authors are indebted to Prof. W. L. Bragg, F.R.S., and to Mr A. P. M. Fleming, C.B.E., Director-Manager, Research and Education Departments, Metropolitan-Vickers Electrical Co. Ltd., for their kind interest and encouragement during the investigation, which was carried out in the Physical Laboratories of the University of Manchester.

IONIZATION CHARTS OF THE UPPER ATMOSPHERE

By G. MILLINGTON, M.A., B.Sc.

Communicated by Prof. S. Chapman, F.R.S., May 12, 1932. *Read July* 8, 1932.

ABSTRACT. In this paper Prof. Chapman's theory of the ionization of the upper atmosphere by solar radiation has been applied to construct a set of charts giving contour lines of equal ionic density over the surface of the earth. A simple approximate method of solving the fundamental differential equation of the theory by a rapid arithmetical process is described. Charts are drawn for winter, equinox and summer conditions for the values 0·5 and 1 of the parameter σ_0 and the value 150 of the parameter R. A brief comparison of these charts with existing empirical charts is given with a short discussion of its practical and theoretical significance.

§ 1. INTRODUCTION

It is well known that the propagation of wireless waves, especially of short waves below 100–200 metres, is intimately connected with the state of ionization of the upper atmosphere, and that this ionization goes through a well-defined diurnal cycle. It is therefore a matter of great importance in attempting to explain or to predict the behaviour of such waves to have a knowledge of the way in which the ionization varies with latitude and season and with local time, or, what comes to the same thing, to know at a given time the variation of ionic density over the surface of the earth at any given season.

The problem has been approached in a semi-empirical way by Eckersley and Tremellen*, who from a large amount of experimental data of skip distances, etc., and guided by general theoretical considerations, constructed a series of density charts for winter, equinox and summer conditions. These charts consist of a number of contour lines which divide them up into a series of grades A, A–B, B ... D, D' in order of decreasing ionic density, A being the intense daylight and D' the late night grade. They have proved very useful in studying the layer conditions for any given route and in explaining the observed variations with time and season of the average signal strengths received from stations in various parts of the world.

Now it would obviously be of great use if such charts could be constructed by purely theoretical methods since this procedure would not only help to elucidate uncertainties in the empirical charts but would also test the general soundness and adequacy of the theoretical ideas involved. Fortunately, a theory has been worked out in great detail by S. Chapman† on "the absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth," and the

* T. L. Eckersley and K. Tremellen, *Proceedings of the World Engineering Congress, Tokyo*, 29, 177 (1929).

† S. Chapman, *Proc. Phys. Soc.* **43**, 26, 483 (1931).

present paper represents the application of this theory to the above problem. Its chief purpose is to develop a general method of constructing such charts for any given values of the parameters involved in Chapman's theory, but in the practical illustrations given here values have been chosen to make the charts conform as nearly as possible to the empirical ones.

§ 2. THEORETICAL PRINCIPLES

Only a brief outline of the theory will be given here, as reference should be made to Chapman's paper for the detailed description of the notation and the development of the equations.

The rate of production I of ions per cm^3, at a place in the upper atmosphere where the sun's zenithal angle is χ, is given by the expression

$$I/I_0 = \exp\,[1 - z - \exp\,(-z).f\,(R,\,\chi)] = F(z,\,\chi) \qquad \text{.....(1),}$$

where z is a linear function of the height, and $f\,(R,\,\chi)$ is a complicated function of a parameter R and the angle χ which is discussed in great detail in part 2 of Chapman's paper.

If α is the recombination coefficient of the ions, and the value of I were to remain constant at its maximum possible value I_0, the ionic density n would reach a maximum steady value n_0 given by

$$n_0 = \sqrt{(I_0/\alpha)} \qquad \text{.....(2),}$$

so that it is convenient to define a ratio ν such that

$$\nu = n/n_0 \qquad \text{.....(3).}$$

If the time is measured in terms of longitude ϕ expressed in radians, the differential equation for determining ν is

$$\sigma_0.d\nu/d\phi + \nu^2 = F\,(z,\,\chi) \qquad \text{.....(4),}$$

where

$$1/\sigma_0 = 1{\cdot}37 \times 10^4\,(I_0\,\alpha)^{\frac{1}{2}}$$

$$= 1{\cdot}37 \times 10^4\,\alpha n_0 \qquad \text{.....(5).}$$

σ_0 is thus another parameter which with R has to be chosen to fit the experimental data. χ for a given latitude and season is a function of ϕ, so that when the form of the function $F\,(z,\,\chi)$ is known equation (4) may be solved as a relation between ν and ϕ for a given value of z by approximate arithmetical methods.

This paper is partly concerned with the development of such a method which is accurate enough for the purpose in hand and at the same time adapted to rapid calculation. But before proceeding to this, there is a further theoretical point of great importance to be discussed. The contours on the charts may be interpreted as representing lines of equal maximum ionic density, and for any given point of the earth's surface the density represented is the maximum density in the layer at the time considered, since what really matters in the propagation of short wireless waves is the maximum density available and not the exact height at which it occurs. As is shown by Chapman's curves in figures 6–11, and more explicitly in figures

12*a*–17*b* of part 1, this height varies with the local time ϕ. In order therefore to draw the contours correctly it would be necessary to solve equation (4) for a series of values of z and obtain a set of curves such as figure 6 of part 1. The envelope of these curves would be the required (ν, ϕ) curve. This process, however, would involve a great deal of time and labour in drawing a set of charts, and a simplifying assumption was made to reduce the work to reasonable dimensions.

An examination of figures 6–11 of part 1 shows that at least for $\sigma_0 = 1$ it is approximately true that the envelope is given by the curve corresponding to the height at which the rate of ionization is maximum at noon, and this assumption has been adopted in working out the charts. The approximation is probably near enough to give contours which are correct within the limits of experimental error, and in cases of doubt it should be remembered that the charts give a lower limit to the density, since the true envelope must lie above the chosen curve.

Now Chapman shows that for a given angle χ the height in terms of z at which
z_0 the rate of ionization is greatest is given very nearly by $\log_e f(R, \chi)$, so that if z_0 is
χ_0 the height corresponding to χ_0, the value of χ at noon,

$$z_0 = \log_e f(R, \chi_0) \qquad \text{.....(6).}$$

This is the value of z to be used in equation (4). Equation (1) now gives

$$\begin{aligned} F(z_0\chi) &= \exp\left[1 - z_0 - \exp(-z_0).f(R, \chi)\right] \\ &= \frac{1}{f(R, \chi_0)}.\exp\left[1 - \frac{f(R, \chi)}{f(R, \chi_0)}\right] \qquad \text{.....(7).} \end{aligned}$$

§ 3. APPLICATION TO THE CONSTRUCTION OF THE CHARTS

The chief steps in the construction of the charts can now be formulated.

(*a*) The values of R and σ_0 must first be decided upon.

(*b*) The function $f(R, \chi)$ for the chosen value of R must be evaluated and plotted as a function of χ. This can be done with sufficient accuracy by interpolation from the values given in table 4 of part 2 of Chapman's paper.

(*c*) The function $F(z_0\chi)$ must be worked out from equation (7) and plotted as a function of χ for various values of χ_0, e.g. $\chi_0 = 0, 15, 30, 45, 60, 75, 85$ and $90°$.

(*d*) A series of latitudes must be chosen, e.g. 0, 20, 40, 60 and 80°, and the differential equation solved for these latitudes for the seasons equinox ($\delta = 0$), winter ($\delta = -23{\cdot}5°$), and summer ($\delta = 23{\cdot}5°$). It is only necessary to work out the case of northern latitudes since the equinox chart will be symmetrical about the equator, and the southern hemisphere at the December solstice will be the same as the northern hemisphere at the June solstice and *vice versa*.

(*e*) In solving the differential equation for any given latitude and season, χ_0 is evaluated and the required values of $F(z_0\chi)$ are interpolated from the curves plotted in (*c*).

(*f*) A set of contour values must be decided upon (i.e. a set of ν values); from the (ν, ϕ) graphs obtained by solving the differential equation, the local times ϕ corresponding to a given ν for the various latitudes and seasons can be read off, and from the values obtained the charts can be constructed.

(*g*) By assigning values to α and n_0 consistent with the value of σ_0 chosen, the contours can be given absolute values of density (and hence of minimum transmissible wave-length).

§ 4. THE SOLUTION OF THE DIFFERENTIAL EQUATION

The equation to be solved is

$$\sigma_0 . dv/d\phi + v^2 = F \qquad \text{.....(8)},$$

where F is written for $F(z_0\chi)$.

Assume that the value of v at some time ϕ_1 is v_1 and is known, and that F_1 is the corresponding value of F. The value of v at a neighbouring time ϕ_2 may be expressed by means of Taylor's theorem as

ϕ_1, v_1, F_1
ϕ_2

$$v_2 = v_1 + (\phi_2 - \phi_1)\left(\frac{dv}{d\phi}\right)_1 + \frac{(\phi_2 - \phi_1)^2}{2}\left(\frac{d^2v}{d\phi^2}\right)_1 + \text{etc.},$$

v_2

where $\left(\frac{dv}{d\phi}\right)_1$ is the value of $\frac{dv}{d\phi}$ at ϕ_1, etc. Assume that $(\phi_2 - \phi_1)$ is small and that terms higher than the second order can be neglected, and further that

$$\left(\frac{d^2v}{d\phi^2}\right)_1 = \left[\left(\frac{dv}{d\phi}\right)_2 - \left(\frac{dv}{d\phi}\right)_1\right] \Big/ (\phi_2 - \phi_1),$$

so that

$$v_2 = v_1 + (\phi_2 - \phi_1) . \frac{1}{2}\left[\left(\frac{dv}{d\phi}\right)_1 + \left(\frac{dv}{d\phi}\right)_2\right].$$

Geometrically this is equivalent to assuming that the slope of the chord joining the points is the mean of the slopes of the two tangents, and this is very nearly true provided that $(\phi_2 - \phi_1)$ is small and that there is no sharp bend in the curve between the points.

Now from equation (8)

$$\left(\frac{dv}{d\phi}\right)_1 = \frac{F_1 - v_1^2}{\sigma_0} \quad \left(\frac{dv}{d\phi}\right)_2 = \frac{F_2 - v_2^2}{\sigma_0},$$

so that

$$v_2 = v_1 + \frac{(\phi_2 - \phi_1)}{2\sigma_0}[(F_1 + F_2) - (v_1^2 + v_2^2)].$$

Writing

$$(\phi_2 - \phi_1)/\sigma_0 = k \qquad \text{.....(9)},$$

this becomes

$$kv_2^2 + 2v_2 - \{2v_1 + k(F_1 + F_2) - kv_1^2\} = 0,$$

$$\therefore kv_2 = -1 + \sqrt{\{1 + 2kv_1 + k^2(F_1 + F_2) - k^2v_1^2\}}$$

$$= \sqrt{A} - 1 \qquad \text{.....(10)},$$

where

$$A = [2 + k^2(F_1 + F_2)] - (1 - kv_1)^2 \qquad \text{.....(11)}.$$

v_2 is thus determined, and by a repeated application of the method the whole curve can be traced out.

For a given latitude and season a table is constructed with the values of ϕ decided upon (taking $\phi = 0$ at noon). The corresponding χ values having been worked out, the values of F are interpolated from the graphs described in (*c*) of

§ 3. Adding them in adjacent pairs we obtain a column of $F_1 + F_2$ values from which $2 + k^2(F_1 + F_2)$ can be tabulated.

Take as an example the case for the equinox and latitude 20° and $\sigma_0 = 1$. Then $\chi_0 = 20°$ and we have the results shown in table 1.

Table 1

ϕ (radians)	χ (degrees)	F	F_1+F_2	k	$2+k^2(F_1+F_2)$
0	20	0·94			
			1·83	0·3	2·1647
±0·3	26·1	0·89			
			1·65	0·3	2·1485
±0·6	39·2	0·76			
			1·28	0·3	2·1152
±0·9	54·2	0·52			
			0·71	0·3	2·0639
±1·2	70·5	0·19			
			0·24	0·15	2·0054
±1·35	78·1	0·047			
			0·047	0·15	2·0011
±1·5	86·1	0·000			

Now suppose, for example, it is assumed that $\nu_1 = 0{\cdot}867$ when $\phi_1 = 0{\cdot}6$, then we have the following scheme for determining the next four points:

$\phi_2 = 0{\cdot}9$	$k = 0{\cdot}3$	$2 + k^2(F_1 + F_2) = 2{\cdot}1152$
$k\nu_1 = 0{\cdot}2601$	$(1 - k\nu_1) = 0{\cdot}7399$	$(1 - k\nu_1)^2 = 0{\cdot}5474$
$\sqrt{A} = 1{\cdot}2521$		$A = 1{\cdot}5678$
$k\nu_2 = 0{\cdot}2521$	$\nu_2 = 0{\cdot}841$	
$\phi_2 = 1{\cdot}2$	$k = 0{\cdot}3$	$2 + k^2(F_1 + F_2) = 2{\cdot}0639$
$k\nu_1 = 0{\cdot}2521$	$(1 - k\nu_1) = 0{\cdot}7479$	$(1 - k\nu_1)^2 = 0{\cdot}5593$
$\sqrt{A} = 1{\cdot}2265$		$A = 1{\cdot}5046$
$k\nu_2 = 0{\cdot}2265$	$\nu_2 = 0{\cdot}755$	
$\phi_2 = 1{\cdot}35$	$k = 0{\cdot}15$	$2 + k^2(F_1 + F_2) = 2{\cdot}0054$
$k\nu_1 = 0{\cdot}1133$	$(1 - k\nu_1) = 0{\cdot}8867$	$(1 - k\nu_1)^2 = 0{\cdot}7862$
$\sqrt{A} = 1{\cdot}1041$		$A = 1{\cdot}2192$
$k\nu_2 = 0{\cdot}1041$	$\nu_2 = 0{\cdot}694$	
$\phi_2 = 1{\cdot}5$	$k = 0{\cdot}15$	$2 + k^2(F_1 + F_2) = 2{\cdot}0011$
$k\nu_1 = 0{\cdot}1041$	$(1 - k\nu_1) = 0{\cdot}8959$	$(1 - k\nu_1)^2 = 0{\cdot}8026$
$\sqrt{A} = 1{\cdot}0948$		$A = 1{\cdot}1985$
$k\nu_2 = 0{\cdot}0948$	$\nu_2 = 0{\cdot}632$	

The ease and rapidity of the method is obvious. The value of $k\nu_1$ is given directly by the previous value of $k\nu_2$, except where the value of $(\phi_2 - \phi_1)$ is altered, and by means of a set of four-figure tables of squares and a set of five-figure tables of square-roots the whole work is carried out with uniform accuracy.

There remains the problem of finding rapidly a correct value for the starting-point of these calculations. If ν_s is the value of ν at sunset (or more exactly when F becomes negligible), say ϕ_s, the decay of ionization throughout the night is represented by the equation

ν_s

ϕ_s

$$\frac{1}{\nu} - \frac{1}{\nu_s} = \frac{\phi - \phi_s}{\sigma_0} \qquad \text{.....(12).}$$

If then some value ν_1 is assumed for ν for a particular time ϕ_1 during the day, the above process can be used to trace the curve round to the sunset value ν_s, and from equation (12) the value ν_r to which the ionization has decayed by sunrise (i.e. when F again becomes appreciable) can be determined. With this value of ν_r the above process can again be applied to trace the curve back to the original time ϕ_1, when the same value should be obtained if the correct value was chosen initially.

ν_r

A little preliminary work shows that the maxima of the curves for a given value of σ_0 all occur at about the same time after noon except for the high latitudes, when it is noticeably later; and this value of ϕ can soon be found. At this value the relation $\nu = \sqrt{F}$ holds, and by choosing ϕ_1 as the nearest convenient value to the time of the maximum a near approximation can be made to the value of ν_1. For a given change in the value of ν_1 the change in the deduced ν_s is much smaller, and the value of ν_r is very insensitive to changes in ν_s, except of course in the very short summer nights in high latitudes, so that an accurate value of ν_r is in general obtained. The value of ν_1 obtained with this value of ν_r is often near enough to the original one, but if necessary the process can be continued to revise the original portion of the curve between ν_1 and ν_s, and the final curve is very near the true one and amply accurate enough for the construction of the charts.

§ 5. PRACTICAL ILLUSTRATIONS OF THE METHOD

(*a*) The empirical charts constructed to deal with the propagation of short waves really combine two effects, the effect of attenuation in the intense-daylight regions and the effect of electron limitation in the night regions. The first of these is mainly connected with the lower or E layer of the ionized regions and the second with the upper or F layer, and as the parameters R and σ_0 are quite different for these two layers it is impossible to represent the charts by a single set of values for R and σ_0. Some kind of composite chart would have to be constructed from two separate charts worked out with values of R and σ_0 corresponding to the E and F layers respectively.

Chapman has suggested for the E layer the values $R = 650$ and $\sigma_0 = 0{\cdot}04$ and has taken these values as the basis of the detailed calculations involved in his paper. In the present paper attention is concentrated on the F layer for the study of the night grades, since they are perhaps the more important in the problem of long-distance transmissions.

For this layer the value of R will be taken as 150 for reasons which will be given in § 6, and charts will be constructed for $\sigma_0 = 1$ and $\sigma_0 = 0{\cdot}5$.

(*b*) In interpolating from table 4 of part 2 of Chapman's paper for the evaluation of $f(R, \chi)$ with R equal to 150, the value of $\log_e f(150, \chi)$ was taken as the average of the values of $\log_e f(100, \chi)$ and $\log_e f(200, \chi)$, as inspection of the table showed that this method of interpolation was very close and better than averaging the actual values of $f(100, \chi)$ and $f(200, \chi)$. The values of $\log_e f(150, \chi)$ and of $f(150, \chi)$ are given in table 2.

Table 2. Values of $\log_e f(150, \chi)$ and $f(150, \chi)$

χ	$\log_e f(150, \chi)$	$f(150, \chi)$	χ	$\log_e f(150, \chi)$	$f(150, \chi)$
30	0·1414	1·152	87	2·267	9·64
45	0·3394	1·404	90	2·705	15·00
60	0·6723	1·96	93	3·293	26·92
75	1·275	3·57	95	3·802	44·67
80	1·570	4·80	97	4·431	83·56
83	1·836	6·25	100	5·622	275·4
85	2·035	7·62			

$f(150, \chi)$ is plotted against χ in figure 1, and $\log_e f(150, \chi)$ is plotted against χ in figure 2, as by equation (6) this gives, for $\chi = \chi_0$, the value of z_0, the height at which the maximum ionization is assumed to occur. z_0

(*c*) From equation (7) the values of $F(z_0\ \chi)$ can be worked out for various values of χ_0 from the results of table 2. The values are given in table 3 and plotted in figure 3.

Table 3. Values of $F(z_0\ \chi)$

χ	$\chi_0 = 0$	$\chi_0 = 15°$	$\chi_0 = 30°$	$\chi_0 = 45°$	$\chi_0 = 60°$	$\chi_0 = 75°$	$\chi_0 = 85°$	$\chi_0 = 90°$
0	1·000							
15	0·965	0·966						
30	0·859	0·863	0·868					
45	0·668	0·675	0·697	0·712				
60	0·383	0·396	0·427	0·479	0·510			
75	0·0765	0·0834	0·107	0·153	0·224	0·280		
80	0·0223	0·0254	0·0365	0·0635	0·120	0·199		
83	0·0053	0·0063	0·0103	0·0227	0·0571	0·132		
85	0·0013	0·0017	0·0032	0·0085	0·0284	0·090	0·131	
87	0·0002	0·0002	0·0005	0·0020	0·0102	0·0512	0·101	
90					0·0007	0·0114	0·0499	0·0667
93						0·0004	0·0105	0·0302
95							0·0010	0·0092
100								0·0002

(*d*) The latitudes suggested in § 3 were chosen, and table 4 gives the values of χ_0 and ϕ_s, where ϕ_s is the sunset time measured in radians from noon (so that $-\phi_s$ is sunrise time). ϕ_s

(*e*) From the data of table 4 tables of the form of table 1 can be drawn up, the F values being interpolated from the curves of figure 3. The value of $(\phi_2 - \phi_1)$ has been made 0·3 in most cases except near sunrise, where the (ν, ϕ) curve bends rather sharply and the value 0·15 has been taken.

Broken line, see χ. Full line, $f(150\ \chi)$

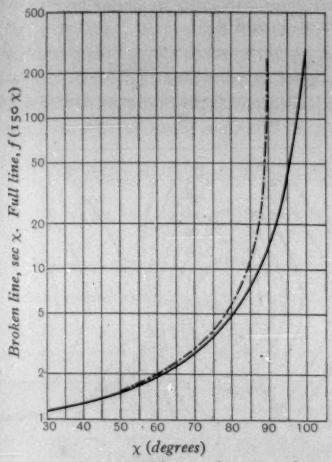

Fig. 1. $f(150, \chi)$ with sec χ for comparison.

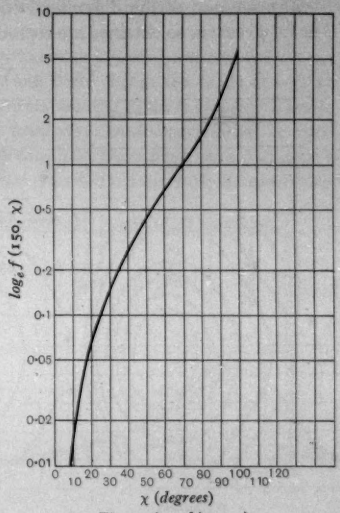

Fig. 2. $\log_e f(150, \chi)$.

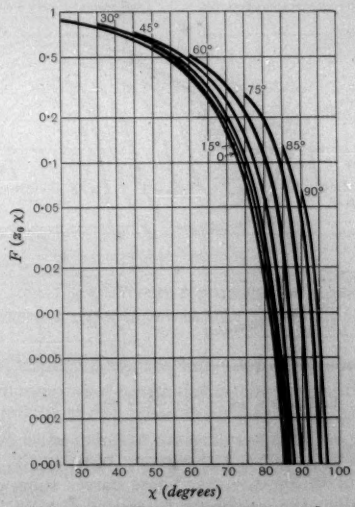

Fig. 3. $F(z_0 \chi)$; $\chi_0 = 0, 15, 30, 45, 60, 75, 85, 90°$.

The solution of the differential equation then follows along the lines of § 4. The (ν, ϕ) curves so obtained are reproduced in figures 4–9. Figures 4–6 are for the

Table 4. Values of χ_0 and ϕ_s

Latitude (°)	Equinox $\delta = 0$		Winter $\delta = -23{\cdot}5°$		Summer $\delta = 23{\cdot}5°$	
	χ_0 (°)	ϕ_s	χ_0 (°)	ϕ_s	χ_0 (°)	ϕ_s
0	0	1·57	23·5	1·57	23·5	1·57
20	20	1·57	43·5	1·41	3·5	1·73
40	40	1·57	63·5	1·20	16·5	1·95
60	60	1·57	83·5	0·719	36·5	2·42
80	80	1·57			56·5	
90					66·5	

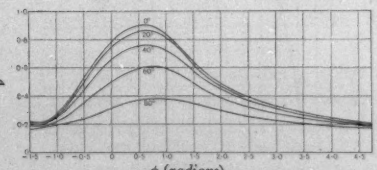

Fig. 4. Equinox: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60, 80°.

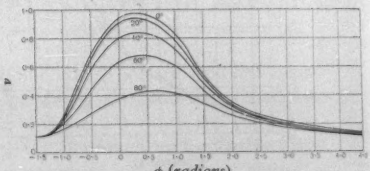

Fig. 7. Equinox: $\sigma_0 = 0{\cdot}5$. Latitudes 0, 20, 40, 60, 80°.

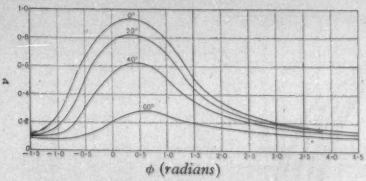

Fig. 5. Winter: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60°.

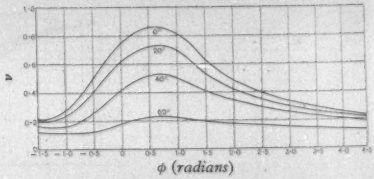

Fig. 8. Winter: $\sigma_0 = 0{\cdot}5$. Latitudes 0, 20, 40, 60°.

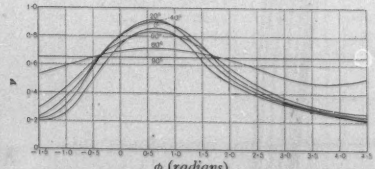

Fig. 6. Summer: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60, 80, 90°.

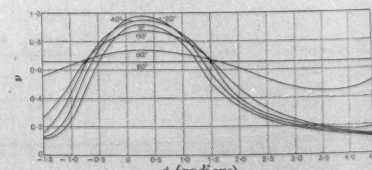

Fig. 9. Summer: $\sigma_0 = 0{\cdot}5$. Latitudes 0, 20, 40, 60, 80, 90°.

value $\sigma_0 = 1$ for equinox, winter, and summer respectively, and figures 7–9 and the corresponding curves for $\sigma_0 = 0{\cdot}5$. These curves show up well the approximately constant position of the maximum ionization with the gradual shift towards

sunset for high latitudes, and the smaller shift with greater maximum ionization and the more rapid decay for the smaller value of σ_0.

(*f*) The values of ν decided upon for the construction of the charts were 0·75, 0·65, 0·55, 0·45, 0·35, 0·30, 0·25, 0·20, 0·15 and 0·10. From the (ν, ϕ) curves in figures 4–9 tables were made of the ϕ values corresponding to these ν values*. These ϕ values were then converted to longitudes from which the charts could be constructed. In keeping with the empirical charts they have been drawn on a Mercator's projection with a G.m.t. scale and the scale was adjusted for use with the same map

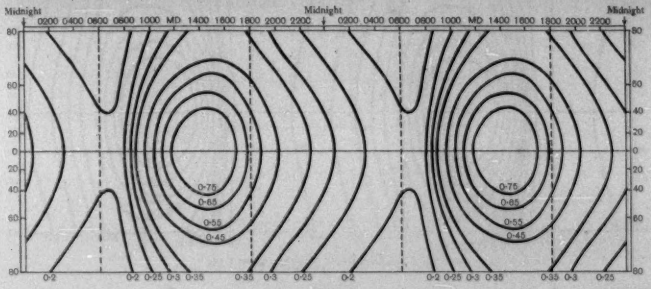

Fig. 10. Equinox chart. $\sigma_0 = 1$.

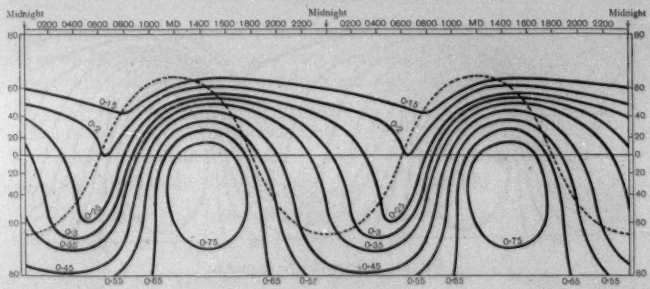

Fig. 11. Winter chart. $\sigma_0 = 1$.

as for the empirical charts. The charts are reproduced on a reduced scale in figures 10–13. The summer charts are not shown as they are simply mirror images of the winter charts†.

(*g*) For an assumed value of n_0 the absolute value of n (or νn_0) is known, and for a given density the minimum wave-length that can be transmitted (i.e. that does not escape through the layer) is known when the height of the position of maximum density is known. If R_0 is the radius of the earth and h the height above the surface R_0, h

* For the region between sunset and sunrise it was found best to use a set of linear graphs of ν^{-1} against ϕ; see equation (12).

† The broken lines are the sunrise-sunset great-circles.

λ_{min} of the earth to which the limiting wave-length λ_{min} penetrates, then from transmission theory

$$\lambda_{min} = \lambda_0 \sqrt{\{h\,(2R_0 + h)\}}/(R_0 + h) \qquad \text{.....(13),}$$

where λ_0 is the critical wave-length corresponding to the density n and is given by

λ_0
$$\lambda_0^2 = 1{\cdot}117 \times 10^9/n \text{ metres}^2 \qquad \text{.....(14).}$$

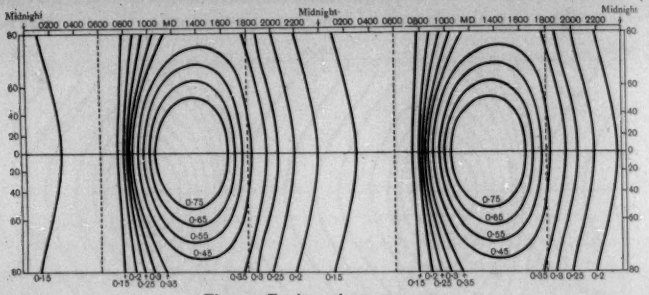

Fig. 12. Equinox chart. $\sigma_0 = 0{\cdot}5$.

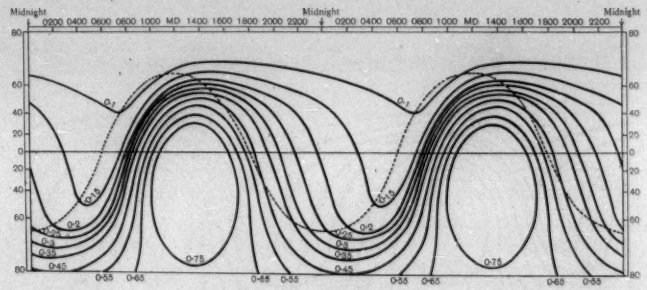

Fig. 13. Winter chart. $\sigma_0 = 0{\cdot}5$.

Taking R_0 equal to 6366 km. and h to 300 km., for the upper layer (13) becomes

$$\lambda_{min} = \lambda_0 . 0{\cdot}297 \qquad \text{.....(15).}$$

p If equation (13) is written $\lambda_{min} = p\lambda_0$, for a given density, and therefore from (14) for a given critical wave-length λ_0, λ_{min} is a function of p, i.e. of h. Taking h_0 equal to 300 km. as datum, in the general case h may be written as $300 + z_0 H$, and if p_0 is the value of p when $h_0 = 300$ km. and $(\lambda_{min})_0$ the value of λ_{min} for p_0

$$\lambda_{min} = (\lambda_{min})_0\, p/p_0 \qquad \text{.....(16),}$$

p/p_0 is a function of z_0 and is plotted in figure 14. This relation is of some importance in assigning minimum wave-length values to the contour lines, since in higher

latitudes in winter z_0 is of the order of 2, and the minimum wave-length may be increased by more than 10 per cent over the equator value for the same contour.

The limits of the value of σ_0, 0·5 to 1·0, are fixed by the facsimile measurements* of the coefficient of recombination and of the ionic density. The values of α and n_0 are not very accurately known, and in adjusting them to satisfy equation (5) α may be taken between the limits of 7×10^{-11} and 10^{-10}, giving n between 10^6 and 7×10^5 approximately for $\sigma_0 = 1$, or between 2×10^6 and $1{\cdot}4 \times 10^6$ for $\sigma_0 = 0{\cdot}5$. In attempting to make the charts agree with the empirical ones, n_0 can be adjusted to give the best fit within these limits.

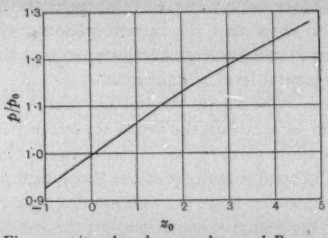

Fig. 14. p/p_0 when $h_0 = 300$ km. and $R = 150$.

§ 6. DISCUSSION OF RESULTS

The value of the parameter R in Chapman's theory has been made 150. R depends upon H, and H itself is only very imperfectly known since it depends upon the constitution of the upper atmosphere in a region concerning which the physical data are very scanty. H depends upon the temperature T of the atmosphere and upon the mean atomic or molecular mass m, being given by $H = kT/mg$, where k is Boltzmann's constant $1{\cdot}37 \times 10^{-16}$, and g is the acceleration due to gravity, 981 cm./sec.^{2} If M is the mean atomic or molecular weight

H

T

m, k

g

M

$$m = M \times (\text{mass of the hydrogen atom})$$
$$= 1{\cdot}66 \times 10^{-24} \times M \text{ gm.},$$

and
$$H = 1{\cdot}37 \times 10^{-16} \times T/1{\cdot}66 \times 10^{-24} \times 981 \times M \text{ cm.}$$
$$= 0{\cdot}84\, T/M \text{ km.}$$

The value of T is very uncertain at the height of the upper layer, a value of 300° K. being possible and of 1000° K. not impossible, while M depends upon the state of dissociation and ionization of the atmosphere. Assuming with Chapman that the ionization is due to the absorption of wave-lengths below 910 Å. by atomic oxygen, an idea of the possible limits of H can be obtained by taking T equal to 300° K. and M to 16 for ionized atomic oxygen, and T equal to 1000° K. and M to 8 for wholly ionized atomic oxygen, giving $H = 15{\cdot}7$ km. and 105 km. respectively. It was felt that it would be best to take a value of about 50 km., and since the cal-

* T. L. Eckersley, *J.I.E.E.*

culations in the earlier part of the paper are not greatly affected by a moderate change in the value of the parameter R, R was actually taken as 45·1 km. as this makes R equal to 150 if h_0 is 300 km., which is convenient for the easy interpolation of $f(R, \chi)$ as described in paragraph (b) of § 5.

The selection of the two values of σ_0, 0·5 and 1·0, is to take account of the fact established by facsimile experiments and by signal-measurements and commercial traffic experience that the general level of ionization is now markedly less than it was three years ago. This effect seems to be connected with the sunspot cycle and to be more marked in the higher latitudes than in the tropics. The lower value of σ_0 may be regarded as a sunspot-maximum value, and the upper one as a sunspot-minimum value. Facsimile results show that the recombination coefficient does not alter within the limits 7×10^{-11} and 10^{-10} given above, so that the value of σ_0 must be varied inversely as the general level of ionization.

The theoretical charts show a good general agreement with the empirical ones. By a suitable adjustment of n_0 within the limits set out in paragraph (g) of § 5, the agreement can be made quite good for the summer chart, especially in the tropics, and gives weight to the general soundness of the theoretical principles involved. In the northern latitudes, however, there is a definite discrepancy between the charts, especially for winter. In the region of noon for $\sigma_0 = 1$ the less dense grades extend too far towards the equator, while if σ_0 is altered to 0·5 to increase the general level of ionization in the noon region, the more rapid decay during the night makes the late night regions too thin. These discrepancies are apart from those arising from the attenuating effect of the lower layer, and they lead to the conclusion that they must be due to some additional cause not taken into account in the theory. Some extra source of ionization which will follow the sunspot cycle is needed in these latitudes. Chapman has outlined a theory of magnetic storms suggesting the projection from the sun of a neutral ionized stream of particles, which, when it gets within a distance from the earth equal to the earth's diameter, begins to get separated out by the earth's magnetic field, the electrons being deflected towards the polar regions. Such a theory throws light on the problems of magnetic storms and aurorae, and indicates an explanation of scattering. If an estimate could be made of the magnitude and diurnal variation of the rate of ionization, this effect could be included in the differential equation by the necessary modification of the F function. That the effect must be diurnal is seen by working out the case of a constant source of ionization. Figures 6 and 9 show the amount of ionization at the north pole in the summer due to quite a small constant rate of ionization. Owing to the comparatively slow rate of recombination the steady value of the ionization is quite large, and similarly quite a small additional constant ionization will have the effect of flattening the (ν, ϕ) curves considerably and of destroying the contrast between day and night regions which actually exists in the higher latitudes.

§7. CONCLUSION

As was stated in the introduction, the main object of this paper has been to develop a technique for the construction of charts from Chapman's theory, but the short discussion given above shows the usefulness of such charts, and indicates the main conclusion obtained from comparison with the empirical charts. It has certainly helped to bring out the general properties of such charts by showing more clearly the effect of, say, a larger recombination coefficient or an increase in the maximum ionic density, and it has in turn suggested possible ways of modifying the theory to fit all the experimental facts.

§8. ACKNOWLEDGMENTS

My thanks are due to Mr T. L. Eckersley who suggested the problem to me and outlined the general method of its solution, and also to Marconi's Wireless Telegraph Company Limited for allowing this paper to be published.

FURTHER INVESTIGATION OF THE ARC SPECTRUM OF ARSENIC

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ABSTRACT. By photographing the spectrum of arsenic by the method of the hollow-cathode discharge in helium and in neon about 100 new lines have been recorded. In the light of the experimental data, the analysis of As I published by previous investigators has been considerably altered and extended. Several new levels have been added and the higher members of the chief groups of the *ms* series of terms have been identified. A mean value of 85,000 cm.$^{-1}$ has been suggested for the deepest term $4p\ ^4S_2$ which leads to a first ionization potential of approximately 10·5 V. for arsenic.

§ 1. INTRODUCTION

In spite of numerous investigations of the spectra of the elements of the fifth group of the periodic table, our knowledge of the arc spectra of these elements remains incomplete; it is only in the case of N I that the absolute values of the characteristic energy levels are known with a fair degree of accuracy. Phosphorus presents many experimental difficulties and the spectrum itself is not known completely, while arsenic, antimony and bismuth, although the excitation of their spectra is easy, contain several important lines in spectral regions which are difficult to explore.

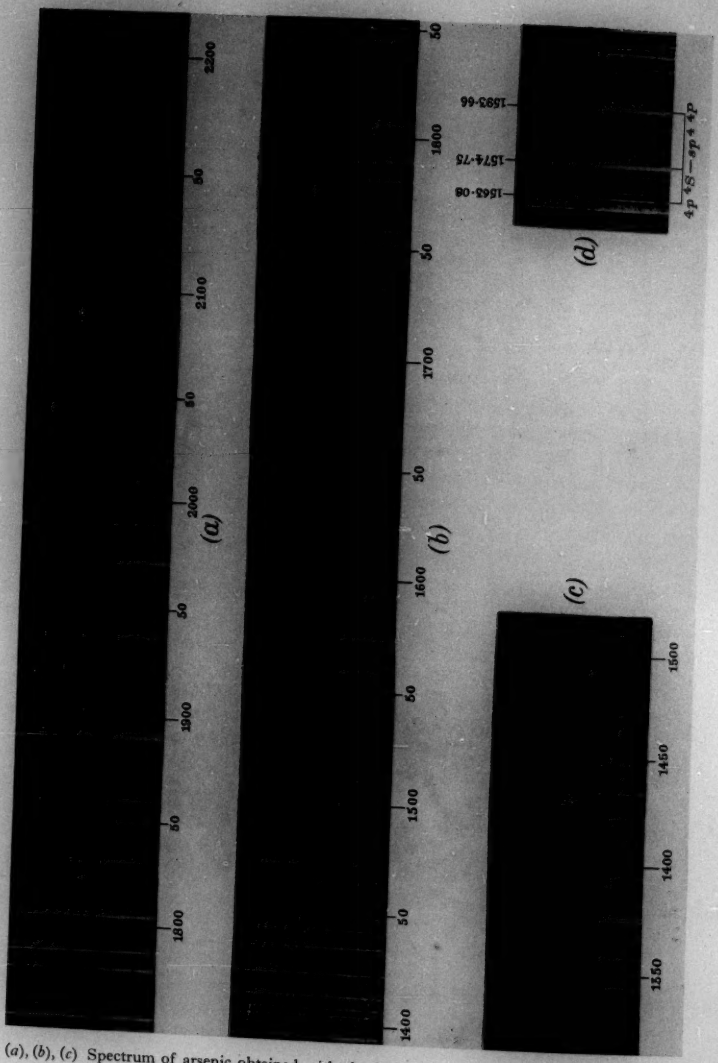
In a recent paper*, K. R. Rao has described experiments on the arc spectrum of arsenic, particularly in the region λ 2200 to λ 1563, which have led to the identification of the combinations between the deepest set of terms $4p\ ^4S$, ^2D and ^2P of As I and the higher $5s$ and $4p$ terms. Combinations with some of the terms of the $4d$ state have also been suggested. The interesting feature of his analysis is the location of the $4p'$ terms, chiefly $4p'\ ^4P$, which involve the displacement of an inner s electron.

Later, Meggers and de Bruin† have independently investigated this spectrum in the region λ 10024 to λ 1890 and have suggested a similar scheme. The most important contribution towards the analysis of As I in the work of these authors is the identification, though partially, of the multiplets in the infra-red, arising from the addition of a $5p$ electron to the s^2p^2 configuration of the core. However, in extending the analysis into the Schumann region, they have utilized the somewhat incomplete data published by L. and E. Bloch‡. For a complete bibliography of the previous work on the arc spectrum of arsenic, reference must be made to these two papers.

There is a large measure of agreement between the schemes published by K. R. Rao and by Meggers and de Bruin. Their conclusions regarding the fundamental multiplets resulting from the combination of the $4p$ terms with the $5s\ ^4P$, ^2P and ^2D are identical. Of the 30 levels assigned to the $4p$, $5s$, or $4d$ states by Meggers and

* *Proc. R. S.* A, **125**, 238 (1929). † *Bur. Standards J. Res.* **3**, 765 (1929).

‡ *J. Phys.* **4**, 622 (1914); and *Comptes Rendus*, **171**, 709 (1920).



(*a*), (*b*), (*c*) Spectrum of arsenic obtained with the hollow-cathode discharge in helium in the region λ 2200 to λ 1300.

(*d*) Portion of the same showing the group $4p\,^4S - sp\,^4P$.

To face p. 595]

de Bruin and 35 assigned by Rao, there are as many as 25 common levels. The chief point of disagreement lies in the designation of these levels, particularly of those ascribed to the $4d$ state; there is, in some cases, a difference in the assignments even of the inner quantum number to the level. It must be stated at the outset that, as remarked both by Rao and by Meggers and de Bruin, it is difficult to define with certainty the $4d$ group of terms, as there are no means of verifying the correctness of the location either by other combinations such as $4d \leftarrow 5p$ or by comparison with other spectra.

In the present communication, these two schemes are discussed in detail and considerably altered and extended in the light of further experimental work. Many additional energy levels of As I have been discovered, as a result of the extension of the spectrum further into the ultra-violet. The higher members of the chief groups have been found and a probable estimate made of the absolute values of the characteristic terms of As I.

§ 2. EXPERIMENTAL

All the previous investigators have utilized the ordinary arc in air between metallic arsenic contained in electrodes of carbon, aluminium or copper. In his investigation of the Schumann region, Rao had recourse to the special method* designed for Prof. Fowler's work on silicon, which uses essentially an arc in an atmosphere of nitrogen.

The experimental work which formed the basis of the present scheme was carried out by Dr K. R. Rao and Dr J. S. Badami in Prof. Paschen's laboratory at the Physikalisch-Technische Reichsanstalt. The author is greatly indebted to them for placing their plates at his disposal. The mode of excitation of the arc spectrum was the hollow cathode discharge in helium or neon. The details of its use have been previously described by various writers†. In the present experiments the hollow cathode was of carbon or tungsten; the latter was found better. To start with, the system was exhausted to a hard vacuum and helium or neon was then kept in circulation by a mercury vapour pump and purified by passing over hot copper oxide and over charcoal cooled by liquid air. A constant potential difference of about 800 V. was applied to the electrodes and a steady current of 200 to 300 mA. was maintained throughout the exposure. The bright negative glow filling the inside of the hollow cathode was photographed by a vacuum spectrograph of radius 1 m. and giving a dispersion of about 17 Å./mm. Exposures varying from six to eight hours were found necessary. The entire region of the spectrum between λ 2800 and λ 500 was obtained at a single exposure. The usual lines of He, Ne, C were used as standards in reducing the plates. It is thought that the wave-lengths are accurate to about 0·03 Å.

§ 3. STRUCTURE OF THE SPECTRUM

The characteristic terms of the arc spectrum of As are built up, according to Hund's theory, on the three terms ^3P, ^1D, ^1S, of the s^2p^2 configuration of As^+. A further group of terms is based on the state sp^3 of the core. The most important of

* A. Fowler, *Proc. R. S.* A, **123**, 422 (1929). E. W. H. Selwyn, *Proc. Phys. Soc.* **41**, 392 (1929).

† F. Paschen, *Sitzungsb. d. Preus. Akad. d. Wiss.* p. 207 (1927). R. Frerichs, *Ann. d. Phys.* **85**, 362 (1928).

these predicted terms are set out in table 1. To avoid the use of fractions, an integral inner quantum number, greater by half a unit than the true value, has been assigned to each term throughout this paper. The notation is in accordance with that used in the previous paper on As I*.

Table 1. Theoretical terms of As I

3_1	3_2	3_3	4_1	4_2	4_3	4_4	5_1	5_2	5_3	6_1	Term prefix	Terms: Limit ^3P	Terms: ^1D	Terms: ^1S
2	6	10	2	3	.	.	.	.	.	.	$4p$	^4S	^2D	^2P
2	6	10	2	2	.	.	1	.	.	.	$5s$	$^4P\ ^2P$	^2D	^2S
2	6	10	2	2	1	.	.	.	.	.	$4d$	$^4F\ ^4D\ ^4P$	$^2G\ ^2F^a\ ^2D^a$	$^2D^b$
.	.	.	.	.	.	.	.	.	.	.		$^2F\ ^2D\ ^2P$	$^2P^a\ ^2S^a$	
2	6	10	2	2	.	.	.	1	.	.	$5p$	$^4D\ ^4P\ ^4S$	$^2F\ ^2D^a$	$^2P^b$
.	.	.	.	.	.	.	.	.	.	.		$^2D\ ^2P\ ^2S$	$^2P^a$	.
2	6	10	2	2	.	.	.	.	.	1	$6s$	$^4P\ ^2P$	^2D	^2S
2	6	10	1	4	.	.	.	.	.	.	sp^4	$^4P\ ^2P$	^2D	^2S

The principal multiplets of As I which contain most of the prominent lines are presented in table 2, in which the classifications made by K. R. Rao and by Meggers and de Bruin have also been indicated in the first two columns for purposes of comparison.

Table 2. Multiplets in As I

M & B	R	$4p$.....	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
			10590·3	*322·3*	*7270·7*	*461·16*	
$5s\ ^4P_1$	$5s\ ^4P_1$	$5s\ ^4P_1 = 34308·0$	50693·5 (10)	40102·0 (5)	—	32507·56 (2)	32046·38 (4)
		915·8					
$^4P_2$	$^4P_2$	$^4P_2 = 33392·2$	51608·1 (12)	41018·0 (4)	40695·6 (5)	33423·9 (2)	32962·77 (4)
		1287·7					
$^4P_3$	$^4P_3$	$^4P_3 = 32104·5$	52895·2 (15)	42305·4 (3)	41983·0 (6)	—	34250·3 (0)
$^2P_1$	$^2P_1$	$^2P_1 = 31866·8$	53134·1 (4)	42543·2 (9)	—	34949·27 (7)	34488·12 (5)
		1469·6					
$^2P_2$	$^2P_2$	$^2P_2 = 30397·2$	54603·3 (6)	44012·4 (3)	43690·5 (10)	36419·24 (6)	35958·09 (8)
$^2D_2$	$^2D_2$	$^2D_2 = 24167·8$	60833·2 (5)	50241·9 (9)	49920·6 (4)	42648·6 (5)	42187·1 (7)
		−19·1					
$^2D_3$	$^2D_3$	$^2D_3 = 24186·9$	60812·5 (5)	50224·0 (7)	49901·4 (10)	—	42167·0 (7)
$4d\ ^4P_1$	$5s\ ^2S_1$	$^2S_1 = 19505·2$	65492·2 (3)	54901·6 (4)	—	47311·3 (5)	46850·6 (4)
$4d\ ^4P_3$	$sp^4\ ^2P_2$	$6s\ ^4P_1 = 17995·4$	67003·9 (6)	56413·3 (2)	—	48822·4 (2)	48360·7 (1)
		912·2					
$4d\ ^2P_1$	α	$^4P_2 = 17083·2$	67915·9 (7)	—	57004·7 (3)	49733·8 (3)	49272·6 (0)
		1392·1					
—	—	$^4P_3 = 15691·1$	69399·2 (8)	58718·2 (1)	58397·2 (2)	—	50663·7 (0)
$4d\ ^2P_2$	$sp^4\ ^2P_1$	$^2P_1 = 16689·0$	68310·1 (5)	57719·7 (6)	—	50128·6 (5)	49667·4 (4)
		1382·5					
—	β	$^2P_2 = 15306·5$	69695·1 (8)	59105·5 (5)	58781·4 (8)	51506·0 (2)	51048·8 (5)

* K. R. Rao, *loc. cit.*

Table 2. Multiplets in As I (*continued*)

M & B	R	4p.....	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
			10590·3	*322·3*	*7270·7*	*461·16*	
$4d\ ^4D_1$	$sp^4\ ^4P_1$	$sp^4\ ^4P_1 = 21022·5$	63976·3 (7)	53387·7 (4)	—	45794·6 (2)	45333·2 (2)
		−477·3					
$4d\ ^4F_2$	$^4P_2$	$^4P_2 = 21499·8$	63502·2 (8)	[52910]	52587·6 (2)	45315·9 (2)	44854·8 (3)
		−752·1					
$4d\ ^4F_3$	$^4P_3$	$^4P_3 = 22251·9$	62748·6 (9)	52158·3 (3)	51835·7 (1)	—	44102·4 (2)
—	$4d\ ^2P_2$	$^2P_2 = 12224·2$	72784·4 (4)	62182·4 (6)	61859·8 (5)	54591·1 (5)	54130·4 (5)
—	$4d\ ^2P_1$	$^2D_2 = 12486·7$	72513·7 (4)	61922·6 (5)	61599·9 (4)	54330·7 (4)	—
		89·0					
—	—	$^2D_3 = 12397·7$	72602·1 (3)	62012·0 (7)	61689·6 (6)	—	53958·1 (4)
—	δ	$^2S_1 = 11371·9$	—	—	—	55444·7 (2)	54983·7 (2)
$5s\ ^2S_1$	$sp^4\ ^2D_{2,3}$	$4d\ ^4F_2 = 20192·9$	64808·4 (4)	54215·5 (5)	53894·4 (3)	46624·0 (5)	46162·6 (6)
		−468·2					
$4d\ ^4D_3$	$4d\ ^2F_3$	$^4F_3 = 20661·1$	64337·6 (4)	53749·6 (6)	53426·2 (4)	—	45694·7 (1)
		618·4					
—	—	$^4F_4 = 20042·7$	—	—	54044·7 (6)	—	—
—	—	$^2F_3 = 19045·2$	—	55364·5 (9)	55042·1 (4)	—	—
		1820·6					
—	—	$^2F_4 = 17224·6$	—	—	56862·8 (9)	—	—
$4d\ ^4P_2$	$sp^4\ ^2S_1$	$4d\ ^4D_1 = 18416·7$	66583·7 (3)	55992·0 (3)	—	48400·8 (3)	47938·7 (3)
		−102·3					
$4d\ ^4P_1$	$4d\ ^4D_2$	$^4D_2 = 18519·0$	66479·7 (5)	55889·4 (7)	55567·6 (3)	48298·6 (3)	47838·9 (2)
		239·5					
—	$4d\ ^4D_3$	$^4D_3 = 18279·5$	—	56130·9 (6)	55807·9 (5)	—	48075·2 (2)
		59·9					
—	$4d\ ^4D_4$	$^4D_4 = 18219·6$	—	—	55867·8 (6)	—	—
$4d\ ^2D_2$	$4d\ ^2D_2$	$^2D_2 = 16701·7$	68300·0 (5)	57707·7 (6)	57385·8 (5)	50114·0 (7)	49653·5 (6)
		100·4					
$4d\ ^2D_3$	$^2D_3$	$^2D_3 = 16601·3$	68397·6 (4)	57808·5 (5)	57486·8 (6)	—	49754·6 (8)
—	—	$4d\ ^4P_1 = 13791·7$	71208·3 (4)	60618·9 (3)	—	—	—
		−158·3					
—	—	$^4P_2 = 13950·0$	71049·0 (4)	60460·0 (4)	60139·2 (3)	52865·6 (1)	—
		−281·6					
—	—	$^4P_3 = 14231·6$	70767·9 (5)	—	59855·2 (1)	—	52124·9 (0)
—	—	$^2P_1 = 13076·8$	71922·3 (4)	61332·0 (4)	—	53741·2 (1)	53279·1 (0)
		196·2					
—	γ	$^2P_2 = 12880·6$	—	61528·6 (4)	61207·8 (5)	—	53474·5 (4)

Other regularities

4p.....	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
a 12740·0	72264·8 (1)	—	—	54074·2 (0)	53613·3 (5)
b 11758·7	73241·3 (2)	62650·7 (2)	62330·5 (1)	—	—
c 10956·2	74046·1 (2)	63452·6 (2)	63129·7 (2)	—	—
d 10863·7	74133·4 (2)	—	—	55953·5 (4)	55494·2 (1)
e 10761·3	—	63646·8 (4)	63326·4 (0)	—	55595·4 (4)
f 10687·9	—	63720·7 (0)	63400·7 (0)	—	—
g 10259·9	74737·9 (2)	64149·8 (3)	—	56558·9 (2)	—
h 10196·4	74801·6 (3)	64211·6 (3)	63891·2 (0)	—	56160·5 (4)
i 9916·7	—	—	—	56899·3 (1)	56439·5 (0)
j 9429·1	75573·8 (3)	64978·9 (4)	64657·1 (0)	—	—
k 8926·5	—	—	65159·3 (3)	57890·1 (2)	57430·7 (5)
l 8193·6	—	—	—	58622·5 (0)	58162·5 (5)

It will be noticed that there is perfect agreement between the two schemes under consideration as regards the identification of the fundamental multiplets arising from the transition $4p \leftarrow 5s$ except those involving the term $5s\,^2S$. Obviously an inner quantum number 2 must be assigned to the level which is designated as $5s\,^2S$ by Meggers; hence the more probable identification made by Rao has been retained.

In considering the other higher levels, the principle guiding the designation of each has been that a level is assumed to be a quartet or a doublet according as its combinations with $4p\,^4S$ or with ^2D and ^2P are the more intense. However, on account of the relatively large intensities of the intercombination lines, this procedure may not always lead to the correct assignment. The similarity in appearance and behaviour of the lines forming one multiplet has also been of great help in this assignment. The photographing of the whole region from λ 2800 to λ 500 at once on a single Schumann plate has considerably facilitated the comparison of the behaviour and intensity of the lines although they are widely separated.

Three levels designated as forming $sp^4\,{}^4P_{1,2,3}$ by Rao have been denominated $4d\,{}^4F_3$, $^4F_2$, $^4D_1$ by Meggers. Examination of the plates showed that the three lines $\nu 63976, 63502, 62749$ have an identical appearance and must belong to the same group; particularly on a neon plate taken with a short exposure these lines stand out very prominently. Reference may be made to (*d*) on the plate. It might be supposed that they form the group $4p\,^4S - 4d\,^4P$ instead of $4p\,^4S - sp^4\,{}^4P$. The large magnitude of the term supports the latter assignment. Such groups have been discovered to be markedly present in the analogous spectra of N I, O II, F III, S II, Cl III*. In all these the $sp^4\,{}^4P$ term is inverted. The following table of the magnitudes of the intervals of this term shows clearly that the present assignment is in keeping with the other spectra.

Table 3. Deep quartet term intervals in five-electron systems

	N I	O II	F III	P I	S II	Cl III	As I
$sp^4\,{}^4P_1$							
$^4P_2$	−19·6	− 82	−177	—	−210·1	−345	−477·3
$^4P_3$	−43·9	−163	−342	—	−363·1	−610	−752·1
$ms\,{}^4P_1$							
$^4P_2$	33·8	105·3	211·3	151	270·83	357·8	915·8
$^4P_3$	46·7	158·6	318·9	249	437·00	520·1	1287·7

This alteration in Meggers's scheme necessitated a complete reassignment of the characteristic terms. Rao has suggested only some of the quartet terms of the $4d$ state. Of his $4d\,^4D$ group, the component $^4D_1$ (marked as doubtful) has been based on the arbitrary choice of a single line. The selection made by the writer seems to be more plausible. Rao and Meggers agree in the identification of the $4d\,^2D$ term, which is well supported by the strong combination lines it gives with $4p\,^2P$ and $4p\,^2D$ and therefore has been adopted by the author.

* I. S. Bowen, *Phys. Rev.* **29**, 237 (1927); **31**, 34 (1928). S. B. Ingram, *Phys. Rev.* **32**, 172 (1928).

The intervals exhibited by the triad of terms $^{4,2}F$, $^{4,2}D$, $^{4,2}P$, arising from the $4d$ state in all spectra of the type under consideration, appear to be very anomalous. Excepting in S II, in which the term is normal, the ^4D term is partially inverted in all these spectra; while the ^4P term shows complete inversion in all except in N I. The scheme now suggested for As I exhibits similar anomalies; the ^4P term is completely inverted and ^4D and ^4F show only partial inversion. In spite of these irregularities in the term intervals, it is curious that the usual order of increasing term values, i.e. ^4P, ^4D, ^4F, is preserved in all cases. This feature is found to hold good in As I also, and perhaps supports the correctness of the proposed classification.

With the uncertainty that is likely to exist in the identification of even the $4d$ terms, on account of the irregularities in the term intervals, it appears nearly impossible, at the present stage, to locate the higher members of this set of terms. There are several characteristic energy levels in the region in which the $5d$ terms may be expected, but these are designated only by arbitrary symbols a, b, etc. It is likely that some at least of these terms belong to the group arising from the addition of a $4d$ electron to the ^1D state of the core.

For the detection of the higher members of the *ms* series of terms, however, the following table of limits has afforded a valuable clue*.

Table 4

	N I	O II	F III	P I	S II	Cl III	As I
$ms\ ^4P_1 - {}^4P_3$	80·5	263·8	530·2	400	707·8	877·9	2203·5
$(m+1)s\ ^4P_1 - {}^4P_3$	118·7	266·6	—	—	737·8	1185·7	2304·3
...	...	...	...	...	...	...	...
Limit	N II	O III	F IV	P II	S III	Cl IV	As II
$mp\ ^3P_0 - {}^3P_2$	133·9	309	637	469·9	835	1341	2542

Obviously the difference $ms\ ^4P_1 - {}^4P_3$ in each of these spectra tends towards the limiting value $mp\ ^3P_0 - {}^3P_2$ of the next higher ion. Assuming a corresponding feature in the case of As I the location of the group $4p\,^4S - 6s\,^4P$ giving the total interval of 2304·3 cm.$^{-1}$ seems to be very satisfactory. The view that the levels are presumably quartets receives support from the faintness of their combinations with $4p\,^2P$ and $4p\,^2D$. The line λ1472 might be mistaken for the second-order neon line λ 736, but on a He plate on which Ne lines are very faint the two are distinctly resolved. Attempts to locate the third member of the series were not successful but it seemed clear that it lies beyond ν 76,000. In this region several lines of As II also occur so that the isolation of the lines belonging to As I has, in itself, been difficult.

For completeness the following supermultiplet in the infra-red, due to the transition $5s \leftarrow 5p$ of As I, is shown also. The identification of these 25 lines is due entirely to Meggers and de Bruin.

* I. S. Bowen, *loc. cit.* S. B. Ingram, *loc. cit.* A. S. Rao (As II), *Proc. Phys. Soc.* **44**, 343 (1932).

Table 5. As I, supermultiplet $5p \rightarrow 5s$

$5s$... / $5p$ ⋮	$^4P_1$ 34308·0	$^4P_2$ 33392·2	$^4P_3$ 32104·5	$^2P_1$ 31866·8	$^2P_2$ 30397·2
	915·8	*1287·7*	*237·7*	*1469·6*	
$^4P_1$ 24141·7	10166·27 (80)	—	—	—	—
723·5					
$^4P_2$ 23418·2	—	9973·35 (100)	—	—	—
1389·2					
$^4P_3$ 22029·0	—	—	10074·81 (150)	—	—
$^4D_1$ 22975·5	11332·50 (150)	10416·05 (10)	—	—	—
371·6					
$^4D_2$ 22603·9	11704·13 (50)	10787·68 (25)	—	—	—
885·0					
$^4D_3$ 21718·9	—	11672·62 (100)	10384·94 (8)	—	—
1167·0					
$^4D_4$ 20551·9	—	—	11551·96 (150)	—	—
$^4S_2$ 21354·8	—	12036·73 (50)	10749·03 (50)	—	—
$^2D_2$ 22447·4	11860·63 (100)	10944·13 (15)	—	9418·9 (25)	—
1614·8					
$^2D_3$ 20832·6	—	12558·95 (25)	11271·26 (100)	—	95640·0 (60)
$^2P_1$ 20749·7	—	—	—	11116·62 (20)	—
71·7					
$^2P_2$ 20678·0	—	12713·57 (4)	—	11188·15 (50)	—
$^2S_1$ 19900·5	—	13491·27 (8)	—	11965·60 (10)	—

Meggers and de Bruin have called attention to a considerable number of fairly strong lines of As I, which they have been able to detect in the blue and green regions of the spectrum; two of these, λ5361·12 (10) and λ5497·10 (4), have been interpreted as the forbidden combinations $4p\,^4S_2 - 4p\,^2P_{1,2}$ and the suggestion has been made that these lines might be of the same character as the nebular lines. Two other lines in the extreme ultra-violet, λ1612 and λ1558, have also been suggested as being probably the forbidden combinations $4p\,^4S_2 - 5p\,^4D_{1,3}$. Neither the writer nor K. R. Rao has been able to photograph any of the lines in the visible region. The lines λ1612 and λ1558 have been otherwise classified in the present scheme and it is believed that the relations mentioned by Meggers are only accidental.

§ 4. TERM VALUES

The question of the determination of the absolute term values and the ionization potential of As I has an additional importance on account of the recent interesting observation made by K. R. Rao and J. S. Badami* of the occurrence of the Lyman series of hydrogen in a hollow cathode discharge in He, as a resonance spectrum, in the presence of arsenic. The earliest estimate of this ionization potential was made by Ruark and others† from experiments on low-voltage arcs in arsenic vapour: they

* K. R. Rao and J. S. Badami, *Nature*, October 3, 1931.

† A. E. Ruark and others, *Bur. Standards Sci. Papers*, **19**, 463 (1924).

arrived at the value 11·54 ± 0·5 V. Rao has provisionally adopted this experimental value and based all the terms on $4p^4S_2 = 93{,}500$ cm.$^{-1}$ Meggers and de Bruin made an interesting comparison of the centres of gravity of the $5s$ terms in the arc spectra of Ga, Ge, As, etc. and, by interpolation, assumed an approximate value of 30,000 cm.$^{-1}$ for $5s^4P_1$ which led to $4p^4S_2 = 80{,}697$. In the present work two members of several series are available for the calculation of limits. The series $4p^4S - ms^4P$ gives a limit equal to 81,980, but if the third member lies, as suggested in a preceding paragraph, beyond ν 76,000, the limit $4p^4S_2$ probably will be larger than 84,000. The series $5s^4P - mp^4S$ gives $4p^4S = 88{,}150$. If the identification of the doublets due to transition $4p \leftarrow 6s$ be correct, they lead to the value $4p^4S_2 = 81{,}820$ cm.$^{-1}$ Considering these various estimates, a mean value $4p^4S_2 = 85{,}000$ cm.$^{-1}$ appears to be fairly probable; the error may not perhaps exceed about 2000 units. This mean value is also in keeping with the following variation of the ionization potential with atomic number; the number below each element is the value of the first ionization potential of the element*.

Table 6

B	C	N	O	F
8·34	11·2	14·48	13·56	18·6
Al	Si	P	S	Cl
5·95	8·12	—	10·31	12·96
Ga	Ge	As	Se	Br
5·97	8·09	10·5	9·70	11·80

In table 7 the term values in As I are calculated on the assumption that $4p^4S_2 = 85{,}000$ cm.$^{-1}$ The ionization potential is approximately equal to 10·5 V.

Finally, in table 8 is given a catalogue of all the lines which are ascribed to the neutral atom of arsenic, together with the classification of each, as it is proposed in the present investigation. The first 25 lines in the infra-red are due to Meggers and de Bruin and the intensities of these are on the scale of 1000 for the maximum. Of the remaining lines, the wave-lengths down to λ1563, as measured by K. R. Rao, have been adopted as they were obtained from larger dispersion plates. There are, however, several additional lines in this region, not previously recorded. These and all the lines below λ1563 were measured by the writer. For the sake of uniformity in comparison, the intensities of all the lines (except those in the infra-red) are visual estimates made carefully by the writer from the new plates. Lines which are probably to be ascribed to As I are present on the plates even below λ1319, extending as

* B I: I. S. Bowen, *Phys. Rev.* **29**, 231 (1927). N I: S. B. Ingram, *Phys. Rev.* **34**, 421 (1929). S I: J. J. Hopfield and G. H. Dieke, *Phys. Rev.* **27**, 638 (1926). Se I: R. C. Gibbs and Ruedy, *Phys. Rev.* **37**, 1704 (1931). Al and Ga: A. Fowler, *Report on Line Spectra*. C I and Si I: A. Fowler and E. W. H. Selwyn; and A. Fowler, *Proc. R. S.* **118**, 34 (1928); **123**, 422 (1929). Ge I: K. R. Rao, *Proc. R. S.* **124**, 465 (1929). F I: H. Dingle, *Proc. R. S.* **117**, 407 (1928). Cl I and Br I: C. C. Kiess and T. L. de Bruin, *Bur. Standards J. Res.* **2**, 1117 (1929); **4**, 668 (1930). O I: J. J. Hopfield, *Nature*, **112**, 437 (1923).

far down as λ1250. But as they could not confidently be distinguished from those of As II occurring in that region, they have been omitted from the table.

Table 7. Term values of As I

Term	Term value	Term	Term value
$4p\ ^4S_2$	85000	$4d\ ^4D_2$	18519·0
$^2D_2$	74409·7	$4d\ ^4D_1$	18416·7
$^2D_3$	74087·4	$4d\ ^4D_3$	18279·5
$^2P_1$	66816·7	$4d\ ^4D_4$	18219·6
$^2P_2$	66355·5	$6s\ ^4P_1$	17995·4
$5s\ ^4P_1$	34308·0	$4d\ ^2F_1$	17224·6
$^4P_2$	33392·2	$6s\ ^4P_2$	17083·2
$^4P_3$	32104·5	$4d\ ^2D_2$	16701·7
$^2P_1$	31866·8	$6s\ ^2P_1$	16689·0
$^2P_2$	30397·2	$4d\ ^2D_3$	16601·3
$^2D_3$	24186·9	$6s\ ^4P_3$	15691·1
$^2D_2$	24167·8	$6s\ ^2P_2$	15306·5
$5p\ ^4P_1$	24141·7	$4d\ ^4P_3$	14231·6
$^4P_2$	23418·2	$4d\ ^4P_2$	13950·0
$^4D_1$	22975·5	$4d\ ^4P_1$	13791·7
$^4D_2$	22603·9	$4d\ ^2P_1$	13076·8
$^2D_2$	22447·4	$4d\ ^2P_2$	12880·6
$sp^4\ ^4P_3$	22251·9	*a*	12740·0
$5p\ ^4P_3$	22029·0	$sp^4\ ^2D_2$	12486·7
$5p\ ^4D_3$	21718·9	$sp^4\ ^2D_3$	12397·7
$sp^4\ ^4P_2$	21499·8	$sp^4\ ^2P_2$	12224·2
$5p\ ^4S_2$	21354·8	*b*	11758·7
$sp^4\ ^4P_1$	21022·5	$sp^4\ ^2S_1$	11371·9
$5p\ ^2D_2$	20832·6	*c*	10956·2
$5p\ ^2P_1$	20749·7	*d*	10863·7
$5p\ ^2P_2$	20678·0	*e*	10761·3
$4d\ ^4F_3$	20661·1	*f*	10687·9
$5p\ ^4D_4$	20551·9	*g*	10259·9
$4d\ ^4F_2$	20192·9	*h*	10196·4
$4d\ ^4F_4$	20042·7	*i*	9916·7
$5p\ ^2S_1$	19900·5	*j*	9429·1
$5s\ ^2S_1$	19505·2	*k*	8926·5
$4d\ ^2F_3$	19045·2	*l*	8193·6

§ 5. SUMMARY OF CONCLUSIONS

About 240 lines have been ascribed to the neutral atom of arsenic, of which nearly 100 lines have been newly recorded, the majority of these having been classified. The spectrum was excited by the method of the hollow cathode discharge in He or Ne. The doublet and quartet systems, characteristic of the spectrum, which have been previously published by K. R. Rao and by Meggers and de Bruin, have been discussed in detail and considerably altered and extended in the light of the more complete data obtained by the above method in the Schumann region.

Several new levels have been added; the ^4P term involving a transition of one of the inner electrons has been confirmed. Higher members of the chief groups of the *ms* series of terms have been identified. Interesting anomalies have been detected in the intervals of the $4d$ terms, although the usual order of increasing term

Table 8. Catalogue of lines of As I

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
10614		9418·9	$5s\ ^2P_1 - 5p\ ^2D_2$	—	—
10453		9564·0	$5s\ ^2P_2 - 5p\ ^2D_3$	—	—
10023·98	(4)	9973·35	—	—	$5s\ ^4P_2 - 5p\ ^4P_2$
9923·03	(5)	10074·81	—	—	$5s\ ^4P_3 - 5p\ ^4P_3$
9833·76	(5)	10166·27	—	—	$5s\ ^4P_1 - 5p\ ^4P_1$
9826·69	(8)	10384·94	—	—	$5s\ ^4P_3 - 5p\ ^4D_3$
9597·94	(10)	10416·05	—	—	$5s\ ^4P_2 - 5p\ ^4D_1$
9300·62	(50)	10749·03	—	—	$5s\ ^4P_3 - 5p\ ^4S_2$
9267·29	(25)	10787·68	—	—	$5s\ ^4P_2 - 5p\ ^4D_2$
9134·81	(15)	10944·13	—	$5s\ ^4P_2 - 5p\ ^2D_2$	—
8993·08	(20)	11116·62	$5s\ ^2P_1 - 5p\ ^2P_1$	—	—
8935·58	(50)	11188·15	$5s\ ^2P_1 - 5p\ ^2P_2$	—	—
8869·69	(100)	11271·26	—	$5s\ ^4P_3 - 5p\ ^2D_3$	—
8821·76	(150)	11332·50	—	—	$5s\ ^4P_1 - 5p\ ^4D_1$
8654·16	(100)	11551·96	—	—	$5s\ ^4P_2 - 5p\ ^4D_2$
8564·71	(100)	11672·62	—	—	$5s\ ^4P_3 - 5p\ ^4D_3$
8541·65	(50)	11704·13	—	—	$5s\ ^4P_1 - 5p\ ^4D_2$
8428·94	(100)	11860·63	—	$5s\ ^4P_1 - 5p\ ^2D_2$	—
8355·00	(10)	11965·60	$5s\ ^2P_1 - 5p\ ^2S_1$	—	—
8305·62	(50)	12036·73	—	—	$5s\ ^4P_1 - 5p\ ^4S_2$
8055·72	(5)	12410·13	—	—	$5s\ ^2D_2 - X$
8042·95	(10)	12429·83	—	—	$5s\ ^2D_3 - X$
7960·26	(25)	12558·95	—	$5s\ ^4P_3 - 5p\ ^2D_3$	—
7863·45	(4)	12713·57	—	$5s\ ^4P_2 - 5p\ ^2P_2$	—
7410·07	(8)	13491·43	—	$5s\ ^4P_2 - 5p\ ^2S_1$	—
3119·576	(4)	32046·38	—	$4p\ ^2P_2 - 5s\ ^4P_1$	—
3075·317	(2)	32507·56	—	$4p\ ^2P_1 - 5s\ ^4P_1$	—
3032·845	(4)	32962·77	—	$4p\ ^2P_2 - 5s\ ^4P_2$	—
2990·991	(2)	33423·91	—	$4p\ ^2P_1 - 5s\ ^4P_2$	—
2918·850	(0)	34250·3	—	$4p\ ^2P_2 - 5s\ ^4P_3$	—
2898·702	(5)	34488·12	$4p\ ^2P_2 - 5s\ ^2P_1$	—	—
2860·452	(7)	34949·27	$4p\ ^2P_1 - 5s\ ^2P_1$	—	—
2780·197	(8)	35958·09	$4p\ ^2P_2 - 5s\ ^2P_2$	—	—
2744·991	(6)	36419·24	$4p\ ^2P_1 - 5s\ ^2P_2$	—	—
2492·89	(5)	40102·0	—	$4p\ ^2D_2 - 5s\ ^4P_1$	—
2456·53	(5)	40695·6	—	$4p\ ^2D_3 - 5s\ ^4P_2$	—
2437·22	(4)	41018·0	—	$4p\ ^2D_2 - 5s\ ^4P_2$	—
2381·19	(6)	41983·0	—	$4p\ ^2D_3 - 5s\ ^4P_3$	—
2370·80	(7)	42167·0	$4p\ ^2P_2 - 5s\ ^2D_3$	—	—
2369·67	(7)	42187·1	$4p\ ^2P_2 - 5s\ ^2D_2$	—	—
2363·04	(3)	42305·4	—	$4p\ ^2D_2 - 5s\ ^4P_3$	—
2349·83	(9)	42543·2	$4p\ ^2D_2 - 5s\ ^2P_1$	—	—
2344·02	(5)	42648·6	$4p\ ^2P_1 - 5s\ ^2D_2$	—	—
2288·12	(10)	43690·5	$4p\ ^2D_3 - 5s\ ^2P_2$	—	—
2271·39	(3)	44012·4	$4p\ ^2D_2 - 5s\ ^2P_2$	—	—
2266·75	(2)	44102·4	—	$4p\ ^2P_2 - sp^4\ ^4P_2$	—
2228·72	(3)	44854·8	—	$4p\ ^2P_2 - sp^4\ ^4P_2$	—
2206·04	(2)	45315·9	—	$4p\ ^2P_1 - sp^4\ ^4P_2$	—
2205·20	(2)	45333·2	—	$4p\ ^2P_2 - sp^4\ ^4P_1$	—
2187·83	(1)	45694·7	—	$4p\ ^2P_2 - 4d\ ^4F_3$	—
2182·98	(2)	45794·6	—	$4p\ ^2P_1 - sp^4\ ^4P_1$	—
2176·26	(1)	45935·9	—	—	—
2165·57	(6)	46162·6	—	$4p\ ^2P_2 - 4d\ ^4F_2$	—
2144·14	(5)	46624·0	—	$4p\ ^2P_1 - 4d\ ^4F_2$	—
2138·53	(2)	46746·3	—	—	—
2133·77	(4)	46850·6	$4p\ ^2P_2 - 5s\ ^2S_1$	—	—
2124·06	(1)	47064·7	—	—	—
2112·99	(5)	47311·3	$4p\ ^2P_1 - 5s\ ^2S_1$	—	—
2089·68	(2)	47838·9	—	$4p\ ^2P_2 - 4d\ ^4D_2$	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
2085·33	(3)	47938·7	—	$4p\ ^2P_2 - 4d\ ^4D_1$	—
2079·41	(2)	48075·2	—	$4p\ ^2P_2 - 4d\ ^4D_3$	—
2069·79	(3)	48298·6	—	$4p\ ^2P_1 - 4d\ ^4D_2$	—
2068·34	(3)	48332·4	—	—	—
2067·13	(1)	48360·7	—	$4p\ ^2P_2 - 6s\ ^4P_1$	—
2065·42	(3)	48400·8	—	$4p\ ^2P_1 - 4d\ ^4D_1$	—
2047·58	(2)	48822·4	—	$4p\ ^2P_1 - 6s\ ^4P_1$	—
2028·96	(0)	49272·6	—	$4p\ ^2P_2 - 6s\ ^4P_2$	—
2025·36	(1)	49358·0	—	—	—
2013·30	(6)	49653·5	$4p\ ^2P_2 - 4d\ ^2D_2$	—	—
2012·74	(4)	49667·4	$4p\ ^2P_2 - 6s\ ^2P_1$	—	—
2010·05	(8)	49733·8	—	$4p\ ^2P_1 - 6s\ ^4P_2$	—
2009·21	(8)	49754·6	$4p\ ^2P_2 - 4d\ ^2D_3$	—	—
2003·28	(10)	49901·4	$4p\ ^2D_3 - 5s\ ^2D_3$	—	—
2002·53	(4)	49920·6	$4p\ ^2D_3 - 5s\ ^2D_2$	—	—
λ (vac.)					
1995·45	(7)	50114·0	$4p\ ^2P_1 - 4d\ ^2D_2$	—	—
1994·87	(5)	50128·6	$4p\ ^2P_1 - 6s\ ^2P_1$	—	—
1991·08	(7)	50224·0	$4p\ ^2D_2 - 5s\ ^2D_2$	—	—
1990·37	(9)	50241·9	$4p\ ^2D_2 - 5s\ ^2D_2$	—	—
1973·80	(0)	50663·7	—	$4p\ ^2P_2 - 6s\ ^4P_3$	—
1972·64	(10)	50693·5	—	—	$4p\ ^4S_2 - 5s\ ^4P_1$
1958·91	(5)	51048·8	$4p\ ^2P_2 - 6s\ ^2P_2$	—	—
1941·52	(2)	51506·0	$4p\ ^2P_1 - 6s\ ^2P_2$	—	—
1937·68	(12)	51608·1	—	—	$4p\ ^4S_2 - 5s\ ^4P_2$
1929·17	(1)	51835·7	—	$4p\ ^2D_3 - sp^4\ ^4P_3$	—
1918·47	(0)	52124·9	—	$4p\ ^2P_2 - 4d\ ^4P_3$	—
1917·24	(3)	52158·3	—	$4p\ ^2D_2 - sp^4\ ^4P_2$	—
1902·48	(2)	52563·0	—	$4p\ ^2P_2 - 4d\ ^4P_1$	—
1901·59	(2)	52587·6	—	$4p\ ^2D_3 - sp^4\ ^4P_2$	—
1900·02	(0)	52631·0	—	—	—
1891·59	(1)	52865·6	—	$4p\ ^2P_1 - 4d\ ^4P_2$	—
1890·53	(15)	52895·2	—	—	$4p\ ^4S_2 - 5s\ ^4P_3$
1882·03	(4)	53134·1	—	$4p\ ^4S_2 - 5s\ ^2P_1$	—
1876·91	(0)	53279·1	$4p\ ^2P_2 - 4d\ ^2P_1$	—	—
1873·09	(4)	53387·7	—	$4p\ ^2D_2 - sp^4\ ^4P_1$	—
1871·74	(4)	53426·2	—	$4p\ ^2D_3 - 4d\ ^4F_3$	—
1870·05	(4)	53474·5	$4p\ ^2P_2 - 4d\ ^2P_2$	—	—
1865·21	(5)	53613·3	—	—	$4p\ ^2P_2 - a$
1860·77	(1)	53741·2	$4p\ ^2P_1 - 4d\ ^2P_1$	—	—
1860·48	(6)	53749·6	—	$4p\ ^2D_2 - 4d\ ^4F_3$	—
1855·48	(3)	53894·4	—	$4p\ ^2D_3 - 4d\ ^4F_2$	—
1853·29	(4)	53958·1	$4p\ ^2P_2 - sp^4\ ^2D_3$	—	—
1850·32	(6)	54044·7	—	$4p\ ^2D_3 - 4d\ ^4F_4$	—
1849·31	(0)	54074·2	—	—	$4p\ ^2P_1 - a$
1847·39	(5)	54130·4	$4p\ ^2P_2 - sp^4\ ^2P_2$	—	—
1844·49	(5)	54215·5	—	$4p\ ^2D_2 - 4d\ ^4F_2$	—
1840·58	(4)	54330·7	$4p\ ^2P_1 - sp^4\ ^2D_2$	—	—
1831·80	(5)	54591·1	$4p\ ^2P_1 - sp^4\ ^2P_2$	—	—
1831·39	(6)	54603·3	—	$4p\ ^4S_2 - 5s\ ^2P_2$	—
1821·44	(4)	54901·6	$4p\ ^2D_2 - 5s\ ^2S_1$	—	—
1818·72	(2)	54983·7	$4p\ ^2P_2 - sp^4\ ^2S_1$	—	—
1816·79	(4)	55042·1	$4p\ ^2D_3 - 4d\ ^2F_3$	—	—
1813·18	(1)	55151·8	—	—	—
1810·73	(1)	55042·1	—	—	—
1806·21	(9)	55364·5	$4p\ ^2D_2 - 4d\ ^2F_3$	—	—
1803·60	(2)	55444·7	$4p\ ^2P_1 - sp^4\ ^2S_1$	—	—
1801·99	(1)	55494·2	—	—	$4p\ ^2P_2 - d$
1799·61	(3)	55567·6	—	$4p\ ^2D_3 - 4d\ ^4D_2$	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
1798·71	(4)	55595·4	—	—	$4p\ ^2P_2 - e$
1791·86	(5)	55807·9	—	$4p\ ^2D_3 - 4d\ ^4D_3$	—
1789·94	(6)	55867·8	—	$4p\ ^2D_3 - 4d\ ^4D_4$	—
1789·25	(7)	55889·4	—	$4p\ ^2D_3 - 4d\ ^4D_2$	—
1787·20	(4)	55953·5	—	—	$4p\ ^2P_1 - d$
1785·97	(3)	55992·0	—	$4p\ ^2D_2 - 4d\ ^4D_1$	—
1781·55	(6)	56130·9	—	$4p\ ^2D_2 - 4d\ ^4D_3$	—
1780·61	(4)	56160·5	—	—	$4p\ ^2P_2 - h$
1775·11	(1)	56334·5	—	—	—
1772·63	(2)	56413·3	—	$4p\ ^2D_2 - 6s\ ^4P_1$	—
1771·81	(0)	56439·5	—	—	$4p\ ^2P_2 - i$
1768·07	(2)	56558·9	—	—	$4p\ ^2P_1 - g$
1760·39	(1)	56805·6	—	—	—
1758·62	(9)	56862·8	$4p\ ^2D_3 - 4d\ ^2F_4$	—	—
1757·49	(1)	56899·3	—	—	$4p\ ^2P_1 - i$
1756·37	(00)	56935·6	—	—	—
1754·24	(3)	57004·7	—	$4p\ ^2D_3 - 6s\ ^4P_2$	—
1746·15	(1)	57268·8	—	—	—
1742·59	(5)	57385·8	$4p\ ^2D_3 - 4d\ ^2D_2$	—	—
1741·23	(5)	57430·7	—	—	$4p\ ^2P_2 - k$
1739·53	(6)	57486·8	$4p\ ^2D_3 - 4d\ ^2D_3$	—	—
1735·52	(1)	57619·6	—	—	—
1732·87	(6)	57707·7	$4p\ ^2D_2 - 4d\ ^2D_2$	—	—
1732·51	(6)	57719·7	$4p\ ^2D_2 - 6s\ ^2P_1$	—	—
1729·85	(5)	57808·5	$4p\ ^2D_2 - 4d\ ^2D_3$	—	—
1727·41	(2)	57890·1	—	—	$4p\ ^2P_1 - k$
1724·84	(3)	57976·4	—	—	—
1719·32	(5)	58162·5	—	—	$4p\ ^2P_2 - l$
1712·41	(2)	58397·2	—	$4p\ ^2D_3 - 6s\ ^4P_3$	—
1710·25	(0)	58471·0	—	—	—
1709·42	(1)	58499·4	—	—	—
1705·83	(0)	58622·5	—	—	$4p\ ^2P_1 - l$
1703·05	(1)	58718·2	—	$4p\ ^2D_2 - 6s\ ^4P_2$	—
1701·22	(8)	58781·4	$4p\ ^2D_3 - 6s\ ^2P_2$	—	—
1697·35	(4)	58915·4	—	—	—
1691·89	(5)	59105·5	$4p\ ^2D_2 - 6s\ ^2P_2$	—	—
1680·50	(2)	59506·1	—	—	—
1670·70	(1)	59855·2	—	$4p\ ^2D_3 - 4d\ ^4P_3$	—
1662·81	(3)	60139·2	—	$4p\ ^2D_3 - 4d\ ^4P_2$	—
1653·99	(4)	60460·0	—	$4p\ ^2D_2 - 4d\ ^4P_2$	—
1649·65	(3)	60618·9	—	$4p\ ^2D_2 - 4d\ ^4P_1$	—
1644·40	(5)	60812·5	—	$4p\ ^4S_2 - 5s\ ^2D_3$	—
1643·84	(5)	60833·2	—	$4p\ ^4S_2 - 5s\ ^2D_2$	—
1633·78	(5)	61207·8	$4p\ ^2D_3 - 4d\ ^2P_2$	—	—
1630·47	(4)	61332·0	$4p\ ^2D_2 - 4d\ ^2P_1$	—	—
1625·26	(4)	61528·6	$4p\ ^2D_2 - 4d\ ^2P_2$	—	—
1623·38	(4)	61599·9	$4p\ ^2D_3 - sp^4\ ^2D_2$	—	—
1621·02	(6)	61689·6	$4p\ ^2D_3 - sp^4\ ^2D_3$	—	—
1618·06	(1)	61802·4	—	—	—
1616·56	(5)	61859·8	$4p\ ^2D_3 - sp^4\ ^2P_2$	—	—
1614·92	(5)	61922·6	$4p\ ^2D_2 - sp^4\ ^2D_2$	—	—
1612·59	(7)	62012·0	$4p\ ^2D_2 - sp^4\ ^2D_3$	—	—
1608·17	(6)	62182·4	$4p\ ^2D_2 - sp^4\ ^2P_2$	—	—
1604·35	(1)	62330·5	—	—	$4p\ ^2D_2 - b$
1598·59	(2)	62555·1	—	—	—
1597·58	(2)	62594·7	—	—	—
1596·15	(2)	62650·7	—	—	$4p\ ^2D_2 - b$
1593·66	(9)	62748·6	—	—	$4p\ ^4S_2 - sp^4\ ^4P_3$
1587·36	(5)	62997·7	—	—	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
1584·04	(2)	63129·7	—	—	$4p\ ^2D_3-c$
1579·12	(0)	63326·4	—	—	$4p\ ^2D_3-e$
1577·27	(0)	63400·7	—	—	$4p\ ^2D_3-f$
1575·98	(2)	63452·6	—	—	$4p\ ^2D_2-c$
1574·75	(8)	63502·2	—	—	$4p\ ^4S_2-sp^4\ ^4P_2$
1573·97	(6)	63533·6	—	—	—
1571·17	(4)	63646·8	—	—	$4p\ ^2D_2-e$
1569·35	(0)	63720·7	—	—	$4p\ ^2D_2-f$
1568·29	(1)	63763·7	—	—	—
1566·51	(2)	63836·2	—	—	—
1565·16	(0)	63891·2	—	—	$4p\ ^2D_3-h$
1563·08	(7)	63976·3	—	—	$4p\ ^4S_2-sp^4\ ^4P_1$
1559·53	(6)	64121·9	—	—	—
1558·85	(3)	64149·8	—	—	$4p\ ^2D_3-g$
1557·35	(3)	64211·6	—	—	$4p\ ^2D_2-h$
1556·25	(3)	64257·0	—	—	—
1554·30	(4)	64337·6	—	—	$4p\ ^4S_2-4d\ ^4F_3$
1547·60	(2)	64616·2	—	—	—
1546·62	(0)	64657·1	—	—	$4p\ ^2D_3-j$
1543·01	(4)	64808·4	—	—	$4p\ ^4S_2-4d\ ^4F_2$
1538·94	(4)	64978·9	—	—	$4p\ ^2D_2-j$
1534·70	(3)	65159·3	—	—	$4p\ ^2D_3-k$
1533·74	(2)	65200·1	—	—	—
1528·95	(0)	65404·4	—	—	—
1526·90	(3)	65492·2	—	$4p\ ^4S_2-5s\ ^2S_1$	—
1515·58	(4)	65981·3	—	—	—
1510·71	(3)	66194·0	—	—	—
1509·77	(1)	66235·3	—	—	—
1504·22	(5)	66479·7	—	—	$4p\ ^4S_2-4d\ ^4D_2$
1501·87	(3)	66583·7	—	—	$4p\ ^4S_2-4d\ ^4D_1$
1492·45	(6)	67003·9	—	—	$4p\ ^4S_2-6s\ ^4P_1$
1472·41	(7)	67915·9	—	—	$4p\ ^4S_2-6s\ ^4P_2$
1464·13	(5)	68300·0	—	$4p\ ^4S_2-4d\ ^2D_2$	—
1463·91	(5)	68310·1	—	$4p\ ^4S_2-6s\ ^2P_1$	—
1462·04	(4)	68397·6	—	$4p\ ^4S_2-4d\ ^2D_3$	—
1442·81	(8)	69309·2	—	—	$4p\ ^4S_2-6s\ ^4P_3$
1434·82	(8)	69695·1	—	$4p\ ^4S_2-6s\ ^2P_2$	—
1423·03	(6)	70272·6	—	—	—
1413·07	(5)	70767·9	—	—	$4p\ ^4S_2-4d\ ^4P_3$
1407·48	(4)	71049·0	—	—	$4p\ ^4S_2-4d\ ^4P_2$
1404·33	(4)	71208·3	—	—	$4p\ ^4S_2-4d\ ^4P_1$
1402·67	(0)	71292·6	—	—	—
1390·39	(4)	71922·3	—	$4p\ ^4S_2-4d\ ^2P_1$	—
1389·72	(0)	71956·9	—	—	—
1383·80	(1)	72264·8	—	—	$4p\ ^4S_2-a$
1379·05	(4)	72513·7	—	$4p\ ^4S_2-sp^4\ ^2D_2$	—
1377·37	(3)	72602·1	—	$4p\ ^4S_2-sp^4\ ^2D_3$	—
1375·91	(1)	72679·2	—	—	—
1373·92	(4)	72784·4	—	$4p\ ^4S_2-sp^4\ ^2P_2$	—
1365·35	(2)	73241·3	—	—	$4p\ ^4S_2-b$
1363·35	(0)	73348·7	—	—	—
1350·51	(2)	74046·1	—	—	$4p\ ^4S_2-c$
1348·92	(2)	74133·4	—	—	$4p\ ^4S_2-d$
1338·36	(2)	74718·3	—	—	—
1338·01	(2)	74737·9	—	—	$4p\ ^4S_2-g$
1336·87	(3)	74801·6	—	—	$4p\ ^4S_2-h$
1325·36	(2)	75451·2	—	—	—
1323·21	(3)	75573·8	—	—	$4p\ ^4S_2-j$
1319·48	(2)	75787·4	—	—	—

values, i.e. ^4P, ^4D, ^4F, has been found to exist. A mean value of 85,000 cm.$^{-1}$ has been suggested for the deepest term $4p\ ^4S_2$ which leads to a first ionization potential of approximately 10·5 V. for arsenic.

§ 6. ACKNOWLEDGMENTS

My thanks are due to Prof. A. Fowler, F.R.S., and to Dr A. L. Narayan, for their continual interest and encouragement. I wish also to express my deep indebtedness to Dr K. R. Rao for placing his plates at my disposal and for his valuable guidance throughout the progress of the work.

OBITUARY NOTICES

D. W. DYE, D.Sc., F.R.S.

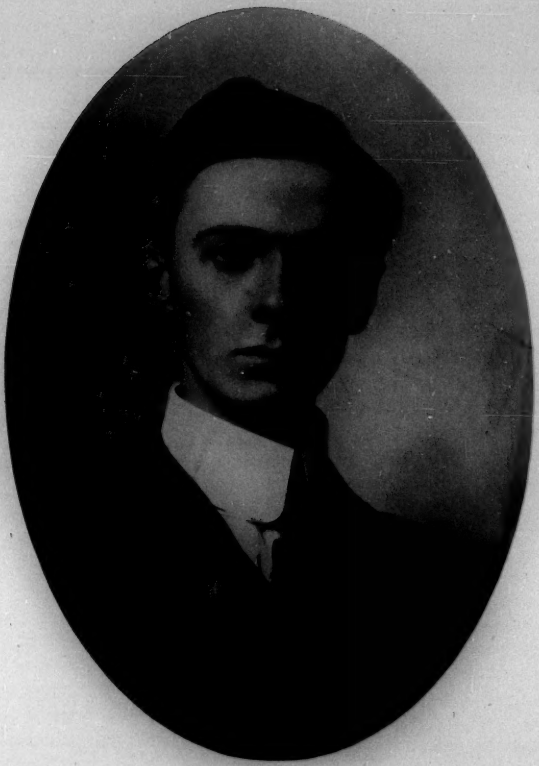
DAVID WILLIAM DYE, whose death occurred on February 18, 1932, was an acknowledged authority on the subject of precision electrical measurements, and it may be said with but little exaggeration that he devoted his whole life to work in this field.

The third son of Charles Dye, J.P., of Portsmouth, he was born on December 30, 1887, and received his education at the Portsmouth Municipal Technical College, and later at the City and Guilds Engineering College, South Kensington, where he graduated at London University. He then took a short apprenticeship course with the British Thomson-Houston Company at Rugby, and in 1910 joined the staff of the National Physical Laboratory, where he worked until his death.

Dye's earliest work at the National Physical Laboratory was done under the direction of Albert Campbell, who was then in charge of the Electrical Measurements Division, and he was associated with Campbell in the development of methods for the magnetic testing of iron and its alloys in various forms, the construction of standards of inductance, and the measurement of currents of radio-frequency. When Campbell retired, Dye took charge of this side of the work of the Laboratory, and soon after the retirement of Mr F. E. (now Sir Frank) Smith in 1919 he became head of the Electrical Standards and Measurements Division. In this position he was responsible for the fundamental standards of current, e.m.f., resistance, inductance, capacity and frequency, as well as for the magnetic work. He soon showed that he had a wonderful instinct for measurements of the very highest accuracy, and especially for the attainment of this accuracy by means of perfection of the mechanical construction of his instruments.

His most notable achievement was undoubtedly the setting up of the fundamental standards of frequency at present in use at the National Physical Laboratory, and its extent may be judged from the fact that when he started this work about ten years ago radio-frequencies could not be measured with an accuracy better than one part in a thousand, but in the year of his death he completed and installed standardizing equipment, which operates continuously, and by means of which it is possible to determine the frequency of the Laboratory standards at any time with an accuracy of one part in ten million. This accuracy is obtained simply by inspection of a chronograph chart on which a record is made automatically.

The course followed by this investigation throws considerable light on his methods of working. He took the Eccles valve-maintained tuning-fork, carefully studied its possibilities and limitations by a long series of observations, and finally converted it into a precision time-standard[1], by means of which an audible frequency could be produced with an accuracy previously unobtainable. He then investigated the Abraham-Bloch multivibrator circuit for the generation of har-



D. W. DYE, D.SC., F.R.S.

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monics of a high order from a known fundamental frequency, determined its properties by a long series of careful measurements, and then set himself to control its fundamental frequency by means of his tuning-fork standard. In this way all the harmonics up to the 120th, i.e. frequencies up to 120,000, became known with the same high accuracy that had been obtained for the fundamental frequency of 1000. Then by making the 20th harmonic of the first multivibrator control a second one of higher frequency, he obtained standardized harmonic frequencies up to 1,200,000 ~. In each case the advance was made by means of perfection of instrumental technique based on a long series of accurate observations covering a wide range of possible conditions of use. The result was a standard wave-meter [2] which now forms our national standard, and has no superior. In recent years he had made many improvements in the original model. Although the standard tuning-fork was made of elinvar it was found to have an appreciable temperature coefficient. Later, as the accuracy of measurement increased, it was found to vary slightly in frequency with atmospheric pressure. It was, therefore, mounted in a massive air-tight enclosure, the temperature of which was accurately controlled. The idea followed of obtaining a continuous record of the frequency in terms of the standard clock. Current from the standard tuning-fork was made to drive a chronograph by means of a small synchronous motor. Signals from the pendulum of the standard clock were made to operate the pencil of this chronograph, and at intervals of one hour a contact was closed and these signals were recorded as dots on the chronograph chart. By means of suitable gearing, the chronograph drum was caused to make one revolution per second, and the marking pencil was traversed 1 mm. per hour in a direction parallel to the axis of the drum. Thus the record consists of lines of dots 1 mm. apart, and if the line is parallel with the axis of the drum, the rate of the tuning-fork is zero with reference to the standard clock. A rate of one part in a million tilts the line so that it makes an angle of about 30° to the axis.

At the time of his death, Dye was working on a new form of standard vibrator which was intended to supplement the tuning-fork. The new standard took the form of a quartz ring, cut so that it could be made to vibrate radially by piezo-electric excitation. He considered that the simple nature of the stresses in such a ring would make it a standard of the greatest possible stability, and the experimental results support this view. His important paper in the *Proceedings of the Physical Society* on "The Piezo-Electric Quartz Resonator and its Equivalent Electrical Circuit" [3] will be recognized as forming the groundwork of accurate measurements on which this standard was based. He had also made a detailed study, as yet unpublished, of the vibrations of piezo-electric quartz crystals, using for this purpose the methods of optical interferometry.

Ranking in importance with his frequency standards is the magnetometer [4] which he devised for the measurement of the vertical component of the earth's magnetic field. Writing of this instrument, Mr Albert Campbell says: "His beautiful magnetometer was a great step in advance of its predecessors, for it showed that the methods which had been trusted as standard for many years back were in very

considerable error. This puzzled the authorities very much, but in the end the sources of error were tracked down, and the Dye instrument will, without doubt, be the unassailable standard for many years to come." The instrument now forms a principal part of the equipment of the Abinger Magnetic Observatory.

His investigations of the properties of inter-valve transformers at telephonic frequencies [5], the use of the Schering bridge at radio-frequencies, the measurement of the effective resistances of standard condensers at radio-frequencies [6], the use of an electrically-driven sonometer as an audio-frequency meter, and the design and construction of a standard variable air condenser, though not of the same fundamental importance as those previously mentioned, are characterized by the same precision of measurement and refinement of instrumental construction. Dye himself had acquired great skill in mechanical construction, and he delighted in doing the more refined mechanical work with his own hands: indeed, he seemed never so happy as when giving the last touches to the grinding of a quartz crystal, or rolling out a piece of the finest Wollaston wire for the purpose of making a delicate suspension, unless perhaps it was when demonstrating the finished instrument.

Dye was awarded the degree of Doctor of Science of London University in 1926, and was elected a Fellow of the Royal Society in 1928 and of the City and Guilds Institute in 1931. He was a member of the Council of the Physical Society, of the Radio Research Board and of the Comité Consultatif d'Électricité of the Bureau Internationale des Poids et Mesures, and also secretary of the British National Committee of the Union Radio-Scientifique Internationale. He served as chairman of the Commission on Radio Standards at the congresses of the last-named body at Washington, Brussels and Copenhagen. He had a considerable amount of administrative work to do in connection with these and his other official duties, and it is perhaps typical of his outlook that, after dealing for a few hours with such matters, he would suddenly break off, push the papers aside and remark that he really must go and do some work.

L. HARTSHORN

REFERENCES

(1) *Proc. R. S.* A, **103**, 240 (1923).
(2) *Phil. Trans.* A, **224**, 259 (1924).
(3) *Proc. Phys. Soc.* **38**, 399 (1926).
(4) *Proc. R. S.* A, **117**, 434 (1928).
(5) *N.P.L. Collected Researches* **18**, 273 (1924).
(6) *Proc. Phys. Soc.* **40**, 285 (1928).

ERNEST HOWARD GRIFFITHS, D.Sc., F.R.S.

Principal E. H. Griffiths, who died on March 3, 1932, at the age of 80, was the last of a band of four workers who, collaborating together in the early 'nineties, did yeoman service in the development of the technique of high-temperature measurements. The names of Callendar, Griffiths, Heycock and Neville have now become household words in the extensive literature which has grown around the resistance thermometer. Griffiths's interest in the subject arose out of an enquiry from Heycock and Neville, who were searching for some better instrument for the measurement of the freezing points of alloys than the so-called "fixed zero" mercury thermometers available about 1890. Griffiths constructed a number of platinum resistance thermometers and proceeded to calibrate them by reference to fixed points. He was unable to reconcile his results with those of Callendar, who had just then published his difference formula. Griffiths had accepted the values of the fixed points given in the literature, one of which was the boiling point of sulphur as determined by Regnault. On the other hand, Callendar had based his formula on a direct comparison between the resistance of a spiral of platinum wire and the readings of an air thermometer.

To clear up the discrepancy they joined forces to redetermine the boiling point of sulphur, and showed that Regnault's value was about 4° too high. In this investigation they so admirably developed the technique of the sulphur determination that it has remained practically unchanged for the past 40 years. In addition they converted the resistance boxes of their day from crude appliances suitable for technical electrical measurements into instruments of precision.

At the same time Griffiths was working on the determination of the mechanical equivalent of heat. The primary object of this work was to check up the electrical standards which did not enjoy in those days the confidence we now have in their accuracy in absolute units. In his work on the mechanical equivalent of heat Griffiths had the assistance of Mr G. M. Clark.

Other investigations made in that period were the determination of the latent heat of evaporation of benzene with Miss Marshall, the influence of temperature on the specific heat of aniline, and the latent heat of evaporation of water; the last of these he regarded as one of his best pieces of work, and in later years he submitted it as a thesis for the D.Sc. degree of the University of Wales.

At Cambridge, Griffiths paid out of his own pocket for all the special apparatus he required. The writer recalls being told how a certain resistance bridge with a new design of plug contacts had cost £100 to build: an unheard-of sum for a bridge in those days. Griffiths protested to Horace Darwin who was the head of the Cambridge Instrument Company, but the latter produced the cost sheets and proved that the profit to the manufacturer was negligible.

For his work on the mechanical equivalent of heat Griffiths went to the extent of building a small laboratory in the grounds of his house. The design of his

equipment was not stinted on the grounds of the cost of the material: the writer once opened a piece of his apparatus and found that it contained two large platinum pots. Griffiths earned his money by coaching, and at that period a successful coach earned large fees. It involved intensive work during term time, so that scientific research had to be confined to vacations.

In 1902 he accepted the principalship of the University College, Cardiff. The change from the academic surroundings of Cambridge to the industrial surroundings of Cardiff was probably not to his liking. The problems of administration in a college of a democratic university were strange to him, and to one of his temperament must have proved irksome. And for some years he had not the solace of active experimental research owing to lack of facilities at that stage in the development of the college.

Griffiths devoted much energy to the furthering of the scheme for the building of a new college in Cathays Park. He was particularly interested in one of the buildings, the Viriamu Jones Memorial Research Laboratory. The details of this building were his special study and he insisted that it should be constructed entirely of non-magnetic materials.

In 1909 the writer was invited by Griffiths to work with him in the new Laboratory. Many months were spent in equipping the place, for all the jobs usually done by laboratory assistants were done by ourselves. There was no money for paying wages to a mechanic. Griffiths's hours of work were unconventional, but one could count upon his coming in during holiday periods and week-ends.

The investigation of the thermal capacities of metals from liquid-air temperatures up to 100° C. was the only scientific work he participated in at Cardiff.

In 1918 he retired to Cambridge and in that year his wife died. The last fourteen years of his life were somewhat lonely, for he had no children or near relatives. During the early part of this period of retirement he threw himself wholeheartedly into the work of the British Association, of which he became treasurer, but during the greater part of the last six years he was confined to his home with arthritis.

Throughout his life, and in the face of many bitter disappointments in connection with college work, he retained a spirit of optimism and youthfulness. One of the wishes he had "For any Boy" was:

> So let him live;
> Love work, love rest, love all that life can give:
> And when he feels too weary to feel joy
> Leave life with laughter—to some other boy.

His work received recognition by the award of the Hughes Gold Medal of the Royal Society: his election to an honorary fellowship of his old College, Sidney Sussex, and of Jesus College, Oxford.

The affection with which he was regarded was strikingly shown by the distinguished gathering in the chapel of Sidney Sussex College to pay the last tribute to an old colleague. The service closed with a hymn whose words were set to the old Welsh air "Ar hyd y nos" beloved by Griffiths.

EZER GRIFFITHS

H. CHAPMAN JONES, F.I.C.*

The death of Henry Chapman Jones on March 7, at the age of seventy-seven years, removed one who has held an honourable place in the teaching of chemistry and in the development of the science and practice of photography. Apart from an early association with Birkbeck College, Chapman Jones's career as a teacher of chemistry was bound up with the Royal College of Science, where he was successively assistant, demonstrator, and lecturer, from 1881 until 1914, when he retired. In the first few years of this long period he was on Sir Edward Frankland's staff, and later he served under T. E. Thorpe, W. A. Tilden, and H. B. Baker. For a number of years he presided over the North Laboratory in the old Royal College of Science buildings, and many who worked there will recall the quiet, serious, and kindly way in which he went about his duties, as well as the high standard of industry and accuracy which he expected of his students.

The main contributions to knowledge made by Chapman Jones were in the domain of photography, and were published principally under the auspices of the Royal Photographic Society. Following Sir William Abney in the application of scientific method to the problems of photography, Chapman Jones dealt with such matters as the densities of negatives, the factors affecting the sensitiveness of plates, media of high refractive power for photomicrography, and the relationship between the size of the particle and the colour of the image.

Besides original papers on these and other topics, Chapman Jones contributed many articles on photographic subjects to the technical press, while two books on photography came from his pen, namely, *Science and Practice in Photography*, which ran to four editions, and *Photography of To-day*, a popular treatise published in 1913. His associations with the Royal Photographic Society were very close, and after being honorary secretary from 1892 until 1897, and vice-president from 1897 until 1902, he became president of the Society in 1912. In addition, he was a Fellow of the Chemical and Physical† Societies and of the Institute of Chemistry.

Chapman Jones was a single-minded man, whose integrity and ideals were of the highest. His quiet, almost shy, old-world courtesy and his modesty were marks of an interesting personality which only those who knew him well could appreciate. At the same time, he held strong opinions on many subjects, and he did not hesitate to express these in his own direct, unimpassioned way. His gentle, warm-hearted nature revealed itself to best advantage in the circle of his own family and of those who knew him intimately. He is survived by a widow and two sons.

J. C. P.

* Reprinted from *Nature*, **129**, 570 (1932), by the courtesy of the Editor.

† Mr Jones was a Fellow of the Physical Society from November 27, 1880, until his death.

REVIEWS OF BOOKS

Revue d'Acoustique. First Number. Pp. 84 + xi. (Paris: Les Presses Universitaires de France, 49 Bd. St.-Michel, Paris.) 25 fr.

From the prospectus which accompanies this journal we learn that it is the aim of a panel of the leading French acousticians to provide for a long-felt want by publishing a review which shall report, co-ordinate, and tabulate information dealing solely with the science of sound. It is proposed that the review shall cover the whole domain of acoustics and contain purely theoretical papers and memoirs beside accounts of the most recent apparatus and methods obtaining in industrial practice.

To the present number M. Canac has contributed a vocabulary of proposed acoustical terms and definitions (based, to a large extent, on that already published in the *Journal of the Acoustical Society of America*); useful abstracts of some half-dozen papers; and an account of experimental work he has performed on the absorption of sound in tubes. Prof. Carrière, who is considering the absolute measurement of fundamental quantities in sound, has dealt in detail with the measurement of frequency, and proposes to continue by treating the measurement of amplitude, speed and pressure. In addition M. Perrin, writing on sound receivers, devotes most of his article to his own instruments and researches. A bibliographical review under nine headings and an eleven-page supplement consisting of a list of abbreviations of journal titles, completes the matter reported.

During the present year five numbers of the review will appear, while in succeeding years a subscription of 150 fr. will purchase the normal complement of six numbers. It is to be hoped that English subscribers will support the publication and thereby further their own interests as well as those of an important branch of physics.

E. J. I.

The Nature of a Gas, by Prof. LEONARD B. LOEB. Pp. x + 153. (London: Chapman and Hall.) 12*s*. 6*d*.

The Committee of the National Research Council of America appointed to deal with electrical insulation has interpreted its terms of reference widely and wisely, and has initiated a series of authoritative monographs on important fundamental properties of dielectric substances. Prof. Loeb's book is one of the first of these monographs and is intended to explain the nature and electric properties of gases to "students and engineers who are concerned with the problems of the behaviour of dielectrics as insulation."

Prof. Loeb is not afraid of elementary exposition. His book is divided into three parts, of which the first is an account of the Rutherford-Bohr atom. The second develops the kinetic picture of a gas and is, as one would expect from Prof. Loeb, a very clear and concise development of the fundamental principles of the kinetic theory. The third part, some forty pages long, is devoted to ionization phenomena, and the book concludes with a series of useful tables.

It is a curious mélange, but is eminently readable and very well fitted for its purpose. University students will find in it a very useful résumé of some of the fundamental properties of gases.

A. F.

A text-book of Thermodynamics, by F. E. HOARE, M.Sc., A.R.C.S., D.I.C. Pp. xii + 265. (London: E. Arnold and Co.) 15*s*.

There is room, in the literature of thermodynamics, for a volume which shall stress the number and the variety of the problems to which the principles of the subject may usefully be applied. Mr Hoare has set out to fill this gap, and has succeeded in producing a volume of much interest to workers. Three chapters of the book are devoted to a consideration of general principles, and the author then proceeds to treat of thermodynamic functions, the absolute scale of temperature, characteristic equations, change of state, problems of equilibrium, the Nernst heat theorem, thermodynamics of liquids and dilute solutions, applications of the principles of thermodynamics to electrical phenomena and to radiation and, finally, the variation of specific heat with temperature.

Thermodynamics is full of subtleties, and in a book which deals with applications rather than with fundamentals the discussion of some of the fundamental difficulties of the subject may incline to sketchiness. Thus, in the book before us, the treatment of the concept of temperature is a trifle naïve, and the handling of that very nice problem the increase of entropy in natural processes does not bring out the difficulties which encumber any adequate treatment of the subject.

These must not, however, be regarded as more than minor matters, and the author is to be congratulated on a volume which will be found very useful by university students of honours grade, and by research workers who look for clear and straightforward applications of the principles of thermodynamics to physical problems.

A. F.

Photoelectric Phenomena, by Prof. A. L. HUGHES and Prof. L. A. DUBRIDGE. Pp. xii + 531. (London: McGraw-Hill Publishing Co.) 30*s*.

Prof. Hughes's treatise on photoelectricity, published in 1913, and his *Report* of 1921, published in the *Bulletins* of the National Research Council, will be well known to most readers of these *Proceedings*. His colleague at St Louis and collaborator in the book under review, Prof. DuBridge, is also actively engaged in the investigation of photoelectric phenomena, and there is much evidence in the work of the close contact of the authors with the problems under discussion.

As would be expected, a great part of the book is taken up with the discussion of surface photoelectric effects, but there are also excellent accounts of volume effects in gases and vapours, of photoconductivity in solids and liquids and of photovoltaic effects. A chapter is devoted to the description of the effects obtained with X- and γ-rays, and there is a short account of the more important scientific and technical applications of photoelectric cells.

The book is directed primarily to physicists, and more particularly to experimental physicists. The sections dealing with experimental work are admirably written, the topics very judiciously selected and the emphasis very justly placed. These sections, and the excellent chapter on photoelectric technique, will be of use and interest both to pure physicists and to those who are more concerned with technical applications. The theoretical treatment is less detailed, and the bibliography somewhat less complete than on the experimental side. Nevertheless, a great many students of physics will find in chapter 6, on "Theories of photoelectric emission," a most useful and sufficiently detailed introduction to the theoretical aspects of photoelectricity.

A great deal of the subject matter of the book is here collected together for the first time, and in an eminently readable form, mainly from the viewpoint of the experimental

photoelectrician. The book fills a real gap in the literature of the subject, and it should be assured of a very warm welcome. There are a few minor slips and misprints—the most surprising oversight is in the section on γ-ray effects, where readers requiring further information are referred, not to the recent treatise of Rutherford, Chadwick and Ellis, but to two works published respectively in 1925 and 1928.

H. R. R.

(1) *L'Idée générale de la Mécanique Ondulatoire et de ses premières applications*, by M. Boll. Pp. 74. (Paris: Hermann et Cie.) 15 fr.

(2) *Exposé Électronique des Lois de l'Électricité*, by M. Boll. Pp. 72. (Paris: Hermann et Cie.) 15 fr.

M. Marcel Boll is the author of an impressive and varied array of treatises and text-books, of which perhaps the best known in this country is the excellent (classical) *Introduction à la théorie des Quanta*, written in collaboration with C. Salomon.

In the first of the books now under review, M. Boll gives a brief general account of wave mechanics, suitable as an introduction to the more detailed and advanced treatises. The treatment is throughout formal and systematic—that is, there is no attempt to create for the reader, by means of models and analogies, an illusion of "understanding" the general outline of the new theories of matter. In other words, the book is elementary, rather than popular. It is clearly written, and can be recommended.

The second book will be of special interest to teachers of electricity, many of whom feel that electronic ideas and terminology should be introduced as early and as frequently as possible in even the most elementary lecture courses on the subject. It happens frequently, however, in practice—possibly through the exigencies of formal degree courses—that the electron is kept unduly in the background, and that an unnecessarily wide gap is left between the older and the more modern parts of electrical theory. For this reason, M. Boll's book should be of service to both teachers and students.

H. R. R.

The Theory of Electric and Magnetic Susceptibilities, by J. H. Van Vleck. Pp. xi + 384. (London: Oxford University Press.) 30*s*.

It is generally acknowledged that physicists who have devoted their attention to magnetic phenomena were greatly assisted by the publication of Debye's excellent article on magnetism in the *Handbuch der Radiologie*. In the opinion of the reviewer, Van Vleck's treatise on the theory of electric and magnetic susceptibilities is certain to be of similar assistance to all who are interested in polar molecules and the magnetic properties of substances. This treatise is no hastily assembled collection of facts and theories. It presents the clearest possible account of the present state of the mathematical theory of susceptibilities, an account which could only have been written by one who had read very widely and who had first-hand knowledge of much that he described.

In the first chapters, Van Vleck gives a very comprehensive review of the classical theories which form the basis of calculation of electrical susceptibilities, and he includes a much more adequate treatment of the Langevin-Debye formula than is to be found elsewhere. He discusses in detail its experimental verification, and, while not concerning himself with accounts of experimental technique, he presents a most valuable and stimulating survey of the phenomena exhibited by polar molecules.

His outline of the classical theory of magnetic susceptibility is masterly. The work of Miss Van Leeuwen, published in 1919, is so often passed over in silence, that it is most satisfactory to find that in this book it is given the prominence it deserves. Her proof of

the theorem that the calculated magnetic susceptibility by the application of classical Boltzmann mechanics to any dynamical system is equal to zero, is reproduced, and, in addition, Van Vleck provides an alternative proof of the theorem.

The author is at great pains to contrast the findings of the old quantum theory with those of the new. This does not represent a waste of space; for he shows, in a most lucid manner, and, truly, with a minimum of mathematics, how the inadequacies of the old quantum theory have been removed by the new, and how every calculation of magnetic susceptibility on the new mechanics has its analogue on classical theory.

As an introduction to the new mechanics, the chapter on quantum-mechanical foundations is likely to be most helpful to all interested in modern magnetism and allied phenomena. It pre-supposes no detailed knowledge of quantum mechanics, or of spectroscopic nomenclature or of the Schrödinger wave equation. The necessary perturbation theory and the theorems of spectroscopic stability are adequately presented, and the correlation and intermingling of wave functions and matrices which the author has successfully achieved make this introduction particularly valuable. Special attention is given to the general derivation of the Langevin-Debye formula and to a discussion of its limits of accuracy.

The treatment of magnetic susceptibility is not restricted to paramagnetic gases, but covers the salts of the rare earths and iron groups. Heisenberg's theory of ferromagnetism, together with its solution of the problem of the Weiss internal field, is fully discussed, and the experimental evidence which supports it is reviewed. The book ends with a brief treatment of optical phenomena, such as the Kerr effect, which are related to the problems discussed in the earlier chapters.

It is, then, a book which is bound to excite the serious attention of all who are interested in theoretical physics, and it is one which experimenters who measure dielectric constants or magnetic susceptibilities will be glad to possess; the author is much to be congratulated.

L. F. B.

Théorie de la Quantification dans la nouvelle Mécanique, by L. DE BROGLIE. Pp. xxviii + 250. (Paris: Hermann et Cie.) 70 fr.

This book may in some respects be regarded as a continuation of an earlier treatise of which an English translation, the *Introduction to the Study of Wave Mechanics*, appeared in 1930. After a brief historical introduction, it is divided into two parts, of which the first and shorter (on general principles and methods of quantization in wave mechanics) covers briefly, but clearly and adequately, the older ground. The second, and more important, part of the book deals with the general theory of quantization. As is usual in de Broglie's writings, the discussion is singularly lucid, and attention is very carefully focussed on fundamental principles. It scarcely needs to be said that the work is indispensable to all who are in any way interested in the wave aspects of the new mechanics.

Printing and paper are excellent, but, as with so many French books, reference would be facilitated by the provision of an alphabetical index and a paper-knife. There is, however, a very detailed table of contents.

H. R. R.

(1) *Sur une Forme plus restrictive des Relations d'Incertitude, d'après MM. Landau et Peierls*, by L. DE BROGLIE. Pp. 24. (Paris: Hermann et Cie.) 1932. 6 fr.

(2) *L'Existence du Neutron*, by I. CURIE and F. JOLIOT. Pp. 22. (Paris: Hermann et Cie.) 1932. 6 fr.

These "exposés de physique théorique," published under the editorship of L. de Broglie, form Nos. 31 and 32 of the general series of *Actualités Scientifiques et Industrielles*, of which the first thirty were spaced over the three years 1929–31. The plan of the series

is one which might well be copied in other countries—a similar series in English would certainly be profitable to students of physics, if not to the publisher.

De Broglie's pamphlet consists mainly of a commentary upon the recent paper of Landau and Peierls, "Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie."* The subject-matter is of considerable theoretical importance, and of immediate interest in connection with nuclear dynamics.

The second "exposé" is even more obviously of topical interest. It deals with recent work on the disintegration of light nuclei, and more particularly with the researches carried out at the Curie Laboratory of the Paris Radium Institute.

H. R. R.

Magnetic, Meteorological and Seismological Observations made at the Government Observatories, Bombay and Alibag in 1928, under the direction of Dr S. K. Banerji. Pp. 160 with 5 Plates. (Calcutta, Government of India.) 22*s*. 6*d*.

This is the fiftieth volume of the series and is on the same lines as the previous volume† but has in addition an appendix on the climatology of Bombay. The rainfall was 85·7 in., 16·2 in. above the normal, and the mean temperature 79·9° F., 0·6° above the normal; mean wind 11·2 miles per hour, but on July 23 a gust of 54 miles per hour was recorded. The deviation of the compass had a mean value 3·8′ west as compared with 1·9′ west last year, and magnetic activity was generally slightly greater than last year.

For the 72 years ending 1920 the means for Bombay were: barometer 29·81 in., temperature 79·2° F., temperature in sunshine 139° F., humidity 75 per cent, wind 6·6 miles per hour, cloud covering 0·4 of the sky.

C. H. L.

The Structure of Wind over Level Country. Meteorological Office Geophysical Memoirs No. 54, by the late M. A. Giblett, M.Sc., and other members of the staff. Pp. 119 with 21 Plates. (London: H.M. Stationery Office.) 10*s*.

This memoir, Dr Simpson tells us in his preface, is the outcome of a large amount of team work on the part of the staff of the Airship Division of the Meteorological Department, undertaken at the request of the Aeronautical Research Committee after the accident to Airship R 33 in 1925. The work was planned by the late Mr M. A. Giblett, who lost his life in the R 101 disaster, and Mr B. C. V. Oddie installed the necessary instruments at Cardington. The results of the six years' observations are embodied in a theory put forward by Mr C. S. Durst, according to which the large eddies, which produce gusts of wind beginning quickly and subsiding slowly, are due to convection currents set up in the air when the temperature near the ground is higher than that overhead. These currents are superposed on the general drift of the air, and take place in cells about a mile long in the direction of the wind, ¼ mile broad across the wind, and about the same height. The upward motion of the air in a cell takes place mainly near the back wall and the downward near the front wall. The air moving backwards near the ground is slowed down by viscosity and heated, while obstacles over which it passes produce the smaller eddies found in winds. The heated air ascending at the back of the cell expands and cools and its moisture-content condenses and forms clouds.

C. H. L.

* *Zs. f. Phys.* **69**, 56 (1931).

† *Proc. Phys. Soc.* **44**, 112 (1932). The deviation of the compass and its increase there given were in minutes, not degrees.

Recent Advances in Physical Chemistry, by SAMUEL GLASSTONE, D.Sc. Pp. viii + 470. (London: J. and A. Churchill.) 15*s*.

There is to-day considerable overlap in the sciences, and physical chemistry takes to itself results in certain branches of physical science—molecular spectra and the theory of surface forces are cases in point—in which the knowledge of the physicist may reasonably be expected to be in advance of, and more critical than, that of the physical chemist.

But it is simply impossible to keep a critical and detailed knowledge of all those branches of physical chemistry which may be supposed to be of special interest to the physicist, and those of us who have to deal with borderline subjects owe a debt of gratitude to anyone who, having the requisite knowledge, is disinterested enough to provide us with accurate and critical summaries of those branches of the subject which incline more to chemistry than to physics.

The different sections of Dr Glasstone's book deal with valency theories, the parachor, dipole moments, molecular spectra, homogeneous gas reactions, photochemical reactions, the properties of surfaces, heterogeneous catalysis, solubility, strong electrolytes, and acid-base catalysis. The author has the teacher's flair for exposition, and handles his subjects in clear, scholarly and adequate fashion. He does not fall into the error of assuming that his readers are likely to possess an expert's knowledge of the subject.

The book can be strongly recommended to those who desire a brief conspectus of some of the more important advances in physical chemistry during the past decade.

Not the least valuable feature of the book is the bibliographical appendix to each chapter. The entries therein amount in the aggregate to nearly five hundred items.

A. F.

Hearing in Man and Animals, by R. T. BEATTY, M.A., B.E., D.Sc. Pp. xi + 223. (London: G. Bell and Sons, Ltd.) 12*s*.

The purpose of this book as set forth by the author in his preface is to co-ordinate work on audition in the different fields of research—physics, anatomy, physiology, etc. He rightly points out that there is a tendency, in this as in other branches of biophysics, for a worker to keep to the path indicated by the usual scope of research in his own branch of science, and to write in the *argot* peculiar to that branch, ignoring papers in the other branches or perhaps not understanding the technical terms affected by his colleagues on the other side. This contention of the author is borne out to a certain extent by the Physical Society's *Discussion on Audition* in which the contributions can be grouped, as to terminology, in three sections—physical, biological and psychological. At any rate the author has ably achieved his purpose in the treatise under review. Simple language has been used; nothing more is demanded of the reader than a lively interest and a certain scientific grounding. The bibliography at the end of each chapter is not extensive, but references to text-books where further information is available are given. Unlike many text-books, this work enlivens the course of the reader by anecdotes, historical curiosities and imaginative pictures of bygone stages in scientific investigation. Nevertheless it is quite up-to-date, and not its least valuable feature is the insight it gives to the scientific investigator into lacunae in research, to which he might with profit direct his attention. Altogether, a very readable book, and one which should be added to every bookshelf which possesses the *Discussion on Audition*.

E. G. R.

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